ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, August 16, 2017 — 12:30 to 3:30 p.m., only

Student Name _Mr. Sibol_

School Name _JMAP_

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II**, **III**, and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The function $f(x) = \dfrac{x-3}{x^2 + 2x - 8}$ is undefined when x equals

(1) 2 or −4   (3) 3, only

(2) 4 or −2   (4) 2, only

Handwritten work:
$x^2 + 2x - 8 = 0$
$(x+4)(x-2) = 0$
$-4 \quad 2$

2 Which expression is equivalent to $(3k - 2i)^2$, where i is the imaginary unit?

(1) $9k^2 - 4$ (3) $9k^2 - 12ki - 4$

(2) $9k^2 + 4$ (4) $9k^2 - 12ki + 4$

Handwritten work:
$9k^2 - 12ki + 4i^2$
$9k^2 - 12ki - 4$

3 The roots of the equation $x^2 + 2x + 5 = 0$ are

(1) −3 and 1   (3) $-1 + 2i$ and $-1 - 2i$

(2) −1, only   (4) $-1 + 4i$ and $-1 - 4i$

Handwritten work:
$x^2 + 2x + 1 = -5 + 1$
$(x+1)^2 = -4$
$x + 1 = \pm 2i$
$x = -1 \pm 2i$

4 The solution set for the equation $\sqrt{x + 14} = \sqrt{2x + 5} + 1$ is

(1) {−6}   (3) {18}

(2) {2}   (4) {2, 22}

Handwritten work:
$x + 14 = 2x + 5 + 2\sqrt{2x+5} + 1$
$-x + 8 = 2\sqrt{2x+5}$
$x^2 - 16x + 64 = 8x + 20$
$x^2 - 24x + 44 = 0$
$(x - 22)(x - 2) = 0$
$\cancel{22} \quad 2$

5 As x increases from 0 to $\frac{\pi}{2}$, the graph of the equation $y = 2 \tan x$ will

(1) increase from 0 to 2   (3) increase without limit

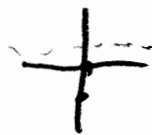(2) decrease from 0 to −2   (4) decrease without limit

Handwritten work:
$\sqrt{22 + 14} - \sqrt{2(22) + 5} = 1$
$6 - 7 \neq 1$

6 Which equation represents a parabola with the focus at $(0, -1)$ and the directrix $y = 1$?

(1) $x^2 = -8y$ (3) $x^2 = 8y$

(2) $x^2 = -4y$ (4) $x^2 = 4y$

(handwritten, right margin:)

