ALGEBRA II

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Monday, August 19, 2024 — 12:30 p.m. to 3:30 p.m., only

Student Name _Mr. Sibol_

School Name _JMAP_

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

1 A grocery store owner wonders how many customers bring reusable bags to the store. An employee stands at the store entrance for two hours and counts the number of people bringing in reusable bags. This type of study is best classified as

(1) a census

(2) an experiment

(3) an observational study

(4) a survey

2 The graph of $y = 2^x - 4$ is positive on which interval?

(1) $(-\infty, \infty)$

(2) $(2, \infty)$

(3) $(0, \infty)$

(4) $(-4, \infty)$

3 Tim deposits \$300 into a savings account. The annual interest rate is 2.7% and compounds monthly. He uses the equation $A = 300\left(1 + \dfrac{0.027}{12}\right)^{12t}$ to determine how much money, A, he will have after t years. Which equation is equivalent to Tim's equation?

(1) $A = 300\left[(1.00225)^{12}\right]^t$

(2) $A = 300(0.08558)^{12t}$

(3) $A = 300\left[1 + \left(\dfrac{0.027}{12}\right)^{12t}\right]$

(4) $A = (300)^{12t}(1)^{12t} + \left(\dfrac{0.027}{12}\right)^{12t}$

4 Which equation is true for all real values of x?

(1) $x^4 + x = (x + 1)(x^3 - x^2 + x)$

(2) $x^4 + x = (x + 1)(x^3 + x)$

(3) $x^4 + x = (x^2 + x)^2$

(4) $x^4 + x = (x - 1)(x^3 + x^2 + x)$

$x^4 + x$

$x(x^3 + 1)$

$x(x+1)(x^2 - x + 1)$

$(x+1)(x^3 - x^2 + x)$

5 The solution of $\dfrac{x}{x + 3} + \dfrac{2}{x - 4} = \dfrac{2x + 27}{x^2 - x - 12}$ is

(1) -3

(2) -7

(3) 3

(4) 7

$\dfrac{x(x-4)}{(x+3)(x-4)} + \dfrac{2(x+3)}{(x-4)(x+3)} = \dfrac{2x+27}{(x-4)(x+3)}$

$x^2 - 4x + 2x + 6 = 2x + 27$

$x^2 - 2x + 6 = 2x + 27$

$x^2 - 4x - 21 = 0$

$(x-7)(x+3) = 0$

$7, -3$

6 The cost, in dollars, of a single-ride fare in the New York City subway in the years since 1904 is listed in the table below.

Years since 1904 (x)	0	49	72	91	99	111
Fare (y)	$0.05	$0.15	$0.50	$1.50	$2.00	$2.75

Which equation best models the cost of a single-ride fare based on these data?

(1) $y = 0.0375(1.0392)^x$

(2) $y = 1.0392(0.0375)^x$

(3) $y = 0.0234x - 0.487$

(4) $y = -0.179 + 0.356 \ln(x)$

7 Which expression is equivalent to $\dfrac{6x^4 + 4x^3 + x + 200}{x + 2}$?

(1) $6x^2 - 8x + 17 + \dfrac{166}{x + 2}$

(2) $6x^2 + 16x + 33 + \dfrac{266}{x + 2}$

(3) $6x^3 + 16x^2 + 32x + 65 + \dfrac{330}{x + 2}$

(4) $6x^3 - 8x^2 + 16x - 31 + \dfrac{262}{x + 2}$

$$-2\begin{array}{|rrrrr} 6 & 4 & 0 & 1 & 200 \\ & -12 & 16 & -32 & 62 \\ \hline 6 & -8 & 16 & -31 & 262 \end{array}$$

8 The solution to the equation $6(2^{x + 4}) = 36$ is

(1) -1

(2) $\dfrac{\ln 36}{\ln 12} - 4$

(3) $\ln(3) - 4$

(4) $\dfrac{\ln 6}{\ln 2} - 4$

$\ln 2^{x+4} = \ln 6$

$\dfrac{(x+4)\ln 2}{\ln 2} = \dfrac{\ln 6}{\ln 2} - 4$

$x + 4 = 0 \qquad x = -4$

9 The asymptote of the graph of $f(x) = 5\log(x + 4)$ is

(1) $y = 6$

(2) $x = -4$

(3) $x = 4$

(4) $y = 5$

10 The probability of having math homework is $\frac{1}{3}$ and the probability of having English homework is $\frac{1}{7}$. The probability of having math homework or having English homework is $\frac{9}{21}$. What is the probability of having math homework and having English homework?

