ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Tuesday, August 16, 2022 — 12:30 to 3:30 p.m., only

Student Name Mr. Sibol

School Name JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice …

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The Hot and Tasty Coffee chain conducts a survey of its customers at its location at the Staten Island ferry terminal. After the survey is completed, the statistical consultant states that 70% of customers who took the survey said the most important factor in choosing where to get their coffee is how fast they are served. Based on this result, Hot and Tasty Coffee can infer that

(1) most of its customers in New York State care most about being served quickly

(2) coffee drinkers care less about taste and more about being served quickly

(3) most of its customers at the Staten Island ferry terminal care most about being served quickly

(4) most of its customers at transportation terminals and stations care most about being served quickly

2 Given that i is the imaginary unit, the expression $(x - 2i)^2$ is equivalent to

(1) $x^2 + 4$

(2) $x^2 - 4$

(3) $x^2 - 2xi - 4$

(4) $x^2 - 4xi - 4$

$x^2 - 4xi + 4i^2$ (-1)

3 The equation below can be used to model the height of a tide in feet, $H(t)$, on a beach at t hours.

$$H(t) = 4.8\sin\left(\frac{\pi}{6}(t + 3)\right) + 5.1$$

Using this function, the amplitude of the tide is

(1) $\frac{\pi}{6}$

(2) 4.8

(3) 3

(4) 5.1

Use this space for computations.

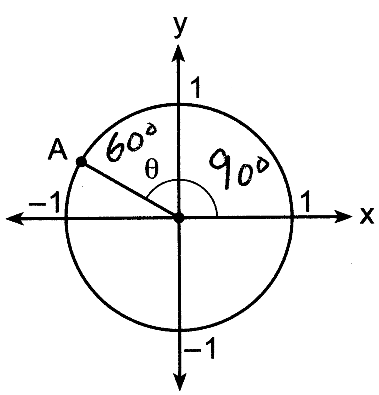
4 In watching auditions for lead singer in a band, Liem became curious as to whether there is an association between how animated the lead singer is and the amount of applause from the audience. He decided to watch each singer and rate the singer on a scale of 1 to 5, where 1 is the least animated and 5 is the most animated. He did this for all 5 nights of auditions and found that the more animated singers did receive louder applause.

The study Liem conducted would be best described as

(1) experimental (3) a sample survey

(2) observational (4) a random assignment

5 In the diagram of a unit circle below, point A, $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, represents the point where the terminal side of θ intersects the unit circle.



What is m$\angle\theta$?

(1) 30° (3) 135°

(2) 120° (4) 150°

6 Consider the function $f(x) = 2x^3 + x^2 - 18x - 9$. Which statement is true?

$0 = 0$

(1) $2x - 1$ is a factor of $f(x)$. (3) $f(3) \neq f\left(-\frac{1}{2}\right)$

(2) $x - 3$ is a factor of $f(x)$. (4) $f\left(\frac{1}{2}\right) = \cancel{0}$ $= -17.5$

$x^2(2x+1) - 9(2x+1)$

$(x^2-9)(2x+1)$

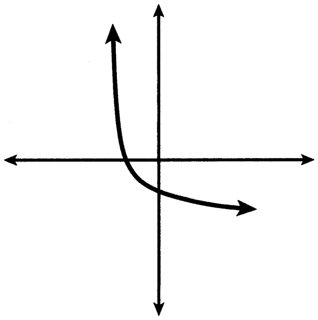
$(x+3)(x-3)(2x+1)$

translate 2 right

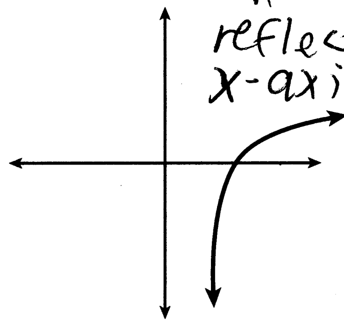
7 Which sketch could represent the function $m(x) = -\log_{100}(x - 2)$?

reflect over x-axis

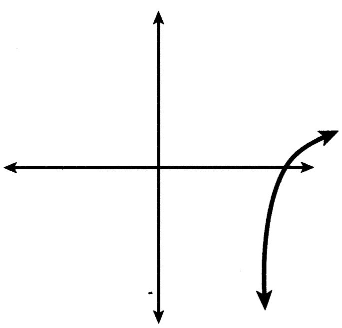
Use this space for computations.