Vertex is $(0,0)$
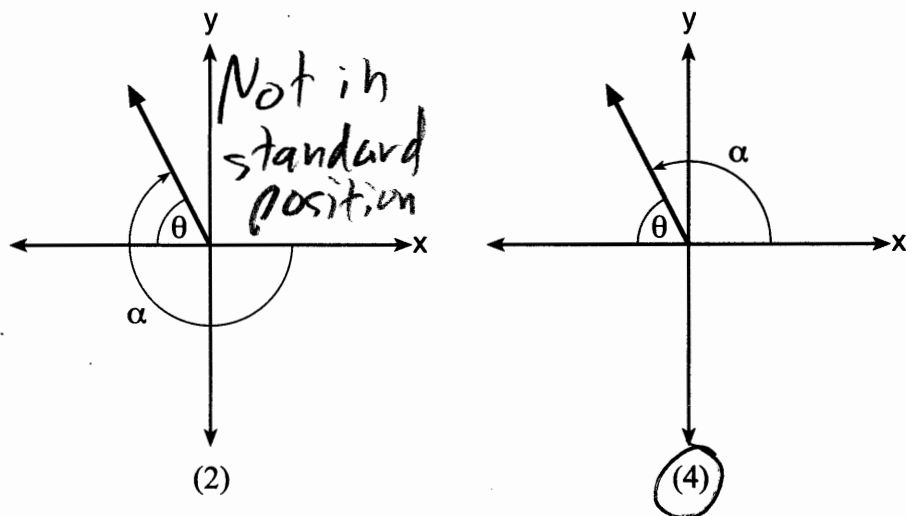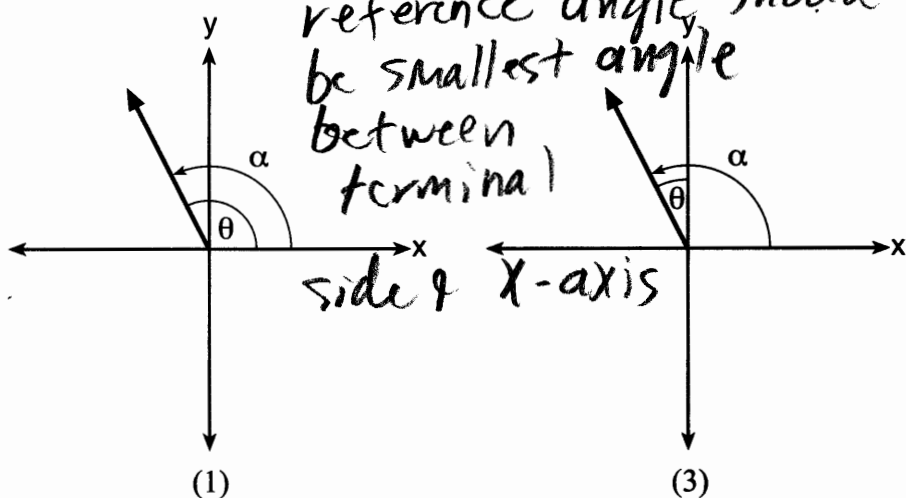
$p = 1$

$y = \frac{-1}{4p}(x-h)^2 + k$

$y = \frac{-1}{4(1)}(x-0)^2 + 0$

$y = \frac{-1}{4}x^2$

$-4y = x^2$

7 Which diagram represents an angle, α, measuring $\frac{13\pi}{20}$ radians drawn in standard position, and its reference angle, θ?

(handwritten:) reference angle, should be smallest angle between terminal side & x-axis

(1)

(3)

(handwritten:) Not in standard position

(2)

(4)

8 What are the zeros of $P(m) = (m^2 - 4)(m^2 + 1)$?

(1) 2 and -2, only

(2) 2, -2, and -4

(3) -4, i, and $-i$

(4) 2, -2, i, and $-i$

9 The value of a new car depreciates over time. Greg purchased a new car in June 2011. The value, V, of his car after t years can be modeled by the equation $\log_{0.8}\left(\frac{V}{17000}\right) = t$.

What is the average decreasing rate of change per year of the value of the car from June 2012 to June 2014, to the *nearest ten dollars per year*?

(1) 1960

(2) 2180

(3) 2450

(4) 2770

$.8^t , \dfrac{V}{17000}$

$V = 17,000\,(.8)^t$

$\dfrac{17,000(.8)^3 - 17,000(.8)^1}{3-1}$

-2448

10 Iridium-192 is an isotope of iridium and has a half-life of 73.83 days. If a laboratory experiment begins with 100 grams of Iridium-192, the number of grams, A, of Iridium-192 present after t days would be

$A = 100\left(\frac{1}{2}\right)^{\frac{t}{73.83}}$.

Which equation approximates the amount of Iridium-192 present after t days?

(1) $A = 100\left(\frac{73.83}{2}\right)^t$

(2) $A = 100\left(\frac{1}{147.66}\right)^t$

(3) $A = 100(0.990656)^t$

(4) $A = 100(0.116381)^t$

$\left(\frac{1}{2}\right)^{\frac{1}{73.83}} \approx .990656$

11 The distribution of the diameters of ball bearings made under a given manufacturing process is normally distributed with a mean of 4 cm and a standard deviation of 0.2 cm. What proportion of the ball bearings will have a diameter less than 3.7 cm?