(1) $\frac{19}{21}$

(3) $\frac{1}{21}$

(2) $\frac{1}{5}$

(4) $\frac{10}{21}$

$$\frac{1}{3} + \frac{1}{7} - \frac{9}{21}$$

$$\frac{7}{21} + \frac{3}{21} - \frac{9}{21}$$

$$\frac{1}{21}$$

11 The solution set of the equation $x - 1 = \sqrt{2x + 6}$ is

(1) $\{5, -1\}$

(3) $\{-1\}$

(2) $\{5\}$

(4) $\{ \}$

$$(x-1)^2 = 2x+6$$
$$x^2 - 2x + 1 = 2x + 6$$
$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$

$$x = 5, \cancel{-1} \qquad -1-1 < 0$$

12 Given $x > 0$, the expression $\left(\frac{1}{x^{-2}}\right)^{-\frac{3}{4}}$ is equivalent to

(1) $x\sqrt{x}$

(3) $\sqrt[3]{x^2}$

(2) $\frac{1}{x\sqrt{x}}$

(4) $\frac{1}{\sqrt[3]{x^2}}$

$$\frac{1^{\frac{3}{4}}}{(x^{-2})^{-\frac{3}{4}}} = \frac{1}{x^{\frac{3}{2}}} = \frac{1}{x^1 \cdot x^{\frac{1}{2}}} = \frac{1}{x\sqrt{x}}$$

13 The graph of which function has a period of 3?

(1) $y = -7\sin\left(\dfrac{2\pi}{3}x\right) - 5$ *(circled)*

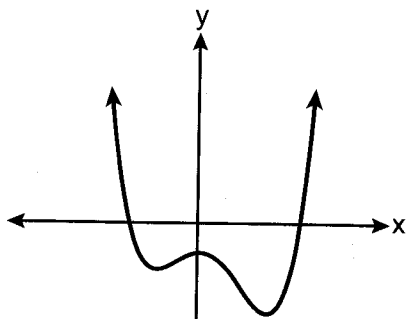(2) $y = -7\sin\left(\dfrac{3\pi}{2}x\right) + 9$
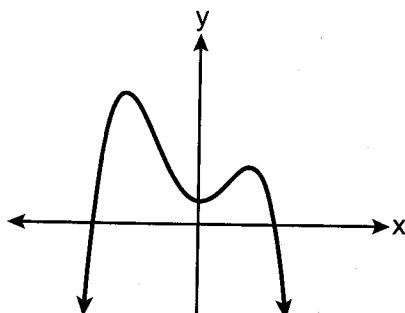
(3) $y = -7\sin(3x) - 5$

(4) $y = 3\sin(\pi x) + 9$
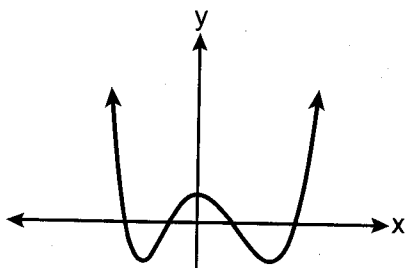
$$\dfrac{\partial\pi}{\dfrac{\partial\pi}{3}} = 3$$

14 Which graph could represent a 4th degree polynomial function with a positive leading coefficient, 2 real zeros, and 2 imaginary zeros?
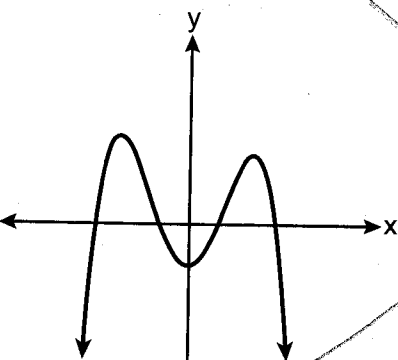

(1) *(circled)*

(3)

(2)

(4)

