The University of the State of New York

317TH HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, January 21, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer two questions from part II.

26 Find to the *nearest degree* the positive values of x between 0° and 360° that satisfy the equation $2 \cos x = \sin^2 x + 1$. [10]

27 Solve the following system of equations and check: [8, 2]

$$x^2 - 3y^2 = 13$$
$$x + 3y = 1$$

28 Write the equations that would be used in solving the following problems. In each case state what the letter or letters represent. [Solution of the equations is not required.]

a Three numbers are in the ratio 2 : 4 : 5. If the first number is decreased by 2, the second number is left unchanged and the third number is increased by 8, the resulting numbers form an arithmetic progression. Find the original numbers. [5]

b The rate of a motor boat in still water is 12 miles per hour. If the rate of the current is 3 miles per hour, how far upstream can the boat travel before returning to its starting point if the time allowed for the entire trip is 4 hours? [5]

29 Using logarithms find to the *nearest tenth* the value of $\dfrac{45.6 \sin 36° 20'}{\sqrt[3]{.574}}$ [10]

Part III

Answer three questions from part III.

30 *a* Draw the graph of $y = \cos x$ as x varies from 0° to 360° at intervals of 30°. [4]

b On the set of axes used in *a* draw the graph of $y = \tan x$ for the same values of x. [4]

c From the graphs made in answer to *a* and *b* estimate to the *nearest degree* the values of x between 0° and 360° which satisfy the equation: $\cos x = \tan x$. [2]

31 *a* Starting with the formula for $\cos (x + y)$ derive the formula for $\cos 2x$ in terms of $\cos x$. [4]

b Prove that the following equality is an identity: [6]

$$\frac{2 \cot x - \sin 2x}{2 \cos^2 x} = \cot x$$

[OVER]

32 Coast Guard Station B is 23.6 miles north of Coast Guard Station A. From B the bearing of a ship in distress is N 75° 40′ E and from A the bearing is N 32° 20′ E. Find to the *nearest tenth of a mile* the distance from the ship to the nearer Coast Guard Station. [6, 4]

33 In parallelogram $ABCD$ side $AB = 25$, side $BC = 15$ and diagonal $AC = 34$.

a Find angle B to the *nearest degree*. [7]

b Find the area of the parallelogram to the *nearest integer*. [3]

34 *a* From two points on level ground in line with the foot of a cliff the angles of elevation of the top of the cliff are x and y. If x is greater than y and the distance between the points is d, derive a formula for h, the height of the cliff, in terms of d, x and y. [6]

b Using the formula derived in *a*, find the height of the cliff to the ***nearest ten feet*** if $d = 125$ feet, $x = 60°\ 40'$ and $y = 58°$. [4]

*35 *a* State in terms of A, B, a and b, the law of tangents for triangle ABC. [2]

b Using the formula stated in answer to *a*, find angle A to the *nearest degree* if angle $C = 96°\ 20'$, $a = 56.7$ and $b = 23.5$. [8]

* This question is based upon one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the positive value of x that satisfies the equation $2x^2 + x - 15 = 0$. 1.....

2 Express in terms of i the sum of $\sqrt{-12}$ and $\sqrt{-3}$. [Write answer in simplest form.] 2.....

3 Express $\frac{5}{3-\sqrt{2}}$ as an equivalent fraction with a rational denominator. 3.....

4 If x varies directly as y and $x = 16$ when $y = 12$, find x when $y = 21$. 4.....

5 Find the value of $2x^{\frac{2}{3}} + x^{-1}$ when $x = 8$. 5.....

6 Find the sum of the first 20 terms of the progression 3, 6, 9, ... 6.....

7 Express $\frac{1 - \frac{1}{a}}{1 + \frac{1}{a}}$ as a fraction reduced to lowest terms. 7.....

8 Write the equation of a straight line whose slope is 3 and which passes through point (2, 5). 8.....

9 If the roots of the quadratic equation $2x^2 + px + q = 0$ are 3 and 5, what is the value of q? 9.....

10 Solve the following formula for a: $K = \frac{1}{2}h\ (a + b)$ 10.....

11 Find the smallest positive angle which satisfies the equation: $\sqrt{4 \sin^2 x + 3} = 2$ 11.....

12 Express $\log \frac{x^3}{y}$ in terms of $\log x$ and $\log y$. 12.....

13 Find n if $\log n = 2.8104$ 13.....

14 Find $\log \tan 39° 24'$ 14.....

[OVER]

15 Express csc 160° as a function of a positive acute angle. 15.....

16 Express tan $(A - B)$ in terms of tan A and tan B. 16.....

17 In a circle whose radius is 9 inches, how many radians are there in a central angle which intercepts an arc 15 inches long? 17.....

18 If $x = \cos^{-1} \frac{1}{2}$, what is the principal value of x? 18.....

19 If x is a positive acute angle whose cosine is a, express sin $\frac{1}{2}x$ in terms of a. 19.....

20 In triangle ABC, $b = 18$, sin $A = .7$ and sin $B = .6$. Find a. 20.....

21 In triangle ABC, $a = 12$, $b = 5$, cos $C = \frac{2}{3}$. Find c. [Answer may be left in radical form.] 21.....

Directions (22–25) — Indicate the correct completion for *each* of the following by writing on the line at the right the letter a, b or c.

22 The positive geometric mean between m and $3m$ is (*a*) $2m$ (*b*) $\sqrt{3m}$ (*c*) $m\sqrt{3}$ 22.....

23 The graph of $4x^2 - 9y^2 = 36$ is (*a*) an ellipse (*b*) a hyperbola (*c*) a circle 23.....

24 The graph of the equation $y = 3x^2 + 5x + 2$ (*a*) cuts the x-axis in two distinct points (*b*) is tangent to the x-axis (*c*) has no points in common with the x-axis 24.....

25 The period of the function 2 sin $\frac{1}{2}x$ is (*a*) 720° (*b*) 360° (*c*) 180° 25.....