

**II**

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

**ALGEBRA II**

Thursday, June 14, 2018 — 1:15 to 4:15 p.m., only

Student Name:

Mr. Sibol

School Name:

JMAP

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice...**

**A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The graphs of the equations  $y = x^2 + 4x - 1$  and  $y + 3 = x$  are drawn on the same set of axes. One solution of this system is

- (1)  $(-5, -2)$   
 (2)  $(-1, -4)$

- (3)  $(1, 4)$   
 (4)  $(-2, -1)$

$$y + 3 = -1$$

$$y = -4$$

$$x^2 + 4x - 1 = x - 3$$

$$x^2 + 3x + 2 = 0$$

$$(x+2)(x+1) = 0$$

$$x = -2, -1$$

2 Which statement is true about the graph of  $f(x) = \left(\frac{1}{8}\right)^x$ ?

- (1) The graph is always increasing.  
 (2) The graph is always decreasing.  
 (3) The graph passes through  $(1, 0)$ .  
 (4) The graph has an asymptote,  $x = 0$ .

exponential decay

3 For all values of  $x$  for which the expression is defined,  $\frac{x^3 + 2x^2 - 9x - 18}{x^3 - x^2 - 6x}$ , in simplest form, is equivalent to

(1) 3

(2)  $-\frac{17}{2}$

(3)  $\frac{x+3}{x}$

(4)  $\frac{x^2-9}{x(x-3)}$

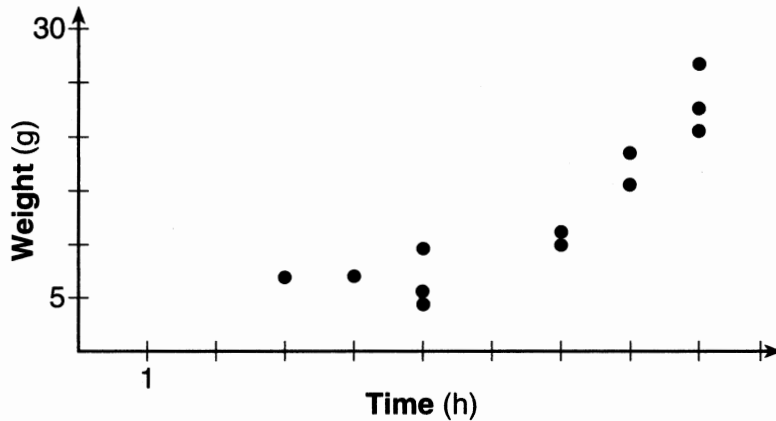
$$\frac{x^2(x+2) - 9(x+2)}{x(x^2 - x - 6)}$$

$$\frac{(x^2 - 9)(x+2)}{x(x-3)(x+2)}$$

$$\frac{(x+3)(x-3)}{x(x-3)}$$

Use this space for computations.

- 4 A scatterplot showing the weight,  $w$ , in grams, of each crystal after growing  $t$  hours is shown below.



The relationship between weight,  $w$ , and time,  $t$ , is best modeled by

- (1)  $w = 4^t + 5$                       (3)  $w = 5(2.1)^t$   
 (2)  $w = (1.4)^t + 2$                 (4)  $w = 8(.75)^t$

- 5 Where  $i$  is the imaginary unit, the expression  $(x + 3i)^2 - (2x - 3i)^2$  is equivalent to

- (1)  $-3x^2$                               (3)  $-3x^2 + 18xi$   
 (2)  $-3x^2 - 18$                       (4)  $-3x^2 - 6xi - 18$

$$x^2 + 6xi + 9i^2 - (4x^2 - 12xi + 9i^2)$$

$$-3x^2 + 18xi$$

- 6 Which function is even?

- (1)  $f(x) = \sin x$                       (3)  $f(x) = |x - 2| + 5$   
 (2)  $f(x) = x^2 - 4$                     (4)  $f(x) = x^4 + 3x^3 + 4$

$$f(-x) = (-x)^2 - 4$$

$$= x^2 - 4$$

Use this space for computations.

7 The function  $N(t) = 100e^{-0.023t}$  models the number of grams in a sample of cesium-137 that remain after  $t$  years. On which interval is the sample's average rate of decay the fastest?