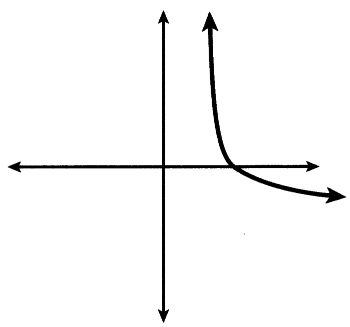
(1)



(3)



(2)



(4)

8 Which equation has roots of $3 + i$ and $3 - i$?

(1) $x^2 - 6x + 10 = 0$ (3) $x^2 - 10x + 6 = 0$

(2) $x^2 + 6x - 10 = 0$ (4) $x^2 + 10x - 6 = 0$

The product of the roots equals $(3+i)(3-i)$

$9 - i^2$

10

$\frac{c}{a} = \frac{10}{1}$

9 A local university has a current enrollment of 12,000 students. The enrollment is increasing continuously at a rate of 2.5% each year. Which logarithm is equal to the number of years it will take for the population to increase to 15,000 students?

(1) $\frac{\ln 1.25}{0.25}$ (3) $\frac{\ln 1.25}{2.5}$

(2) $\frac{\ln 3000}{0.025}$ (4) $\frac{\ln 1.25}{0.025}$

$\frac{15{,}000}{12{,}000} = \frac{12{,}000e^{.025t}}{12{,}000}$

$\ln 1.25 = \ln e^{.025t}$

$\frac{\ln 1.25}{.025} = \frac{.025t}{.025}$

Use this space for computations.

10 What is the total number of points of intersection of the graphs of the equations $y = e^x$ and $xy = 20$?

(1) 1
(2) 2
(3) 3
(4) 0

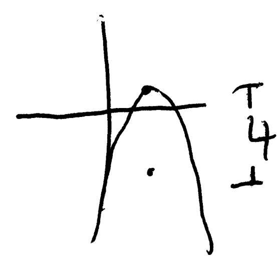
11 The amount of a substance, $A(t)$, in grams, remaining after t days is modeled by $A(t) = 50(0.5)^{\frac{t}{3}}$. Which statement is false?

(1) In 20 days, there is no substance remaining. $A(20) \approx .49$
(2) After two half-lives, there is 25% of the substance remaining. $.5 \cdot .5 = .25$
(3) The amount of the substance remaining can also be modeled by $A(t) = 50(2)^{\frac{-t}{3}}$.
(4) After one week, there is less than 10g of the substance remaining.

$a(7) \approx 9.9$

12 A parabola that has a vertex at (2,1) and a focus of (2,−3) has an equation of

(1) $y = \frac{1}{16}(x - 2)^2 + 1$
(2) $y = -\frac{1}{16}(x + 2)^2 - 1$
(3) $y = -\frac{1}{16}(x - 2)^2 + 1$
(4) $y = -\frac{1}{16}(x - 2)^2 - 3$



Use this space for computations.

13 The expression $\left(a\sqrt[3]{2b^2}\right)\left(\sqrt[3]{4a^2b}\right)$ is equivalent to

(1) $2ab\sqrt[3]{a^2}$ (3) $2ab\sqrt[3]{2a^2}$

(2) $2ab$ (4) $2a^2b\sqrt[3]{2b}$

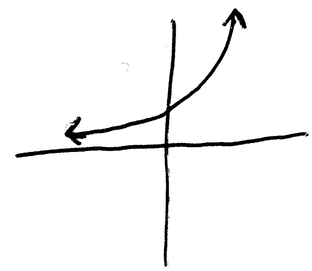
$a\sqrt[3]{8a^2b^3}$

$2ab\sqrt[3]{a^2}$

14 Given $f(x) = 3^{x-1} + 2$, as $x \to -\infty$

(1) $f(x) \to -1$ (3) $f(x) \to 2$

(2) $f(x) \to 0$ (4) $f(x) \to -\infty$



15 For all values of x for which the expression is defined, $\dfrac{x^2 + 3x}{x^2 + 5x + 6}$ is equivalent to

(1) $1 - \dfrac{x}{x+2}$ (3) $\dfrac{3x}{5x+6}$

(2) $\dfrac{x}{x+2}$ (4) $1 + \dfrac{1}{2x+6}$

$$\frac{x(x+3)}{(x+2)(x+3)}$$

16 A recursive formula for the sequence 64, 48, 36, … is

(1) $a_n = 64(0.75)^{n-1}$ NOT RECURSIVE (3) $a_n = 64 + (n-1)(-16)$

(2) $a_1 = 64$ (4) $a_1 = 64$
$a_n = a_{n-1} - 16$ $a_n = 0.75\,a_{n-1}$

geometric

$\dfrac{48}{64} = 0.75$

Use this space for computations.

