The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, January 27, 1960 — 9:15 a.m. to 12:15 p.m., only

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express $\frac{3}{4-\sqrt{3}}$ as an equivalent fraction with a rational denominator. 1.....

2 Factor: $6\sin^2 A - 7\sin A - 10$ 2.....

3 Find the value of $(x+2)^0 + (x+1)^{-\frac{3}{3}}$ if $x = 7$. 3.....

4 Solve the following equation for $\sin x$: $a\sin x + b = b\sin x + c$. 4.....

5 Find the logarithm of 0.2247. 5.....

6 Log $\cos x = 9.9273 - 10$. Find x to the *nearest minute*. 6.....

7 Find the 33rd term in the arithmetic progression 9, 6, 3, ... 7.....

8 Combine into a single fraction: $\frac{4}{x-2} - \frac{3}{x}$ 8.....

9 If b varies inversely as h and if $b = 8$ when $h = 9$, find b when $h = 6$. 9.....

10 Write an equation of the line which is parallel to the line $y = 2x + 9$ and which passes through the point $(0, -4)$. 10.....

11 Express cos 195° as a function of a positive acute angle. 11.....

12 If A is an acute angle, express cos A in terms of tan A. 12.....

13 In triangle ABC, $a = 7$, $\sin A = 0.21$ and $\sin B = 0.36$. Find b. 13.....

14 In triangle ABC, $a = 8$, $b = 10$ and $\cos C = -0.2$. Find c. 14.....

15 In parallelogram $ABCD$, $AB = 12$, $BC = 18$ and angle $A = 120°$. Find the area of $ABCD$. 15.....

16 If $\sin x = \frac{1}{3}$ and x is an acute angle, find the value of $\cos 2x$. 16.....

17 If $\sin x = \frac{3}{5}$ and x is an acute angle, find the value of $\sin (45° - x)$. 17.....

18 Find the value of $\cot \frac{7\pi}{6}$. 18.....

Directions (19–25): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

19 The roots of the equation $x^2 + px + q = 0$ are -1 and 3. The value of p is (1) -2 (2) 2 (3) 3 (4) -3 19.....

20 If $x = \log m$, then $x + 2$ equals (1) $\log m^2$ (2) $\log 2m$ (3) $\log 100m$ (4) $\log (m + 2)$ 20.....

21 An equation of the axis of symmetry of the graph of the equation $y = 2x^2 + 6x - 5$ is

(1) $x = -\frac{3}{2}$ (2) $x = -3$ (3) $y = -\frac{3}{2}$ (4) $y = -3$ 21.....

22 The fraction $\frac{x^{-1}}{x^{-1} + y^{-1}}$ is equal to

(1) y (2) $\frac{y}{x + y}$ (3) $\frac{x + y}{x}$ (4) $\frac{x}{x + y}$ 22.....

23 As angle x increases from 0° to 360°, $\sin x$ and $\cos x$ both increase in (1) Quadrant I (2) Quadrant II (3) Quadrant III (4) Quadrant IV 23.....

24 The expression $3 \sin \frac{1}{2}x$ reaches its maximum value when x, expressed in radians, equals (1) $\frac{\pi}{2}$ (2) $\frac{3}{2}$ (3) 3 (4) π 24.....

25 In a circle of radius 12 inches, the number of radians in an arc 8 inches long is (1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) $\frac{2}{3}\pi$ (4) $\frac{3}{2}\pi$ 25.....

Part II

Answer three questions from this part. Show all work unless otherwise directed.

26 *a* Solve the equation $2 \sin^2 x - 3 \sin x = 4$ for $\sin x$. [Answer may be left in radical form.] [6]

b Using the results obtained in part *a*, determine the quadrant(s) in which angle x lies. [2]

c Express the principal value of angle x in inverse trigonometric form. [2]

27 Solve the following system of equations, group your answers and check them in both equations: [6, 2, 2]

$$x^2 - 3xy = 10$$
$$x + y = 1$$

28 Write the equations that would be used to solve the following problems. In each case state what the letter or letters represent. [*Solution of the equations is not required.*]

a John can do a job in 10 minutes less time than William. One day John worked alone for 15 minutes; then William worked alone for 20 minutes to finish the job. How long would it take each working alone to do the job? [5]

b If a two-digit number is divided by the sum of the digits, the result is 4. If the digits are reversed, the new number exceeds the original number by 36. Find the original number. [5]