- (1) [1,10]                      (3) [15,25]  
 (2) [10,20]                    (4) [1,30]

$$\frac{N(10) - N(1)}{10 - 1} \approx -2.03$$

$$\frac{N(25) - N(15)}{25 - 15} \approx -1.46$$

$$\frac{N(20) - N(10)}{20 - 10} \approx -1.63$$

8 Which expression can be rewritten as  $(x + 7)(x - 1)$ ?

- (1)  $(x + 3)^2 - 16$   
 (2)  $(x + 3)^2 - 10(x + 3) - 2(x + 3) + 20$

$$\frac{N(30) - N(1)}{30 - 1} \approx -1.64$$

(3)  $\frac{(x-1)(x^2 - 6x - 7)}{(x+1)}$

$$\begin{aligned} x^2 + 6x - 7 \\ x^2 + 6x + 9 - 7 - 9 \\ (x+3)^2 - 16 \end{aligned}$$

(4)  $\frac{(x+7)(x^2 + 4x + 3)}{(x+3)}$

9 What is the solution set of the equation  $\frac{2}{x} - \frac{3x}{x+3} = \frac{x}{x+3}$ ?

- (1) {3}                              (3) {-2,3}  
 (2)  $\left\{\frac{3}{2}\right\}$                           (4)  $\left\{-1, \frac{3}{2}\right\}$

$$\frac{2}{x} = \frac{4x}{x+3}$$

$$4x^2 = 2x + 6$$

$$4x^2 - 2x - 6 = 0$$

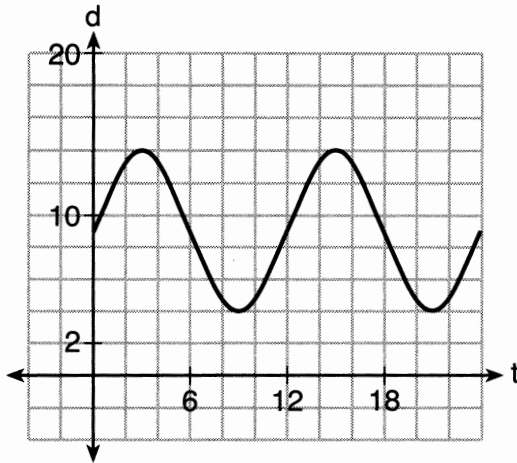
$$2x^2 - x - 3 = 0$$

$$(2x-3)(x+1) = 0$$

$$x = \frac{3}{2}, -1$$

Use this space for computations.

- 10 The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below.



If the depth,  $d$ , is measured in feet and time,  $t$ , is measured in hours since midnight, what is an equation for the depth of the water at the marker?

- (1)  $d = 5\cos\left(\frac{\pi}{6}t\right) + 9$       (3)  $d = 9\sin\left(\frac{\pi}{6}t\right) + 5$   
 (2)  $d = 9\cos\left(\frac{\pi}{6}t\right) + 5$       (4)  $d = 5\sin\left(\frac{\pi}{6}t\right) + 9$

$$q = \frac{14-4}{2} = 5$$

$$d = \frac{14+4}{2} = 9$$

- 11 On a given school day, the probability that Nick oversleeps is 48% and the probability he has a pop quiz is 25%. Assuming these two events are independent, what is the probability that Nick oversleeps and has a pop quiz on the same day?

- (1) 73%      (3) 23%  
 (2) 36%      (4) 12%

$$P(O \text{ and } Q) = P(O) \cdot P(Q)$$

$$= .48 \cdot .25$$

$$= .12$$

- 12 If  $x - 1$  is a factor of  $x^3 - kx^2 + 2x$ , what is the value of  $k$ ?

- (1) 0      (3) 3  
 (2) 2      (4) -3

$$1^3 - k(1)^2 + 2(1) = 0$$

$$3 = k$$

Use this space for computations.

- 13 The profit function,  $p(x)$ , for a company is the cost function,  $c(x)$ , subtracted from the revenue function,  $r(x)$ . The profit function for the Acme Corporation is  $p(x) = -0.5x^2 + 250x - 300$  and the revenue function is  $r(x) = -0.3x^2 + 150x$ . The cost function for the Acme Corporation is

- (1)  $c(x) = 0.2x^2 - 100x + 300$   
 (2)  $c(x) = 0.2x^2 + 100x + 300$   
 (3)  $c(x) = -0.2x^2 + 100x - 300$   
 (4)  $c(x) = -0.8x^2 + 400x - 300$

$$p(x) = r(x) - c(x)$$

$$-0.5x^2 + 250x - 300 = -0.3x^2 + 150x - c(x)$$

$$c(x) = 0.2x^2 - 100x + 300$$

- 14 The populations of two small towns at the beginning of 2018 and their annual population growth rate are shown in the table below.

Town	Population	Annual Population Growth Rate
Jonesville	1240	6% increase
Williamstown	890	11% increase

$$1240(1.06)^x = 890(1.11)^x$$

$$x \approx 7$$

Assuming the trend continues, approximately how many years after the beginning of 2018 will it take for the populations to be equal?

- (1) 7  
 (2) 20  
 (3) 68  
 (4) 125

- 15 What is the inverse of  $f(x) = x^3 - 2$ ?

