The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, January 21, 1959 — 9:15 a.m. to 12:15 p.m., only

Name of pupil..... Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Factor completely: $2ax^2 - 18a$ 1.....

2 Express the sum of $2\sqrt{-9}$ and $3\sqrt{-16}$ as a monomial in terms of i. 2.....

3 Express $\dfrac{5}{4-\sqrt{2}}$ as an equivalent fraction with a rational denominator. 3.....

4 If the number 0.000328 is expressed as 3.28×10^n, what is the value of n? 4.....

5 Express in *simplest* form: $\dfrac{a-\dfrac{1}{b}}{\dfrac{1}{a}-b}$ 5.....

6 Write an equation of the line which is parallel to the line $y = 3x + 5$, and which intersects the y-axis two units below the origin. 6.....

7 If s varies inversely as t^2 and if $s = 4$ when $t = 3$, find s when $t = 6$. 7.....

8 Solve the equation $R = \dfrac{ST}{S+T}$ for S. 8.....

9 Write the discriminant of the equation $nx^2 + px + q = 0$. 9.....

10 Find *two* numbers that, when inserted between 4 and 500, form with those numbers a geometric progression of four terms. 10.....

11 Find the number whose logarithm is 0.6318. 11.....

[OVER]

12 Find $\sin 86° 34'$. 12.....

13 Solve the following set of equations for $\cot B$ in terms of x and y:

$$\tan A + \cot B = x$$
$$2\tan A + \cot B = y$$

13.....

14 Find the numerical value of $\frac{1}{2}\cos\frac{2\pi}{3} - \sin\pi$. 14.....

15 Find the positive value of $\sin \text{arc cos } (-\frac{4}{5})$. 15.....

16 Express $\cot 285°$ as a function of a positive acute angle. 16.....

17 In triangle ABC, $a = 5$, $b = 7$, $\sin A = 0.35$. Find $\sin B$. 17.....

18 In triangle ABC, $a = 9$, $b = 7$, $c = 5$. Find $\cos A$. 18.....

19 Find the area of triangle ABC if $a = 3$, $b = 4$ and $C = 150°$. 19.....

20 Solve the equation $\tan^2 x - 3 = 0$ for the smallest positive value of x. 20.....

Directions (21–25): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b*, *c* or *d*.

21 The maximum value of $3\cos\frac{1}{2}x$ is (*a*) 1 (*b*) $\frac{1}{2}$ (*c*) 3 (*d*) $\frac{3}{2}$ 21.....

22 The tens digit of a certain two-digit number is represented by x and the units digit by y; the number exceeds its tens digit by 34. This leads to the equation
(*a*) $xy - x = 34$ (*b*) $xy - 10x = 34$ (*c*) $10x + y + 34 = x$
(*d*) $9x + y = 34$ 22.....

23 The value of $3x^0 + 16^{-\frac{1}{4}}$ is (*a*) $3\frac{1}{8}$ (*b*) $3\frac{1}{8}$ (*c*) $3\frac{1}{12}$ (*d*) $1\frac{1}{8}$ 23.....

24 If $\log x = n$, then $\log \frac{x^3}{10}$ is (*a*) $3n - 1$ (*b*) $\frac{3n}{10}$ (*c*) $\frac{n^3}{10}$
(*d*) $3\log n - 1$ 24.....

25 The graph of $x^2 = y^2 + 9$ is (*a*) a circle (*b*) an ellipse (*c*) a hyperbola
(*d*) a parabola 25.....

Part II

Answer three questions from this part. Show all work unless otherwise directed.

26 Find to the *nearest tenth* the roots of $2x^2 + 8x + 1 = 0$. [10]

27 Solve the following set of equations and check in both equations: [8, 2]

$$x^2 + 3y^2 = 13$$
$$x - y = 3$$

28 *a* *Sketch* the graph of $y = \tan x$ as x varies from 0 to π radians. [4]

b On the same set of axes used in answer to *a*, *sketch* the graph of $y = \cos 2x$ as x varies from 0 to π radians. [5]

c How many values of x between 0 and π satisfy the equation $\tan x = \cos 2x$? [1]

29 Solve *graphically* the following set of equations: [Estimate the answers to *tenths.*] [6, 2, 2]

$$xy = 4$$
$$y = x - 1$$

30 Write the equation or equations that would be used to solve the following problems. In *each* case state what the letter or letters represent. [*Solution of the equations is not required.*]

a How many ounces of pure acid must be added to 24 ounces of a 10% solution of acid to make it a 40% solution? [5]

b A mechanic's helper requires 4 hours longer than the mechanic to repair a piece of machinery. The mechanic began the job alone and worked on it for 3 hours; then he was called away. The helper finished the job in 5 more hours. How long would it have taken the mechanic alone to do the whole job? [5]

31 *a* Starting with the formula for $\cos 2A$, *derive* the formula for $\sin \frac{1}{2}x$ in terms of x. [5]

b Prove that the following equality is an identity: [5]

$$\frac{\cos (x - y)}{\cos (x + y)} = \frac{\cot x + \tan y}{\cot x - \tan y}$$

Part III

Answer two questions from this part. Show all work.

32 Using logarithms, find, to the *nearest tenth*, the value of d if $d = \dfrac{23.8\sqrt[3]{0.0642}}{\sin 38°}$. [10]

33 In triangle ABC, angle $A = 16° 30'$, angle $B = 28° 10'$ and side $AB = 34.5$ feet. Find, to the *nearest tenth of a foot*, the length of AC. [10]

34 Point P is 40 miles directly south of point R. Point S bears N 59° 20′ E from R and the distance from R to S is 50 miles. Find, to the *nearest mile*, the distance from P to S. [5, 5]

*35 The sides of a triangle are 7.4, 8.7 and 9.5. Using the formula $\sin \frac{1}{2}C = \sqrt{\dfrac{(s-a)(s-b)}{ab}}$, find, to the *nearest degree*, the smallest angle of the triangle. [10]

* This question is based on one of the optional topics in the syllabus and may be used as *one* of the questions in part III only.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. As an answer to question 9, do not allow credit for the square root of the discriminant. Do not allow credit if the answer to question 11 is not expressed to *four significant figures* and if the answer to question 12 is not expressed to *four decimal places*. For questions 21–25, allow credit if the pupil has written the correct answer instead of the letters *a*, *b*, *c* or *d*.

(1) $2a\,(x+3)\,(x-3)$

(2) $18i$

(3) $\frac{5(4+\sqrt{2})}{14}$

(4) -4

(5) $-\frac{a}{b}$

(6) $y=3x-2$

(7) 1

(8) $\frac{RT}{T-R}$

(9) p^2-4nq

(10) 20,100

(11) 4.284

(12) 0.8345

(13) $2x-y$

(14) $-\frac{1}{4}$

(15) $\frac{4}{5}$

(16) $-\cot 75°$ *or* $-\tan 15°$

(17) 0.49

(18) $-\frac{1}{10}$

(19) 3

(20) 60°

(21) *c*

(22) *d*

(23) *b*

(24) *a*

(25) *c*