(1) 0.0668

(2) 0.4332

(3) 0.8664

(4) 0.9500

Use calculator

12 A polynomial equation of degree three, $p(x)$, is used to model the volume of a rectangular box. The graph of $p(x)$ has x intercepts at -2, 10, and 14. Which statements regarding $p(x)$ could be true?

A. The equation of $p(x) = (x - 2)(x + 10)(x + 14)$.
B. The equation of $p(x) = -(x + 2)(x - 10)(x - 14)$.
C. The maximum volume occurs when $x = 10$.
D. The maximum volume of the box is approximately 56.

(1) A and C (3) B and C
(2) A and D (4) B and D

[handwritten: $-(x+2)(x-10)(x-14)$; V=0; at x=12.1, using graphing calculator; B and D circled (4)]

13 Which expression is equivalent to $\frac{4x^3 + 9x - 5}{2x - 1}$, where $x \neq \frac{1}{2}$?

(1) $2x^2 + x + 5$ (3) $2x^2 - x + 5$
(2) $2x^2 + \frac{11}{2} + \frac{1}{2(2x - 1)}$ (4) $2x^2 - x + 4 + \frac{1}{2x - 1}$

[handwritten long division: $2x-1)\overline{4x^3+0x^2+9x-5}$ giving $2x^2+x+5$, circled (1)]

14 The inverse of the function $f(x) = \frac{x + 1}{x - 2}$ is

(1) $f^{-1}(x) = \frac{x + 1}{x + 2}$ (3) $f^{-1}(x) = \frac{x + 1}{x - 2}$
(2) $f^{-1}(x) = \frac{2x + 1}{x - 1}$ (4) $f^{-1}(x) = \frac{x - 1}{x + 1}$

[handwritten: $x=\frac{y+1}{y-2}$; $xy-2x=y+1$; $xy-y=2x+1$; $y(x-1)=2x+1$; $\frac{2x+1}{x-1}$; (2) circled]

15 Which expression has been rewritten correctly to form a true statement?

(1) $(x + 2)^2 + 2(x + 2) - 8 = (x + 6)x$
(2) $x^4 + 4x^2 + 9x^2y^2 - 36y^2 = (x + 3y)^2(x - 2)^2$
(3) $x^3 + 3x^2 - 4xy^2 - 12y^2 = (x - 2y)(x + 3)^2$
(4) $(x^2 - 4)^2 - 5(x^2 - 4) - 6 = (x^2 - 7)(x^2 - 6)$

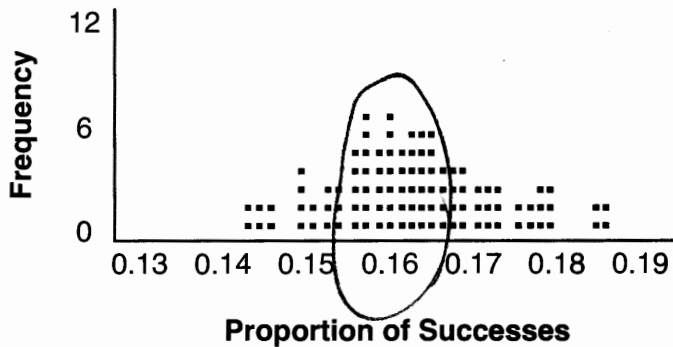*[handwritten: let y=x+2; y^2+2y-8; $(y+4)(y-2)$; $(x+2+4)(x+2-2)$; $(x+6)x$; (1) circled]*

Use this space for computations.

Algebra II – August '17   [5]   [OVER]

16 A study conducted in 2004 in New York City found that 212 out of 1334 participants had hypertension. Kim ran a simulation of 100 studies based on these data. The output of the simulation is shown in the diagram below.

