The University of the State of New York

326TH HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, January 25, 1956 — 9.15 a.m. to 12.15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer three questions from this part. Show all work unless otherwise directed.

26 Given the equation $\cos^2 x + \cos x - 1 = 0$.

a Find to the *nearest tenth* the positive value of $\cos x$ that satisfies this equation. [8]

b Using the result found in answer to part *a*, find to the *nearest degree* the smallest positive value of x that satisfies this equation. [2]

27 Solve the following system of equations and check both sets of roots: [8, 2]

$$x^2 - xy - x = 8$$
$$y + x = -1$$

28 Write the equations that would be used to solve the following problems. In *each* case state what the letter or letters represent. [*Do not solve the equations.*]

a A and B together received a total of \$246 for working 8 days and 10 days respectively. If A had worked for 10 days and B for 8, A's total wages would have equalled B's. Find the daily wages of each. [5]

b A train leaves a certain point and travels at a uniform rate of 55 miles per hour. Two hours later another train leaves the same point and travels in the same direction at a uniform rate of 70 miles per hour. How long after the faster train leaves this point will it be 65 miles behind the slower train? [5]

29 *a* Prove that the following equality is an identity: [5]

$$\frac{\cos\theta \sin^2\theta}{1 - \cos\theta} = \cos\theta + \cos^2\theta$$

b By substituting 45° and 30° for x and y respectively in the following formula, show that $\cot 75° = 2 - \sqrt{3}$. [Do *not* use the trigonometric table in answering this question.] [5]

$$\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

30 *a* Draw the graph of $y = \sin \frac{1}{2}\theta$ as θ varies from 0 to 2π radians at intervals of $\frac{\pi}{3}$ radians. [6]

b On the set of axes used in answer to part *a*, draw the graph of $y = \frac{1}{2}$. [2]

c From the graphs made in answer to parts *a* and *b*, find the values of θ greater than 0 but less than 2π for which $\sin \frac{1}{2}\theta = \frac{1}{2}$. [2]

Part III

Answer two questions from this part. Show all work.

31 The volume of a regular pentagonal pyramid is given by the formula $V = \frac{r^2 h \sin 72°}{1.2}$.

a Solve the formula for r. [3]

b Using logarithms, find r to the *nearest integer* when $V = 465$ and $h = 16.3$ [7]

32 In triangle ABC, $AB = 30$ feet, $BC = 24$ feet and angle $B = 40°$. Find to the *nearest foot* the length of AC. [10]

33 Point B is 70.6 miles directly south of point A. An airplane carrier is located at point C, which is N 75° E from A and N 32° E from B. A plane flies in a straight line from B to C at a ground speed of 150 miles per hour. Find to the *nearest minute* the time required to make the flight. [3, 7]

*34 Point C is 4.4 miles from one end A of an island and 7.6 miles from the other end B. Angle ACB is 110°.

a *Using the law of tangents,* find angle BAC to the *nearest degree.* [7]

b Using the result found in answer to part *a,* find to the *nearest mile* the length of the island. [3]

* This question is based on one of the optional topics in the syllabus and may be used as one of the questions in part III *only.*

ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the value of $4^{\frac{3}{2}} - 4^0$. 1.....

2 Express as a single term in simplest form $\sqrt{27} + 6\sqrt{\frac{1}{3}}$. 2.....

3 Find the sum of the first 10 terms of the progression —8, —6, —4, ... 3.....

4 Find the arithmetic mean between 5 and 8. 4.....

5 Write an equation of the straight line which passes through the point (0, 5) and has a slope of 2. 5.....

6 Factor completely: $x^3 + 7x^2 + 12x$. 6.....

7 Solve the equation $\sec^2 \theta - 2 = 0$ for the positive value of θ less than 90°. 7.....

8 Solve the equation $\sqrt{x^2 + 27} = 2x$ for the *positive* value of x. 8.....

9 Express tan $2A$ in terms of tan A. 9.....

10 Express sin x cot x in terms of cos x. 10.....

11 Express cot 245° as a function of a positive acute angle. 11.....

12 If y varies directly as x and if $y = 8$ when $x = 4$, find the value of y when $x = -5$. 12.....

13 Find to the *nearest minute* the angle whose tangent is 0.8466 13.....

14 Find the antilogarithm of 2.4752 14.....

15 Find the log cos 52° 17′. 15.....

16 In triangle ABC, angle $A = 30°$, side $a = 50$ and angle $B = 67°$. Find side b to the *nearest integer*. 16.....

[OVER]

Directions (17–25): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

17 The roots of the equation $6x^2 - 3x + 1 = 0$ are (*a*) real and rational (*b*) real and irrational (*c*) imaginary 17.....

18 The graphs of the equations $2x + 4y = 7$ and $x + 2y = 7$, when drawn on the same set of axes, (*a*) intersect (*b*) coincide (*c*) are parallel 18.....

19 The graph of the equation $y = \frac{6}{x}$ is a (*a*) straight line (*b*) parabola (*c*) hyperbola 19.....

20 The sum of the first ten terms of the progression 2, 6, 18, ... is (*a*) 3^{10} (*b*) $3^{10} - 1$ (*c*) $3^{10} + 1$ 20.....

21 The equation $\cos^2 \frac{\theta}{2} = \frac{1 + \cos \theta}{2}$ is (*a*) true for all values of θ (*b*) true for only certain values of θ (*c*) not true for any value of θ 21.....

22 The expression $\sin^2 \theta - \cos^2 \theta$ is identically equal to (*a*) $\cos(-2\theta)$ (*b*) $\cos 2\theta$ (*c*) $-\cos 2\theta$ 22.....

23 If *a*, *b* and *c* represent the sides of a triangle and if $\frac{a^2 + b^2 - c^2}{2ab}$ is equal to zero, the triangle is (*a*) acute (*b*) right (*c*) obtuse 23.....

24 The amplitude of the function $\frac{1}{2}\sin 2\theta$ is (*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 24.....

25 The principal value of $\cos^{-1}(-1)$ is (*a*) $-\pi$ (*b*) 0 (*c*) π 25.....

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 14 is not expressed to four significant digits. For questions 17–25, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

(1) 7
(2) $5\sqrt{3}$
(3) 10
(4) $6\frac{1}{2}$
(5) $y = 2x + 5$
(6) $x(x+3)\ (x+4)$
(7) 45°
(8) 3
(9) $\dfrac{2\tan A}{1-\tan^2 A}$
(10) $\cos x$
(11) $\tan 25°$ *or* $\cot 65°$
(12) —10
(13) 40° 15′
(14) 298.7
(15) 9.7866 — 10 *or* $\bar{1}.7866$
(16) 92
(17) *c*
(18) *c*
(19) *c*
(20) *b*
(21) *a*
(22) *c*
(23) *b*
(24) *a*
(25) *c*