29 Three numbers are in the ratio of 1 : 2 : 3. If 2 is added to the smallest number, the resulting number together with the other two numbers form a geometric progression. Find the original numbers. [5, 5]

30 *a* *Sketch* the graph of $y = \cos 2x$ as x varies from 0 to 2π radians. [4]

b On the same set of axes used in part *a*, *sketch* the graph of $y = \frac{1}{2} \sin x$ as x varies from 0 to 2π radians. [4]

c From the graphs made in answer to *a* and *b*, determine the number of values of x for which $\cos 2x = \frac{1}{2} \sin x$. [1]

d From the graphs made in answer to *a* and *b*, determine *one* value of x for which $\cos 2x - \frac{1}{2} \sin x = 1$. [1]

31 *a* Starting with the formulas for $\sin \frac{1}{2}x$ and $\cos \frac{1}{2}x$, *derive* a formula for $\tan \frac{1}{2}x$. [You may assume that x is an angle in the first quadrant.] [4]

b Prove the identity: $\tan 2x \csc x = \dfrac{2 \cos x}{\cos 2x}$ [6]

Part III

Answer two questions from this part. Show all work.

32 Using logarithms, find to the *nearest tenth* the value of $\dfrac{\tan 75^\circ\ (4.66)^2}{\sqrt[3]{0.941}}$. [10]

33 In triangle ABC, $a = 25$, $b = 31$ and $c = 14$. Find angle B to the *nearest ten minutes.* [10]

34 Lighthouse B is 3.7 miles east of lighthouse A. The bearing of a ship C from lighthouse A is S 12° 50′ W and the bearing of C from lighthouse B is S 61° 40′ W. Find to the *nearest tenth of a mile* the distance of the ship from B. [5, 5]

*35 Prove the identity: $\dfrac{\sin 3y + \sin y}{\sin 3y - \sin y} = \dfrac{2\cos^2 y}{2\cos^2 y - 1}$ [10]

* This question is based on one of the optional topics in the syllabus and may be used as *one* of the questions in part III only.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 19–25, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\frac{3(4+\sqrt{3})}{13}$

(2) $(6\sin A + 5)(\sin A - 2)$

(3) $1\frac{1}{4}$

(4) $\frac{c-b}{a-b}$ *or* $\frac{b-c}{b-a}$

(5) $9.3516 - 10$ *or* -0.6484

(6) $32^\circ\ 14'$

(7) -87

(8) $\frac{x+6}{x(x-2)}$

(9) 12

(10) $y = 2x - 4$

(11) $-\cos 15^\circ$ *or* $-\sin 75^\circ$

(12) $\frac{1}{\sqrt{\tan^2 A + 1}}$

(13) 12

(14) 14

(15) $108\sqrt{3}$ *or* 186.8

(16) $\frac{7}{9}$

(17) $\frac{\sqrt{2}}{10}$

(18) $\sqrt{3}$

(19) 1

(20) 3

(21) 1

(22) 2

(23) 4

(24) 4

(25) 1

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(26) *a* $\frac{3 \pm \sqrt{41}}{4}$ *or* $\frac{3 - \sqrt{41}}{4}$ [6]

b quadrants III and IV [2]

c arc sin $\frac{3 - \sqrt{41}}{4}$ [2]

(27)

x	2	$-\frac{5}{4}$
y	-1	$\frac{9}{4}$

[2]

(28) *a* Let x = number of minutes it would take William alone to do the job.

$$\frac{15}{x - 10} + \frac{20}{x} = 1 \quad [5]$$

b Let u = units digit; let t = tens digit.

$$\frac{10t + u}{t + u} = 4$$

$$10u + t = 10t + u + 36 \quad [5]$$

(29) Analysis [5]
6, 12, 18 [5]

(30) *c* four [1]
d 0 *or* π *or* 2π [1]

Part III

(32) 82.7 [10]

(33) 101° 30′ [10]

(34) Analysis [5]
4.8 [5]