At a 95% confidence level, the proportion of New York City residents with hypertension and the margin of error are closest to

(1) proportion ≈ .16; margin of error ≈ .01

(2) proportion ≈ .16; margin of error ≈ .02

(3) proportion ≈ .01; margin of error ≈ .16

(4) proportion ≈ .02; margin of error ≈ .16

Margin of Error

$$z\sqrt{\frac{p(1-p)}{n}}$$

$$1.96\sqrt{\left(\frac{(.16)(.84)}{1334}\right)}$$

.02

17 Which scenario is best described as an observational study?

(1) For a class project, students in Health class ask every tenth student entering the school if they eat breakfast in the morning. *survey*

(2) A social researcher wants to learn whether or not there is a link between attendance and grades. She gathers data from 15 school districts.

(3) A researcher wants to learn whether or not there is a link between children's daily amount of physical activity and their overall energy level. During lunch at the local high school, she distributed a short questionnaire to students in the cafeteria. *survey*

(4) Sixty seniors taking a course in Advanced Algebra Concepts are randomly divided into two classes. One class uses a graphing calculator all the time, and the other class never uses graphing calculators. A guidance counselor wants to determine whether there is a link between graphing calculator use and students' final exam grades. *controlled experiment*
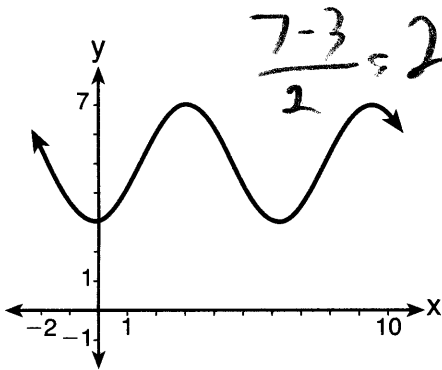
18 Which sinusoid has the greatest amplitude?

$\frac{7-3}{2} = 2$

(1)

(3)

$\frac{3--5}{2} = 4$

$y = ③\sin(\theta-3) + 5$

(2)
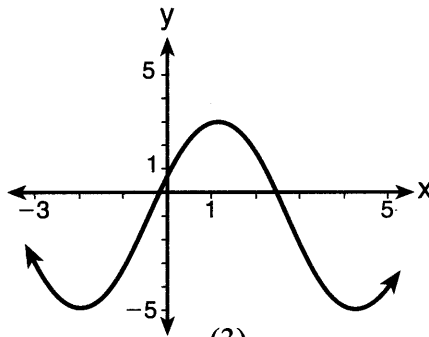
$y = -⑤\sin(\theta-1) - 3$

④

19 Consider the system shown below.

$$2x - y = 4$$
$$(x + 3)^2 + y^2 = 8$$

The two solutions of the system can be described as

(1) both imaginary   (3) both rational

(2) both irrational   (4) one rational and one irrational

$(x+3)^2 + (2x-4)^2 = 8$

$x^2+6x+9 + 4x^2-16x+16 = 8$

$5x^2 -10x +17 = 0$

b^2-4ac

$100 - 4(5)(17) < 0$

20 Which binomial is *not* a factor of the expression $x^3 - 11x^2 + 16x + 84$?

(1) $x + 2$ (3) $x - 6$

(2) $x + 4$ (4) $x - 7$

$-4 \,|\, \begin{array}{cccc} 1 & -11 & 16 & 84 \\ & -4 & 60 & -304 \end{array}$
$\overline{\quad 1 \; -15 \; 76}$

21 A ball is dropped from a height of 32 feet. It bounces and rebounds 80% of the height from which it was falling. What is the total downward distance, in feet, the ball traveled up to the 12th bounce?

(1) 29   (3) 120

(2) 58   (4) 149

$d = 32(.8)^{b-1}$

$S_n = \frac{a_1 - a_1 r^n}{1-r}$

$\frac{32-32(.8)^{12}}{1-.8} \approx 149$

Algebra II – August '17   [7]   [OVER]

22 A public opinion poll was conducted on behalf of Mayor Ortega's reelection campaign shortly before the election. 264 out of 550 likely voters said they would vote for Mayor Ortega; the rest said they would vote for his opponent.