4 real roots

negative leading coefficient

15 Given i is the imaginary unit, which expression is equivalent to
$5i(2x + 3i) - x\sqrt{-9}$?

(1) $15 + 13xi$ (3) $15 + 7xi$

(2) $-15 + 13xi$ (4) $-15 + 7xi$

$10xi + 15i^2 - 3xi$

$-15 + 7xi$

16 What is the focus of the parabola $8(y + 2) = (x + 5)^2$?

(1) $(-5,0)$ (3) $(5,0)$

(2) $(-5,-4)$ (4) $(5,4)$

$y + 2 = \frac{1}{8}(x+5)^2$

$x = \frac{1}{4(2)}(x+5)^2 - 2$

$p = 2$ Vertex $(-5,-2)$

$2 + -2 = 0$

17 Given $q(x) = 2\log(x)$ and $r(x) = (x - 2)^3 - 4$, what is a solution of
$q(x) = r(x)$ to the *nearest tenth*?

(1) 1.1   (3) 3.9

(2) 3.7   (4) 4.3

18 The volume of a cardboard box can be modeled by $V(x)$, which is the product of the length, width, and height, x. If the length can be represented by $L(x) = 18 - 2x$ and the width can be represented by $W(x) = 18 - 2x$, then which function represents $V(x)$?

(1) $V(x) = 4x^2 - 72x + 324$

(2) $V(x) = 4x^3 - 72x^2 + 324x$

(3) $V(x) = -3x + 36$

(4) $V(x) = 4x^3 + 324x$

$V(x) = x(18-2x)(18-2x)$

$x(324 - 72x + 4x^2)$

$324x - 72x^2 + 4x^3$

19 The expression $8^{\frac{x}{2}} \cdot 8^{\frac{x}{3}}$ is equivalent to

(1) $\sqrt[6]{8^{5x}}$

(2) $64^{\frac{5x}{6}}$

(3) $\sqrt[5]{8^{2x}}$

(4) $64^{\frac{x^2}{6}}$

$8^{\frac{x}{2}+\frac{x}{3}} \cdot 8^{\frac{5x}{6}} = \sqrt[6]{8^{5x}}$

20 If θ is an angle in standard position whose terminal side passes through the point $(-3, -4)$, which statement is true?

(1) $\sec \theta > 0$ and $\tan \theta > 0$

(2) $\sec \theta < 0$ and $\tan \theta < 0$

(3) $\sec \theta > 0$ and $\tan \theta < 0$

(4) $\sec \theta < 0$ and $\tan \theta > 0$

Since the terminal side of θ passes through $(-3, -4)$,

$\cos \theta < 0 \rightarrow \sec \theta < 0$

$\sin \theta < 0$

$\tan \theta = \frac{\sin \theta}{\cos \theta} \rightarrow \frac{-}{-} \rightarrow +$

21 What is the value of y for the system shown below?

$$2\begin{pmatrix} 3x + 4y - 5z = -27 \\ 2x + 3y - z = -3 \\ 6x - y + 4z = 3 \end{pmatrix}$$

(1) -27 (3) 3

(2) 6   (4) -3

$6x + 8y - 10z = -54$
$6x + 9y - 3z = -9$
$\overline{\quad y + 7z = 45}$

$6x + 9y - 3z = -9$
$6x - y + 4z = 3$
$\overline{\quad 10y - 7z = -17}$
$y + 7z = 45$
$\overline{\quad 11y = 33}$
$y = 3$

22 The number of employees who work nights and weekends at a department store is summarized in the table below.

	Works Nights	Doesn't Work Nights
Works Weekends	8	40
Doesn't Work Weekends	12	60

48
72
120

20   100

Let N represent the event "works nights" and let W represent the event "works weekends." Based on the table, are N and W independent events?

(1) Yes, because $P(N) \cdot P(W) = P(N \cap W)$.
(2) Yes, because $P(N) \cdot P(W) \neq P(N \cap W)$.
(3) No, because $P(N) \cdot P(W) = P(N \cap W)$.
(4) No, because $P(N) \cdot P(W) \neq P(N \cap W)$.

$\frac{20}{120} \cdot \frac{48}{120} = \frac{8}{120}$

$\frac{1}{6} \cdot \frac{4}{10} = \frac{1}{15}$

$\frac{4}{60} = \frac{1}{15}$

23 Which expression is equivalent to $x^8 - y^8$?

(1) $(x - y)^8$

(3) $(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

(2) $(x^2 + y^2)^2 (x^2 - y^2)^2$

(4) $(x + y)^4 (x - y)^4$

$$(x^4 + y^4)(x^4 - y^4)$$
$$(x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$$
$$(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$$

24 A research assistant receives a first year salary of \$90,000 and a 2% annual raise throughout the first ten years of employment. In total, how much money will be earned over the first ten years, to the *nearest dollar*?