17 Which expression is equivalent to $\frac{x^3 - 2}{x - 2}$?

(1) x^2 (3) $x^2 - 2$

(2) $x^2 + 2x + 4 + \frac{6}{x - 2}$ (4) $x^2 - 2x + 4 - \frac{10}{x - 2}$

$x^2 + 2x + 4$

$x - 2 \overline{) x^3 - 0x^2 + 0x - 2}$

$x^3 - 2x^2$

$2x^2 + 0x$

$2x^2 - 4x$

$4x - 2$

$4x - 8$

6

18 What is the solution set of the equation $\frac{4}{k^2 - 8k + 12} = \frac{k}{k - 2} + \frac{1}{k - 6}$?

(1) $\{-1, 6\}$ (3) $\{-1\}$

(2) $\{1, -6\}$ (4) $\{1\}$

$\frac{4}{k^2 - 8k + 12} = \frac{k(k-6) + (k-2)}{k^2 - 8k + 12}$

$4 = k^2 - 6k + k - 2$

$0 = k^2 - 5k - 6$

$0 = (k-6)(k+1)$

-1 $\cancel{6}$

19 Given the polynomial identity $x^6 + y^6 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$, which equation must also be true for all values of x and y?

(1) $x^6 + y^6 = x^2(x^4 - x^2y^2 + y^4) + y^2(x^4 - x^2y^2 + y^4)$

(2) $x^6 + y^6 = (x^2 + y^2)(x^2 - y^2)(x^2 - y^2)$ $= (x^2 + y^2)(x^4 - 2x^2y^2 + y^4)$

(3) $(x^3 + y^3)^2 = (x^2 + y^2)(x^4 - x^2y^2 + y^4)$ $x^6 + 2x^3y^3 + y^6 \neq x^6 + y^6$

(4) $(x^6 + y^6) \cancel{\div (x^2 + y^2)} = x^4 - x^2y^2 + y^4$

$x^2 + y^2$

20 Given $p(\theta) = 3\sin\left(\frac{1}{2}\theta\right)$ on the interval $-\pi < \theta < \pi$, the function p

(1) decreases, then increases (3) decreases throughout the interval

(2) increases, then decreases (4) increases throughout the interval

Use this space for computations.

21 A company fired several employees in order to save money. The amount of money the company saved per year over five years following the loss of employees is shown in the table below.

Year	Amount Saved (in dollars)
1	59,000
2	64,900
3	71,390
4	78,529
5	86,381.9

$\frac{649}{590} = 1.1$

Which expression determines the total amount of money saved by the company over 5 years?

(1) $\dfrac{59{,}000 - 59{,}000(1.1)^5}{1 - 1.1}$

(2) $\dfrac{59{,}000 - 59{,}000(\cancel{0}.1)^5}{1 - \cancel{0}.1}$

(3) $\sum_{n=1}^{5} 59{,}000(1.1)^{n-1}$

(4) $\sum_{n=1}^{5} 59{,}000(\cancel{0}.1)^{n-1}$

22 A rush-hour commuter train has arrived on time 64 of its first 80 days. As arrivals continue, which equation can be used to find x, the number of consecutive days that the train must arrive on schedule to raise its on-time performance rate to 90%?

(1) $\dfrac{64}{80 + x} = \dfrac{90}{100}$

(2) $\dfrac{64 + x}{80 + x} = \dfrac{90}{100}$

(3) $\dfrac{64 + x}{80} = \dfrac{90}{100}$

(4) $\dfrac{x}{80 + x} = \dfrac{90}{100}$

Use this space for computations.

23 Given $f(x) = -\frac{2}{5}x + 4$, which statement is true of the inverse function $f^{-1}(x)$?

(1) $f^{-1}(x)$ is a line with slope $-\frac{5}{2}$.

(2) $f^{-1}(x)$ is a line with slope $-\frac{2}{5}$.