Use this space for computations.

Which statement is *least* appropriate to make, according to the results of the poll?

(1) There is a 48% chance that Mayor Ortega will win the election.

(2) The point estimate ($\hat{p}$) of voters who will vote for Mayor Ortega is 48%.

(3) It is most likely that between 44% and 52% of voters will vote for Mayor Ortega.

(4) Due to the margin of error, an inference cannot be made regarding whether Mayor Ortega or his opponent is most likely to win the election.

23 What does $\left(\dfrac{-54x^9}{y^4}\right)^{\frac{2}{3}}$ equal?

(1) $\dfrac{9ix^6 \sqrt[3]{4}}{y \sqrt[3]{y^2}}$

(3) $\dfrac{9x^6 \sqrt[3]{4}}{y \sqrt[3]{y}}$

(2) $\dfrac{9ix^6 \sqrt[3]{4}}{y^2 \sqrt[3]{y^2}}$

(4) $\dfrac{9x^6 \sqrt[3]{4}}{y^2 \sqrt[3]{y^2}}$

24 The Rickerts decided to set up an account for their daughter to pay for her college education. The day their daughter was born, they deposited $1000 in an account that pays 1.8% compounded annually. Beginning with her first birthday, they deposit an additional $750 into the account on each of her birthdays. Which expression correctly represents the amount of money in the account n years after their daughter was born?

(1) $a_n = 1000(1.018)^n + 750$

(2) $a_n = 1000(1.018)^n + 750n$

(3) $a_0 = 1000$
$a_n = a_{n-1}(1.018) + 750$

(4) $a_0 = 1000$
$a_n = a_{n-1}(1.018) + 750n$

Algebra II – August '17   [8]

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Explain how $(-8)^{\frac{4}{3}}$ can be evaluated using properties of rational exponents to result in an integer answer.

Rewrite $\frac{4}{3}$ as $\frac{1}{3} \cdot \frac{4}{1}$; using the power of a power rule

26 A study was designed to test the effectiveness of a new drug. Half of the volunteers received the drug. The other half received a sugar pill. The probability of a volunteer receiving the drug and getting well was 40%. What is the probability of a volunteer getting well, given that the volunteer received the drug?

$$P(W \text{ and } D) = P(D) \cdot P(W/D)$$

$$\frac{.4}{.5} = \frac{.8 \cdot P(W/D)}{.8}$$

$$.8 =$$

27 Verify the following Pythagorean identity for all values of x and y:

$$(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$$

$$x^4 + 2x^2y^2 + y^4 = x^4 - 2x^2y^2 + y^4 + 4x^2y^2$$

$$x^4 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4$$

28 Mrs. Jones had hundreds of jelly beans in a bag that contained equal numbers of six different flavors. Her student randomly selected four jelly beans and they were all black licorice. Her student complained and said "What are the odds I got all of that kind?" Mrs. Jones replied, "simulate rolling a die 250 times and tell me if four black licorice jelly beans is unusual."

Explain how this simulation could be used to solve the problem.

Since there are six different flavors, each flavor can be assigned a number, 1-6. Use the simulation to see if the same number is rolled 4 times in a row & how many times.

29 While experimenting with her calculator, Candy creates the sequence 4, 9, 19, 39, 79, ...

Write a recursive formula for Candy's sequence.

$$a_1 = 4$$
$$a_n = 2a_{n-1} + 1$$

Determine the eighth term in Candy's sequence.