(1) \$91,837

(3) \$877,917

(2) \$109,709

(4) \$985,475

$$S_{10} = \frac{90,000 - 90,000 (1.02)^{10}}{1 - 1.02} \approx 985,475$$

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 On the axes below, graph $y = 3.2(1.8)^x$.
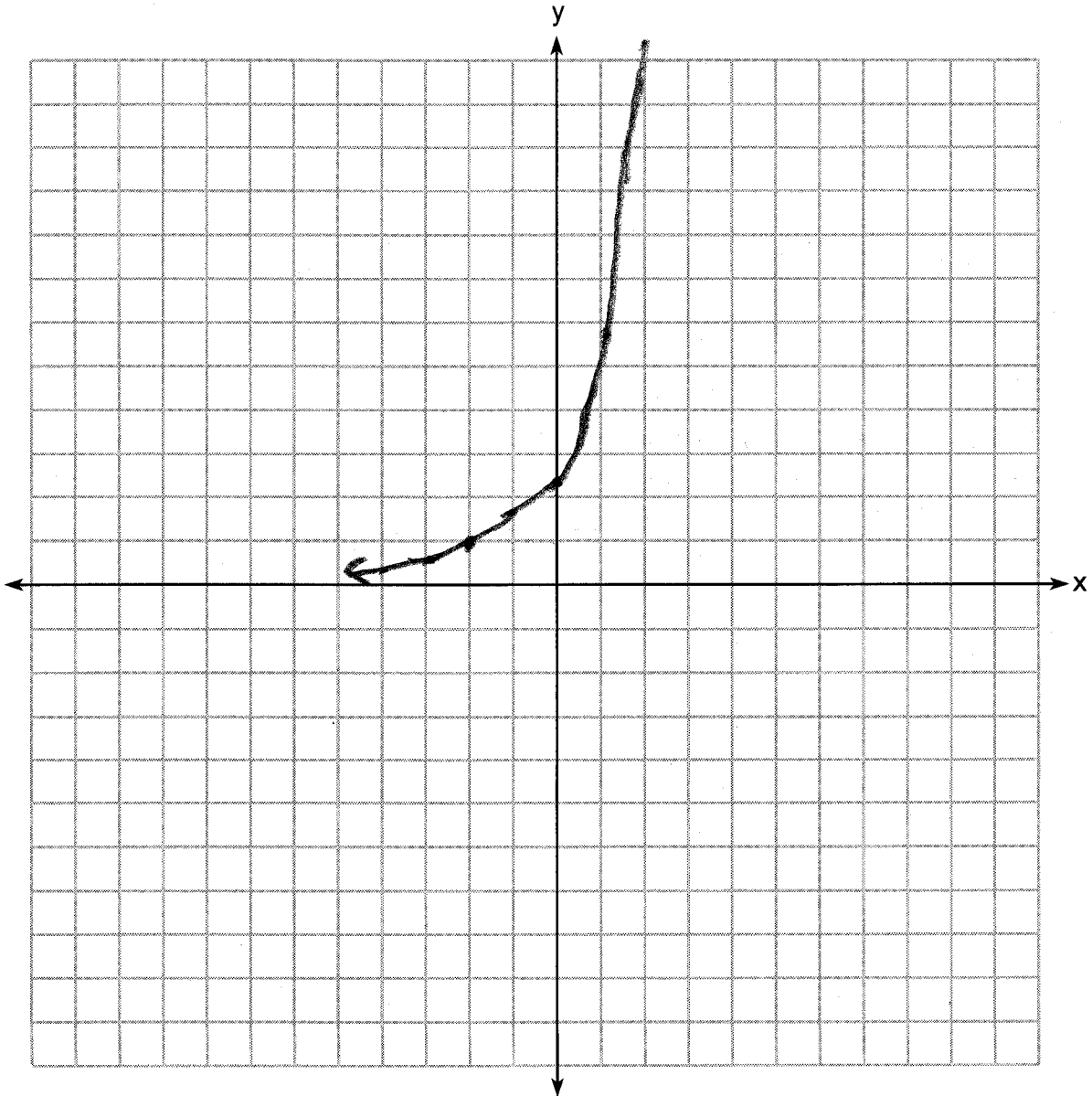
26 Is $x + 3$ a factor of $7x^3 + 27x^2 + 9x - 27$?
Justify your answer.