(3) $f^{-1}(x)$ passes through the point $(6, -5)$.

(4) $f^{-1}(x)$ has a y-intercept at $(0, -4)$.

$(0, 10)$

$x = \frac{-2y}{5} + 4$

$5x = -2y + 20$

$\frac{2y}{2} = \frac{-5x}{2} + \frac{20}{2}$

$y = -\frac{5}{2}x + 10$

$-\frac{5}{2}(6) + 10$

$-15 + 10$

-5

24 The amount of a substance, $A(t)$, that remains after t days can be given by the equation $A(t) = A_0(0.5)^{\frac{t}{0.0803}}$, where A_0 represents the initial amount of the substance. An equivalent form of this equation is

(1) $A(t) = A_0(0.000178)^t$

(2) $A(t) = A_0(0.945861)^t$

(3) $A(t) = A_0(0.04015)^t$

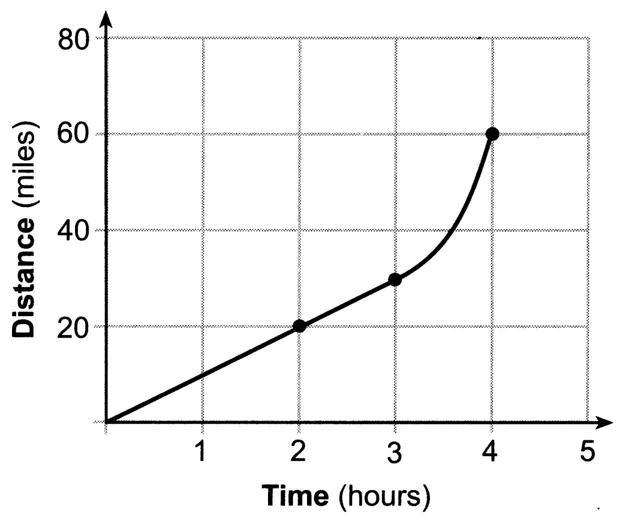
(4) $A(t) = A_0(1.08361)^t$

$0.5^{\frac{1}{.0803}} \approx .000178$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Determine the average rate of change, in mph, from 2 to 4 hours on the graph shown below.



$$\frac{60-20}{4-2} = \frac{40}{2} = 20$$

26 Factor the expression $x^3 - 2x^2 - 9x + 18$ completely.

$x^2(x-2) - 9(x-2)$

$(x^2-9)(x-2)$

$(x+3)(x-3)(x-2)$

27 Solve algebraically for all values of x:

$$\sqrt{4x+1} = 11 - x$$

$4x+1 = 121 - 22x + x^2$

$0 = x^2 - 26x + 120$

$0 = (x-6)(x-20)$

$x = 6, \cancel{20}$

28 Given that $\left(\dfrac{y^{\frac{17}{8}}}{y^{\frac{5}{4}}}\right)^{-4} = y^n$, where $y > 0$, determine the value of n.

$$\left(\frac{y^{\frac{17}{8}}}{y^{\frac{10}{8}}}\right)^{-4} = y^n$$

$$\left(y^{\frac{7}{8}}\right)^{-4} = y^n$$

$$y^{-\frac{7}{2}} = y^n$$

$$n = -7/2$$

29 Given $\cos A = \frac{3}{\sqrt{10}}$ and $\cot A = -3$, determine the value of $\sin A$ in radical form.

$$\cot A = \frac{\cos A}{\sin A}$$

$$-3 = \frac{3/\sqrt{10}}{\sin A}$$

$$\sin A = \frac{3}{-3\sqrt{10}}$$

$$= \frac{-1}{\sqrt{10}}$$

30 According to a study done at a hospital, the average weight of a newborn baby is 3.39 kg, with a standard deviation of 0.55 kg. The weights of all the newborns in this hospital closely follow a normal distribution. Last year, 9256 babies were born at this hospital. Determine, to the *nearest integer*, approximately how many babies weighed more than 4 kg.

$.133696 \cdot 9256 \approx 1{,}237$

31 The table below shows the results of gender and music preference. Based on these data, determine if the events "the person is female" and "the person prefers classic rock" are independent of each other. Justify your answer.