1	2	3	4	5	6	7	8
4	9	19	39	79	159	319	639

30 In New York State, the minimum wage has grown exponentially. In 1966, the minimum wage was $1.25 an hour and in 2015, it was $8.75. Algebraically determine the rate of growth to the nearest percent.

$$2015$$
$$-1966$$
$$\overline{49}$$

$$8.75 = 1.25(x)^{49}$$

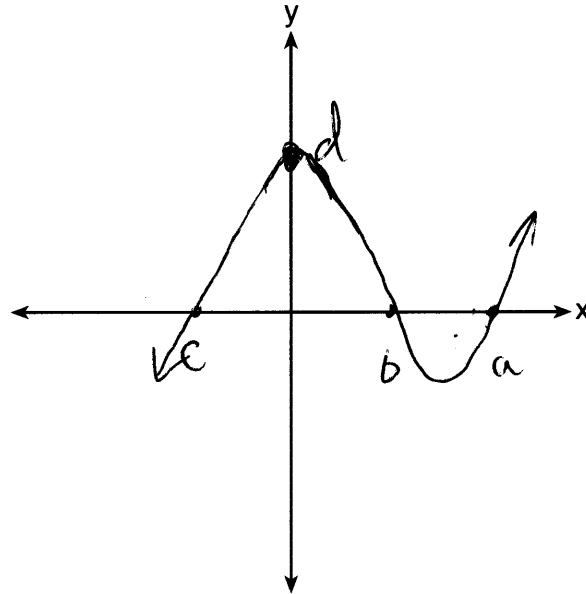$$\sqrt[49]{7} = \sqrt[49]{x^{49}}$$

$$x = 1.0405$$

$$4\% \text{ growth}$$

31 Algebraically determine whether the function $j(x) = x^4 - 3x^2 - 4$ is odd, even, or neither.

$$j(-x) = (-x)^4 - 3(-x)^2 - 4$$

$$= x^4 - 3x^2 - 4$$

Since $j(x) = j(-x)$, $j(x)$ is even

32 On the axes below, sketch a possible function $p(x) = (x - a)(x - b)(x + c)$, where a, b, and c are positive, $a > b$, and $p(x)$ has a positive y-intercept of d. Label all intercepts.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve for all values of p: $\dfrac{3p}{p-5} - \dfrac{2}{p+3} = \dfrac{p}{p+3}$

$$\frac{3p}{p-5} = \frac{p+2}{p+3}$$

$$3p^2 + 9p = p^2 - 3p - 10$$

$$2p^2 + 12p + 10 = 0$$

$$p^2 + 6p + 5 = 0$$

$$(p+5)(p+1) = 0$$

$$p = -5, -1$$

34 Simon lost his library card and has an overdue library book. When the book was 5 days late, he owed $2.25 to replace his library card and pay the fine for the overdue book. When the book was 21 days late, he owed $6.25 to replace his library card and pay the fine for the overdue book.

Suppose the total amount Simon owes when the book is n days late can be determined by an arithmetic sequence. Determine a formula for a_n, the nth term of this sequence.