$$-3 \mid \begin{array}{cccc} 7 & 27 & 9 & -27 \\ & -21 & -18 & 27 \\ \hline 7 & 6 & -9 & 0 \end{array}$$

yes

27 Over the set of integers, factor the expression $2x^4 - 10x^3 + 3x^2 - 15x$ completely.

$2x^3(x-5) + 3x(x-5)$

$(2x^3 + 3x)(x-5)$

$x(2x^2 + 3)(x-5)$

28 The monthly unemployment rate of towns in the United States is approximately normally distributed with a mean rate of 5.2% and a standard deviation of 1.6%. Determine the percentage of towns, to the *nearest integer*, that have a monthly unemployment rate greater than 6%.

$$\text{norm Cdf}(6, 100, 5.2, 1.6) = .31$$

$$31\%$$

29 The function $d(t) = 2\cos\left(\frac{\pi}{6}t\right) + 5$ models the water depth, in feet, at a location in a bay, t hours since the last high tide. Determine the *minimum* water depth of the location, in feet, and justify your answer.

$$2(-1) + 5 = 3$$

30 A brewed cup of coffee contains 130 mg of caffeine. The half-life of caffeine in the bloodstream is 5.5 hours. Write a function, $C(t)$ to represent the amount of caffeine in the bloodstream t hours after drinking one cup of coffee.

$$C(t) = 130(.5)^{\frac{t}{5.5}}$$

31 Markus is a long-distance walker. In one race, he walked 55 miles in t hours and in another race walked 65 miles in $t + 3$ hours. His rates are shown in the equations below.

$$r = \frac{55}{t} \qquad r = \frac{65}{t + 3}$$

Markus walked at an equivalent rate, r, for each race. Determine the number of hours that *each* of the two races took.

$$\frac{55}{t} = \frac{65}{t+3}$$

$$65t = 55t + 165$$

$$10t = 165$$

$$t = 16.5$$

$$t + 3 = 19.5$$

32 Solve the equation $x^2 + 3x + 11 = 0$ algebraically. Express the answer in $a + bi$ form.

$$X = \frac{-3 \pm \sqrt{3^2 - 4(1)(11)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{-35}}{2}$$

$$\frac{-3 \pm i\sqrt{35}}{2}$$

$$-\frac{3}{2} \pm \frac{i\sqrt{35}}{2}$$

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 The population of China, in millions, can be modeled by the function $P(x) = 316.93e^{0.0133x}$, where x is the number of years since 1900.

The population of India since 1900 is summarized in the table below:

Years since 1900	0	10	20	30	40	50	60	70	80	90	100	110	120
Population (millions)	243	254	268	285	324	376.3	450.6	555.1	699	873.3	1056.6	1234.3	1380

Which country's population had a greater average rate of change between 1950 and 2020? Justify your answer.

$$\frac{P(120) - P(50)}{120 - 50} \approx 13.5 \qquad China$$

$$\frac{1380 - 376.3}{120 - 50} \approx 14.3 \qquad India$$

$$India$$

34 In a packaging plant, a machine packs boxes with jars. The machine's manufacturer states that a box is packed, on average, every 42 seconds. To test that claim, the packaging plant randomly selects a sample of 10 boxes and finds the sample mean to be 49.8 seconds.

The company ran a simulation of 1000 trials based on the manufacturer's claim. The approximately normal results are shown below.

