The University of the State of New York

314TH HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, January 23, 1952 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer two questions from part II.

26 Find to the *nearest degree* the smallest positive value of x that satisfies the equation $2 \tan^2 x - 5 \tan x - 1 = 0$.]10]

27 Solve the following system of equations and check the answers: [8, 2]

$$3x - y = 1$$
$$4x^2 - xy = 2$$

28 *a* Draw the graph of the equation $y = x^2 + 3x - 2$ from $x = -4$ to $x = +1$ inclusive. [6]

b From the graph made in answer to *a*, estimate to the *nearest tenth*

(1) the roots of the equation $x^2 + 3x - 2 = 0$ [2]

(2) the coordinates of the minimum point [2]

29 Write the equations that would be used in solving the following problems. In *each* case state what the letter or letters represent. [Solution of the equations is not required.]

a The sum of the digits of a two-digit number is 11. The number obtained by interchanging the digits exceeds twice the original number by 34. Find the original number. [5]

b How many ounces of a 90 per cent solution of disinfectant should be added to 15 ounces of a 10 per cent solution to make the result a 30 per cent solution? [5]

Part III

Answer three questions from part III.

30 *a* Draw the graph of $y = 2 \sin x$ as x varies from 0° to 180° at intervals of 30°. [4]

b On the set of axes used in *a* draw the graph of $y = \cos 2x$ for the same values of x. [4]

c From the graphs made in answer to *a* and *b* estimate the values of x from 0° to 180° that satisfy the equation $2 \sin x = \cos 2x$. [2]

31 *a* Starting with the formulas for $\sin (A + B)$ and $\cos (A + B)$, derive the formula for $\tan (A + B)$ in terms of $\tan A$ and $\tan B$. [5]

b Using the formula derived in answer to *a*, derive the formula for $\tan 2A$ in terms of $\tan A$. [2]

c Prove that the following equality is an identity: [3]

$$\tan 2A = \frac{2}{\cot A - \tan A}$$

[OVER]

32 The pilot of an airplane in flight at position C finds that he must land at the nearer of two airports, A and B. He knows that B is 87.8 miles due north from A. The bearing of A from C is S 85° 50′ E and the bearing of B from C is N 23° 10′ E. How far, to the *nearest tenth of a mile,* is he from the nearer airport? [6, 4]

33 In triangle ABC, $a = 19$, $b = 16$ and $c = 15$.

a Find C to the *nearest degree.* [8]

b Using the answer obtained in *a,* find to the *nearest square unit* the area of triangle ABC. [2]

34 In triangle ABC, $C = 90°$. The bisector of angle A meets CB at D.

a Show $DB = \dfrac{AD \sin \frac{1}{2}A}{\cos A} = \dfrac{AC \tan \frac{1}{2}A}{\cos A}$ [7]

b If $AC = 76$ and $A = 68° 40'$, find DB to the *nearest integer.* [3]

*35 Given the equation $\cos 3\theta + \cos \theta = \cos 2\theta$.

a Using the formula for the sum of two cosines, show that $\cos 2\theta = 0$ and $\cos \theta = \frac{1}{2}$. [7]

b Using the answers obtained in *a,* find the values of θ greater than 0° and less than 180° that satisfy the given equation. [3]

* This question is based upon one of the optional topics in the syllabus.

ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil..... Name of school.....

Part I

Answer all questions in part I. Eact correct answer will receive 2 credits. No partial credit will be allowed.

1 Express $\frac{5}{3-\sqrt{2}}$ as an equivalent fraction with a rational denominator. 1.....

2 Write the expression $3\sqrt{-4} - 2i$ as a single term. 2.....

3 Write an equation of the straight line whose slope is 5 and which passes through the point (0, 1). 3.....

4 If p varies inversely as v and $p = 2$ when $v = 20$, find p when $v = 8$. 4.....

5 Simplify the complex fraction $\dfrac{b - \dfrac{1}{a}}{\dfrac{1}{a}}$ 5.....

6 Find the value of $2b^0 + b^{-\frac{1}{2}}$ when $b = 16$. 6.....

7 The first term of an arithmetic progression is $\frac{3}{2}$ and the sixth term is 14. Find the common difference. 7.....

8 Find two numbers that when inserted between 2 and 250 form with those numbers a geometric progression of four terms. 8.....

9 Find the sum of the roots of the equation $x^2 - 4x + 7 = 0$. 9.....

10 Write the equation of the axis of symmetry of the graph of $y = x^2 - 4x - 12$. 10.....

11 Find log sin 65° 33′ 11.....

12 Find the number whose logarithm is 8.8680 — 10. 12.....

13 Solve the equation $\sqrt{4 \tan x + 5} = 3$ for the smallest positive value of x. 13.....

[OVER]

14 Express sec 310° as a function of a positive acute angle. 14.....

15 Given the equation $m = \frac{\cos x + 1}{\cos x - 1}$, express cos x in terms of m. 15.....

16 In a circle whose radius is 6 inches, find the length of an arc intercepted by a central angle of 3 radians. 16.....

17 Find the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ 17.....

18 Express $\tan^2 A$ in terms of $\sin A$. 18.....

19 If x is an acute angle and $\cos x = \frac{1}{8}$, find $\cos \frac{1}{2}x$. 19.....

20 In triangle ABC, $c = 16$, $\sin C = \frac{2}{3}$ and $\sin B = \frac{\sqrt{5}}{12}$. Find b. 20.....

21 In triangle ABC, $a = 5$, $c = 7$ and $\cos B = \frac{1}{7}$. Find b. 21.....

Directions (questions 22–25) — Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

22 If the roots of a quadratic equation are real, rational and unequal, the discriminant of the equation may be (*a*) 0 (*b*) -4 (*c*) 1 22.....

23 The expression $\log r + \log r^2$ is equal to (*a*) $\log (r + r^2)$ (*b*) $3 \log r$ (*c*) r^3 23.....

24 Using the data $A = 28°$, $b = 16$ and $a = 9$, it is possible to construct (*a*) no triangle (*b*) only one triangle (*c*) two different triangles 24.....

25 The statement $\cos 2x = (\cos x + \sin x)(\cos x - \sin x)$ is true for (*a*) all values of x (*b*) only certain values of x (*c*) no values of x 25.....