$$\frac{6.25 - 2.25}{21 - 5} = \frac{4}{16} = \$.25 \text{ fine/day}$$

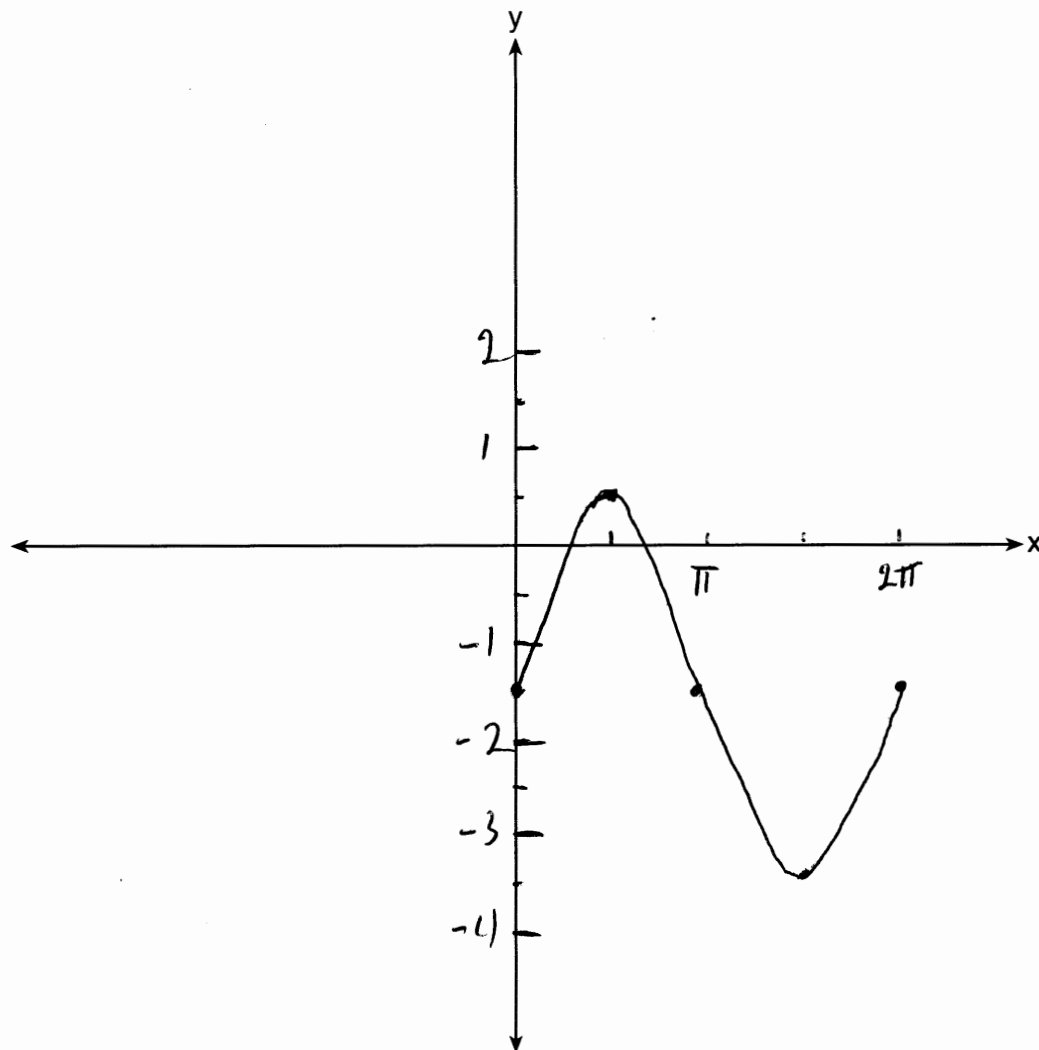$$2.25 - 5(.25) = \$1 \text{ replacement fee}$$

$$a_n = a_1 + (n-1)d$$
$$a_n = 1.25 + (n-1)(.25)$$

Use the formula to determine the amount of money, in dollars, Simon needs to pay when the book is 60 days late.

$$a_{60} = 1.25 + (60-1)(.25) = \frac{5}{4} + \frac{59}{4}$$
$$= \$16$$

35 a) On the axes below, sketch *at least one* cycle of a sine curve with an amplitude of 2, a midline at $y = -\frac{3}{2}$, and a period of 2π.

b) Explain any differences between a sketch of $y = 2 \sin\left(x - \frac{\pi}{3}\right) - \frac{3}{2}$ and the sketch from part *a*.

part a sketch is shifted $\frac{\pi}{3}$ units right

36 Using a microscope, a researcher observed and recorded the number of bacteria spores on a large sample of uniformly sized pieces of meat kept at room temperature. A summary of the data she recorded is shown in the table below.