Mean = 42.029
SD = 3.105
n = 10
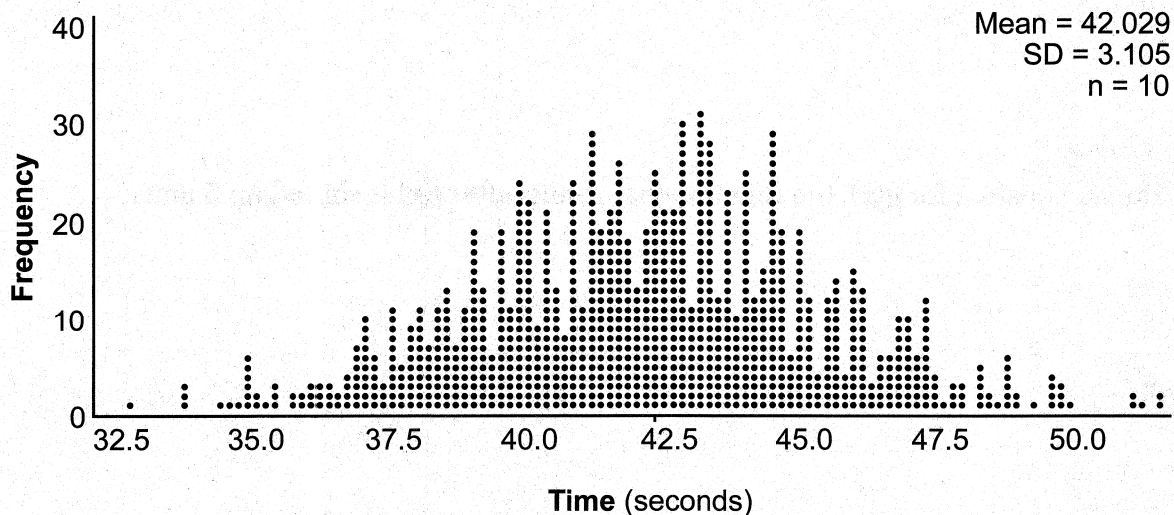
Based on the simulation, determine an interval containing the middle 95% of plausible mean times. Round your answer to the *nearest hundredth*.

$$42.029 \pm 2(3.105)$$
$$35.82 - 48.24$$

Is the time 49.8 seconds unusual? Use statistical evidence to justify your answer.

Yes, because 49.8 falls outside of the confidence interval

35 Consider the function $f(x) = 2^x$.

Is $f(x)$ an even function? Justify your answer.

$$\text{No. } F(-x) = 2^{-x}$$

$$2^{-x} \neq 2^x$$

Write an equation for $g(x)$, the function that results after $f(x)$ is shifted up 5 units.

$$g(x) = f(x) + 5$$

Write an equation for $h(x)$, the inverse of $g(x)$.

$$y = 2^x + 5$$

$$x = 2^y + 5$$

$$\log(x-5) = \log 2^y$$

$$\frac{\log(x-5)}{\log 2} = \frac{y \log 2}{\log 2}$$

$$\log_2(x-5) = g(x)$$

36 Solve the system of equations shown below algebraically:

$$(x - 4)^2 + (y - 1)^2 = 9$$
$$x - y = 6$$

$$y = x - 6$$

$$(x-4)^2 + (x-6-1)^2 = 9$$

$$(x-4)^2 + (x-7)^2 = 9$$

$$x^2 - 8x + 16 + x^2 - 14x + 49 - 9 = 0$$

$$2x^2 - 22x + 56 = 0$$

$$x^2 - 11x + 28 = 0$$

$$(x-7)(x-4) = 0$$

$$x = 7, 4$$

$$7 - y = 6 \qquad 4 - y = 6$$
$$1 = y \qquad\qquad -2 = y$$

$$(7, 1) \qquad (4, -2)$$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Taylor wants to open an investment account with the $1200 she received for her birthday. She has narrowed her choices down to two banks. America's Bank offers 6.4% annual interest compounded quarterly. Barnyard Bank offers 6.35% annual interest compounded continuously.

Write functions for $A(t)$ and $B(t)$ to represent the value of her investment with America's Bank and Barnyard Bank as a function of time, t, in years.

$$A(t) = 1200 \left(1 + \frac{6.4\%}{4}\right)^{4t}$$

$$B(t) = 1200e^{6.35\%t}$$

Taylor would like to invest the $1200 into one bank for ten years making no additional deposits and no withdrawals. With which bank will Taylor earn the most money? Justify your answer.

$$A(10) \approx 2264.28$$
$$B(10) \approx 2264.43$$

$$Barnyard$$

Taylor chooses to invest her money in Barnyard Bank. Algebraically determine how long, to the *nearest tenth of a year*, it will take her initial investment to triple assuming she makes no deposits or withdrawals.

$$\ln 3 = \ln e^{.0635t}$$

$$\frac{\ln 3}{.0635} = \frac{.0635t}{.0635}$$

$$t \approx 17.3$$