	Rap	Techno	Classic Rock	Classical	
Male	39	17	42	12	110
Female	17	37	36	15	105
			78		

$$P(F) = \frac{105}{110+105} = \frac{21}{43}$$

$$P(F \mid CR) = \frac{36}{42+36} = \frac{6}{13}$$

Since the probability of being female does not equal the probability of being female, given the person prefers classic rock, the events are not independent

32 Algebraically determine the solution set for the system of equations below.

$$y = 2x^2 - 7x + 4$$
$$y = 11 - 2x$$

$2x^2 - 7x + 4 = 11 - 2x$

$2x^2 - 5x - 7 = 0$

$(2x-7)(x+1) = 0$

$x = \frac{7}{2}, -1$

$y = 11 - 2\left(\frac{7}{2}\right)$ $= 4$

$y = 11 - 2(-1)$ $= 13$

$\left(\frac{7}{2}, 4\right)$ $(-1, 13)$

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 When observed by researchers under a microscope, a smartphone screen contained approximately 11,000 bacteria per square inch. Bacteria, under normal conditions, double in population every 20 minutes.

a) Assuming an initial value of 11,000 bacteria, write a function, $p(t)$, that can be used to model the population of bacteria, p, on a smartphone screen, where t represents the time in minutes after it is first observed under a microscope.

$$p(t) = 11000(2)^{\frac{t}{20}}$$

b) Using $p(t)$ from part a, determine algebraically, to the *nearest hundredth of a minute*, the amount of time it would take for a smartphone screen that was not touched or cleaned to have a population of 1,000,000 bacteria per square inch.

$$1000000 = 11000(2)^{t/3}$$

$$\log \frac{1000600}{11000} = \log 2^{\frac{t}{3}}$$

$$\log\left(\frac{1000}{11}\right) = \frac{t \log 2}{3}$$

$$\frac{3\log\left(\frac{1000}{11}\right)}{\log 2} = t$$

$$19.52 \approx t$$

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

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$$\frac{1000000}{11000} = \frac{11000(2)^{t/20}}{11{,}000}$$

$$\log \frac{1000}{11} = \log 2^{t/20}$$

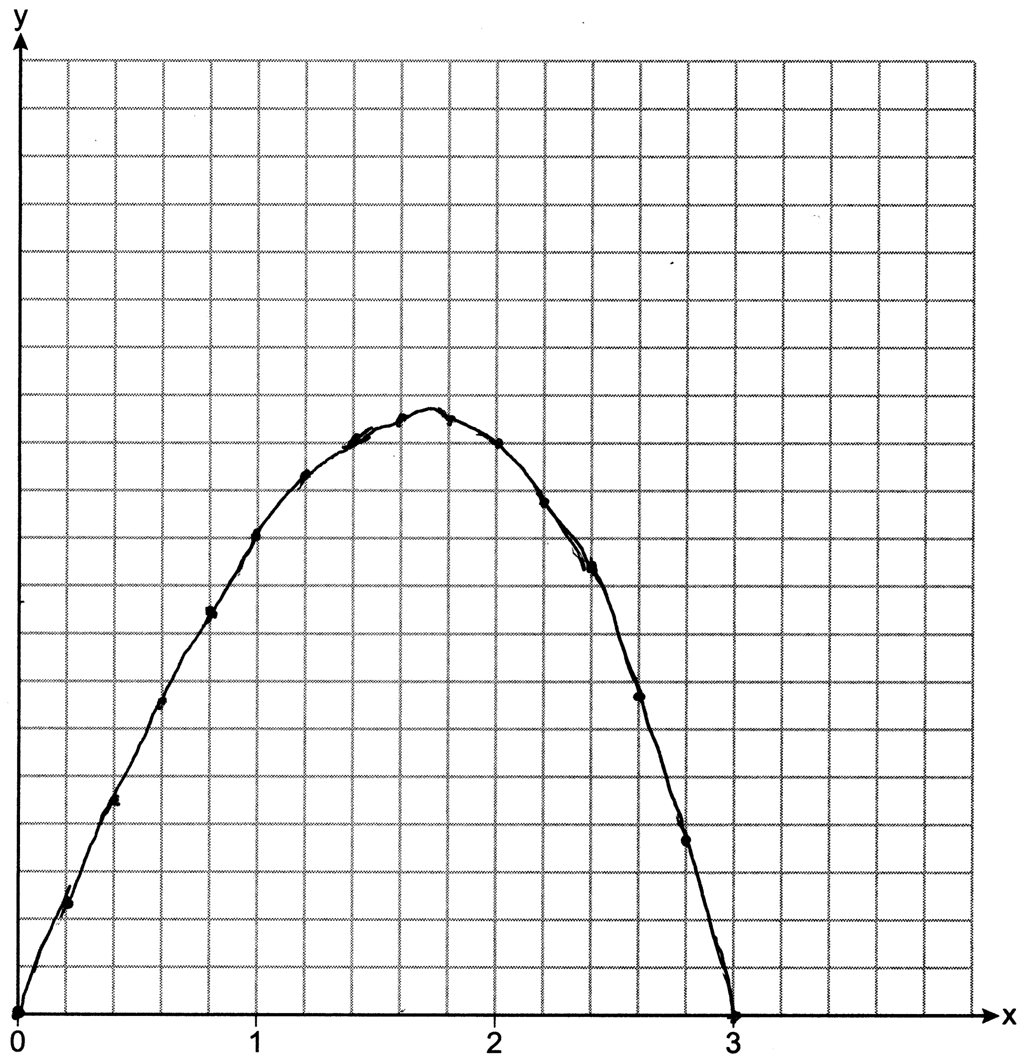
$$\log \frac{1000}{11} = \frac{t \log 2}{20}$$