Hours (x)	Average Number of Spores (y)
0	4
0.5	10
1	15
2	60
3	260
4	1130
6	16,380

Using these data, write an exponential regression equation, rounding all values to the *nearest thousandth*.

$$y = 4.168(3.981)^x$$

The researcher knows that people are likely to suffer from food-borne illness if the number of spores exceeds 100. Using the exponential regression equation, determine the maximum amount of time, to the *nearest quarter hour*, that the meat can be kept at room temperature safely.

$$\frac{\log\left(\frac{100}{4.168}\right)}{\log 3.981} = x \quad \frac{\log 4.168(3.981)^x}{4.168}$$

$$x \approx 2.25$$

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 The value of a certain small passenger car based on its use in years is modeled by $V(t) = 28482.698(0.684)^t$, where $V(t)$ is the value in dollars and t is the time in years. Zach had to take out a loan to purchase the small passenger car. The function $Z(t) = 22151.327(0.778)^t$, where $Z(t)$ is measured in dollars, and t is the time in years, models the unpaid amount of Zach's loan over time.

Graph $V(t)$ and $Z(t)$ over the interval $0 \le t \le 5$, on the set of axes below.
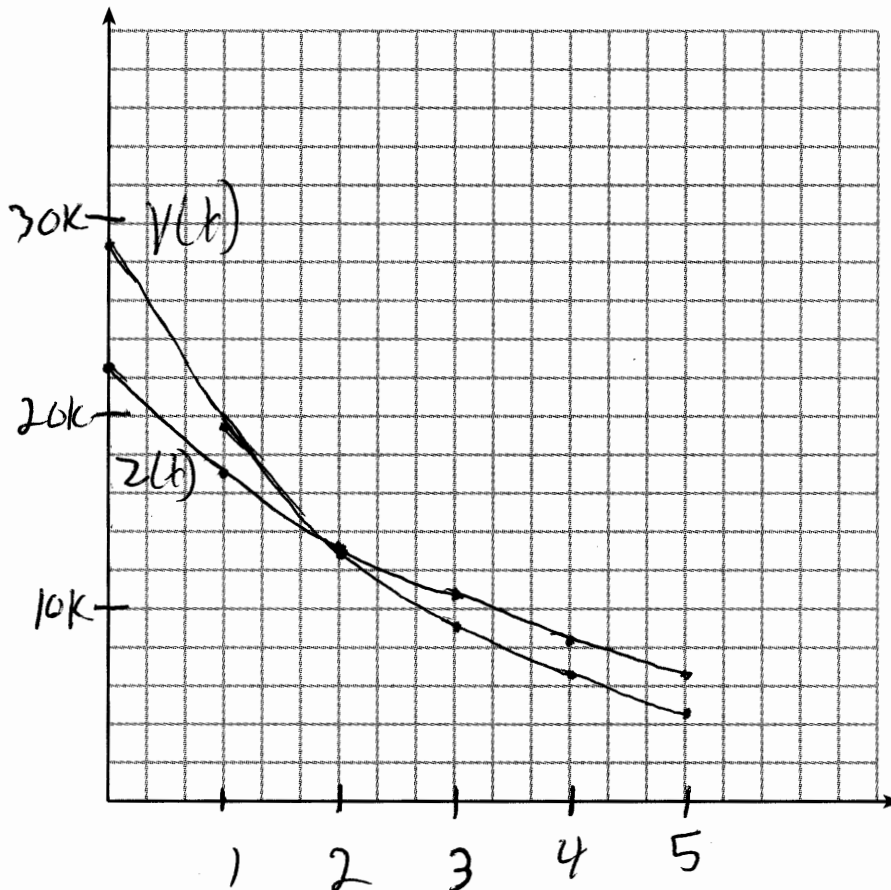
State when $V(t) = Z(t)$, to the *nearest hundredth*, and interpret its meaning in the context of the problem.

$t \approx 1.95$

At 1.95 years, the value of the car equals the loan balance

Zach takes out an insurance policy that requires him to pay a $3000 deductible in case of a collision. Zach will cancel the collision policy when the value of his car equals his deductible. To the *nearest year,* how long will it take Zach to cancel this policy? Justify your answer.

$V(6) \approx \$2916.87$, which is less than \$3000

year 6