The University of the State of New York

329TH HIGH SCHOOL EXAMINATION

# ELEVENTH YEAR MATHEMATICS

Tuesday, January 22, 1957 — 9:15 a.m. to 12:15 p.m., only

## Instructions

**Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.**

**Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.**

**The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.**

**Fill in the following lines:**

Name of pupil.....Name of school.....

## Part I

**Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of $\pi$ or in radical form.**

1 Express as a monomial in terms of $i$: $\sqrt{-8} + \sqrt{-2}$ 1.....

2 Write the fraction $\frac{2}{3-\sqrt{2}}$ as an equivalent fraction with a rational denominator. 2.....

3 Find the value of $2x^{\frac{3}{2}} + x^{-1}$ when $x = 4$. 3.....

4 Combine into a single fraction: $\frac{3}{x-1} - \frac{1}{x}$ 4.....

5 If $d$ varies directly as the square of $t$ and if $d = 12$ when $t = 2$, find the value of $d$ when $t = 5$. 5.....

6 Solve for $n$: $P = T(n-2)$ 6.....

7 If the number 0.0000562 is written in the form $5.62 \times 10^n$, what is the value of $n$? 7.....

8 One root of the equation $2x^2 - 5x - 117 = 0$ is 9. Find the other root. 8.....

9 If the roots of the equation $x^2 + 12x + K = 0$ are equal, find the value of $K$. 9.....

10 Write an equation of the straight line that passes through the origin and that is parallel to a line whose equation is $2x + y = 6$. 10.....

11 Write an equation of the straight line that passes through two points whose coordinates are $(0, -2)$ and $(5, 1)$. 11.....

12 What three numbers can be inserted between 2 and 30 to form with these numbers an arithmetic progression of 5 terms? 12.....

13 When $a = 4$, $r = 3$ and $n = 4$, find the value of $S$ in the formula $S = \frac{a(r^n - 1)}{r - 1}$. 13.....

14 If peanuts and cashew nuts are mixed in the ratio 1 : 3, respectively, how many pounds of peanuts are contained in $n$ pounds of the mixture? 14.....

15 Find $n$ if $\log n = 1.7718$. 15.....

16 Find to the *nearest minute* the positive acute angle whose sine is 0.4129. 16.....

17 Find the numerical value of: $2 \sin \frac{\pi}{6} + \cos \pi$ 17.....

18 Solve the equation $\sqrt{3 \tan x + 1} = 2$ for the smallest positive value of $x$. 18.....

19 Find the number of inches in the radius of the circle in which a central angle of 4 radians intercepts an arc of 30 inches. 19.....

20 Find the positive value of $\cos(\text{arc } \sin \frac{3}{5})$. 20.....

21 In triangle $ABC$, side $a$ is twice as long as side $b$ and angle $C = 30°$. Express the area of the triangle in terms of $b$. 21.....

22 In triangle $ABC$, $a = 6$, $b = 9$ and $\cos C = \frac{2}{3}$. Find $c$. 22.....

23 If $\cos A = k$, express $\cos^2 \frac{1}{2}A$ in terms of $k$. 23.....

24 Express cot 250° as a function of a positive acute angle. 24.....

25 Express $\sin A \cot A + \cos^2 A \sec A$ in terms of $\cos A$. 25.....

## Part II

**Answer three questions from this part. Show all work unless otherwise directed.**

26 Solve the following system of equations and check: [8, 2]

$$\begin{aligned} 3x - 2y - 3z &= -1 \\ 6x + y + 2z &= 7 \\ 9x + 3y + 4z &= 9 \end{aligned}$$

27 *a* Draw the graph of the equation $y = 2x^2 + 4x - 3$ from $x = -4$ to $x = 2$, inclusive. [6]

*b* Draw the axis of symmetry of the graph made in answer to part *a*, and write its equation. [2]

*c* From the graph drawn in answer to part *a*, estimate to *tenths* the roots of the equation $2x^2 + 4x - 3 = 6$. [2]