$$\frac{20 \log \frac{1000}{11}}{\log 2} = t$$

$$130.13 \approx t$$

34 The function $v(x) = x(3 - x)(x + 4)$ models the volume, in cubic inches, of a rectangular solid for $0 \le x \le 3$.

Graph $y = v(x)$ over the domain $0 \le x \le 3$.



To the *nearest tenth of a cubic inch*, what is the maximum volume of the rectangular solid?

12.6

35 Given $f(x) = 3x^3 - 4x^2 + 2x - 1$ and $g(x) = x - 4$, state the quotient and remainder of $\frac{f(x)}{g(x)}$, in the form $q(x) + \frac{r(x)}{g(x)}$.

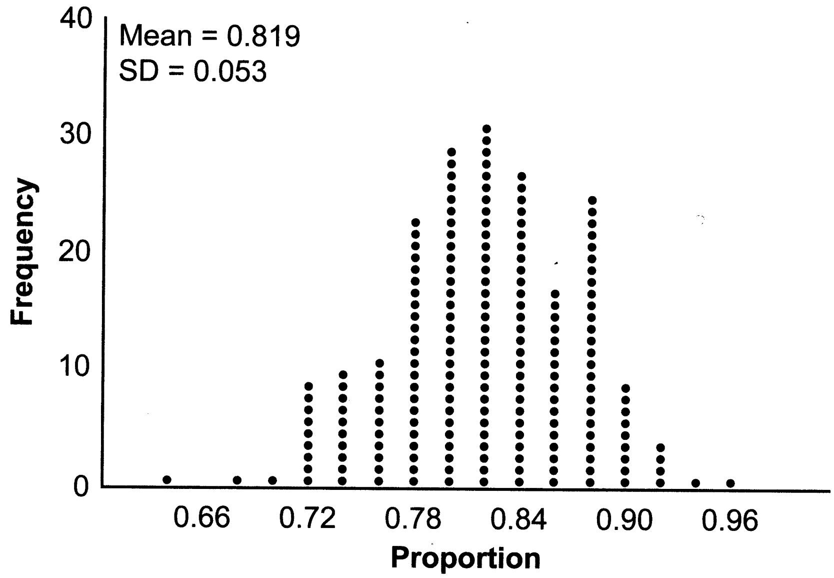
$$\begin{array}{r|rrrr} 4 & 3 & -4 & 2 & -1 \\ & & 12 & 32 & 136 \\ \hline & 3 & 8 & 34 & 135 \end{array}$$

$$3x^2 + 8x + 34 + \frac{135}{x-4}$$

Is $x = 4$ a root of $f(x)$? Explain your answer.

No, because $\frac{f(x)}{g(x)}$ has a remainder

36 State officials claim 82% of a community want to repeal the 30 mph speed limit on an expressway. A community organization devises a simulation based on the claim that 82% of the community supports the repeal. Each dot on the graph below represents the proportion of community members who support the repeal. The graph shows 200 simulated surveys, each of sample size 60.



Based on the simulation, determine an interval containing the middle 95% of plausible proportions. Round your answer to the *nearest thousandth*.