28 Write the equations that would be used to solve the following problems. In *each* case state what the letter or letters represent. [*Solution of the equations is not required.*]

*a* The product of the digits of a two-digit number is 8. The number is 4 times the sum of its digits. Find the number. [5]

*b* A motorist drove from $A$ to $B$ at an average rate of 40 miles per hour. He returned over a different route, which was 10 miles longer, at an average rate of 30 miles per hour. If his total traveling time was 5 hours, what was the distance from $A$ to $B$ by the shorter route? [5]

29 *a* *Sketch* the graph of $y = 2 \sin x$ from $x = 0°$ to $x = 360°$, inclusive. [4]

*b* On the set of axes used in part *a*, *sketch* the graph of $y = \cos 2x$ from $x = 0°$ to $x = 360°$, inclusive. [4]

*c* What is the amplitude of the function $2 \sin x$? [1]

*d* What is the period of the function $\cos 2x$? [1]

30 *a* Without the use of tables, find the exact value of $\tan (A + B)$ if $A$ is a first-quadrant angle for which $\tan A = \frac{1}{2}$ and $B$ is a second-quadrant angle for which $\sin B = \frac{3}{5}$. [5]

*b* In the formula for $\sin (x + y)$, let $y$ be replaced by $2x$ and thus *derive* the formula: $\sin 3x = 3 \sin x - 4 \sin^3 x$. [5]

## Part III

**Answer two questions from this part. Show all work.**

31 *a* If $\log n - 2 \log x = \log y$, express $n$ in terms of $x$ and $y$. [2]

*b* Using logarithms, find to the *nearest hundredth* the value of $\dfrac{762 \times \sqrt[3]{0.364}}{94.4}$. [8]

32 In triangle $ABC$, $B = 53° 20'$, $C = 71° 30'$ and $BC = 26.1$ inches. Find the length of the altitude on side $BC$ to the *nearest tenth of an inch.* [10]

33 *a* Prove the identity $\sin 2x = \dfrac{2 \tan x}{1 + \tan^2 x}$. [4]

*b* Solve for all positive values of $x$ less than 180°: [6]

$$2 \sin^2 x - \cos x - 2 = 0$$

*34 *a* In triangle $ABC$, side $a = 27$, side $b = 31$ and side $c = 36$. Using the formula $\sin \frac{1}{2}C = \sqrt{\dfrac{(s-a)(s-b)}{ab}}$, find the value of angle $C$ to the *nearest degree.* [7]

*b* Given the expression $\sin 75° + \sin 15°$.

(1) Using the proper formula, express this sum as a product. [2]

(2) Without the use of tables, find the value of the answer to part *b* (1). [Answer may be left in radical form.] [1]

* This question is based on one of the optional topics in the syllabus and may be used as one of the questions in part III only.

# FOR TEACHERS ONLY

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## INSTRUCTIONS FOR RATING
## ELEVENTH YEAR MATHEMATICS

**Tuesday,** January 22, 1957 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 15 is not expressed to *four significant digits.*

(1) $3i\sqrt{2}$

(2) $\frac{6+2\sqrt{2}}{7}$

(3) $16\frac{1}{4}$

(4) $\frac{2x+1}{x(x-1)}$

(5) 75

(6) $\frac{P}{T}+2$ *or* $\frac{P+2T}{T}$

(7) $-5$

(8) $-\frac{13}{2}$

(9) 36

(10) $y=-2x$

(11) $3x-5y=10$

(12) 9, 16, 23

(13) 160

(14) $\frac{n}{4}$

(15) 59.13

(16) 24° 23′

(17) 0

(18) 45

(19) $7\frac{1}{2}$

(20) $\frac{4}{5}$

(21) $\frac{b^2}{2}$

(22) $\sqrt{45}$

(23) $\frac{1+K}{2}$

(24) cot 70° *or* tan 20°

(25) 2 cos $A$