$.819 \pm 2(.053)$

$.713 - .925$

The community organization conducted its own sample survey of 60 people and found 70% supported the repeal. Based on the results of the simulation, explain why the organization should question the State officials' claim.

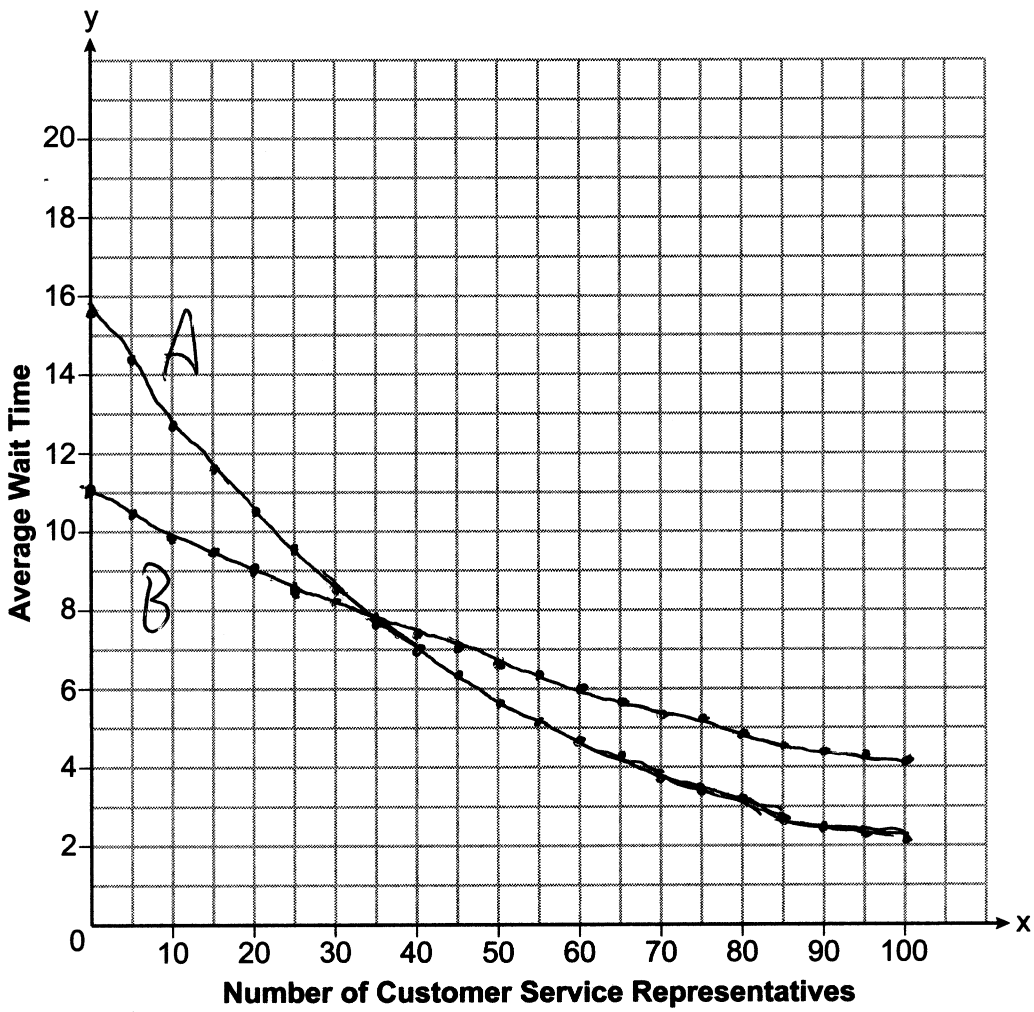
70% does not fall within the interval

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 A technology company is comparing two plans for speeding up its technical support time. Plan A can be modeled by the function $A(x) = 15.7(0.98)^x$ and plan B can be modeled by the function $B(x) = 11(0.99)^x$ where x is the number of customer service representatives employed by the company and $A(x)$ and $B(x)$ represent the average wait time, in minutes, of each customer.

Graph $A(x)$ and $B(x)$ in the interval $0 \leq x \leq 100$ on the set of axes below.



Question 37 is continued on the next page.

Question 37 continued

To the *nearest integer*, solve the equation $A(x) = B(x)$.

35

Determine, to the *nearest minute*, $B(100) - A(100)$. Explain what this value represents in the given context.

2 represents the difference, in minutes, of the averge wait time when there are 100 CSRs, of the plans.