- (1)  $f^{-1}(x) = \sqrt[3]{x} + 2$   
 (2)  $f^{-1}(x) = \pm\sqrt[3]{x} + 2$   
 (3)  $f^{-1}(x) = \sqrt[3]{x+2}$   
 (4)  $f^{-1}(x) = \pm\sqrt[3]{x+2}$

$$y = x^3 - 2$$

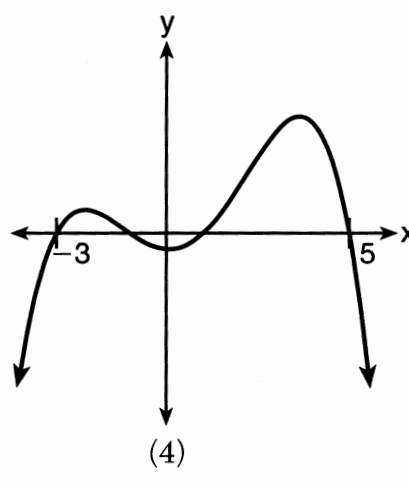
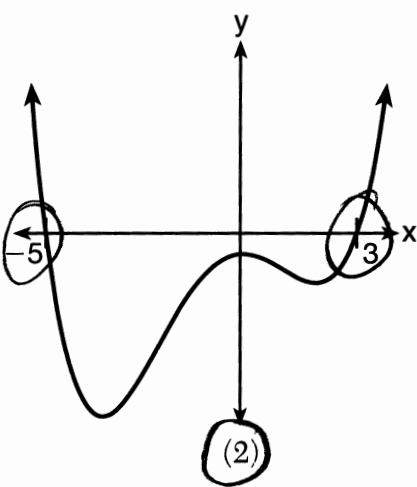
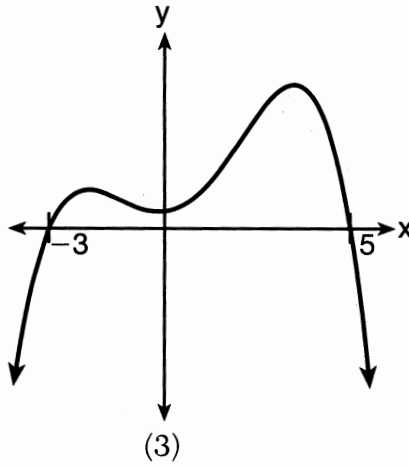
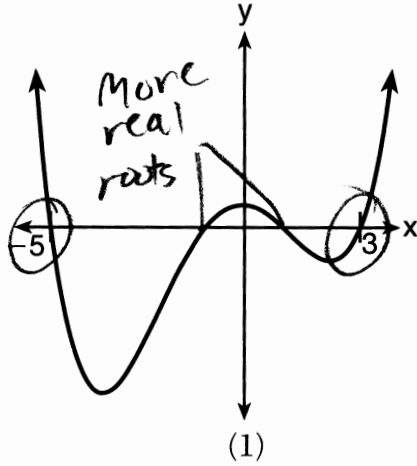
$$x = y^3 - 2$$

$$x + 2 = y^3$$

$$\sqrt[3]{x+2} = y$$

Use this space for computations.

16 A 4th degree polynomial has zeros  $-5$ ,  $3$ ,  $i$ , and  $-i$ . Which graph could represent the function defined by this polynomial?



17 The weights of bags of Graseck's Chocolate Candies are normally distributed with a mean of 4.3 ounces and a standard deviation of 0.05 ounces. What is the probability that a bag of these chocolate candies weighs less than 4.27 ounces?

- (1) 0.2257
- (2) 0.2743
- (3) 0.7257
- (4) 0.7757

Use this space for computations.

18 The half-life of iodine-131 is 8 days. The percent of the isotope left

in the body  $d$  days after being introduced is  $I = 100\left(\frac{1}{2}\right)^{\frac{d}{8}}$ .

When this equation is written in terms of the number  $e$ , the base of the natural logarithm, it is equivalent to  $I = 100e^{kd}$ . What is the approximate value of the constant,  $k$ ?

- (1) -0.087                      (3) -11.542  
(2) 0.087                        (4) 11.542

$$\begin{aligned} 100\left(\frac{1}{2}\right)^{\frac{d}{8}} &= 100e^{kd} \\ \ln\left(\frac{1}{2}\right)^{\frac{1}{8}} &= \ln e^k \\ -0.087 &\approx k \end{aligned}$$

19 The graph of  $y = \log_2 x$  is translated to the right 1 unit and down 1 unit.

The coordinates of the  $x$ -intercept of the translated graph are

- (1) (0,0)                        (3) (2,0)  
(2) (1,0)                        (4) (3,0)

$$\begin{aligned} \log_2(x-1) - 1 &= 0 \\ \log_2(x-1) &= 1 \\ x-1 &= 2 \\ x &= 3 \end{aligned}$$

20 For positive values of  $x$ , which expression is equivalent to

$$\sqrt{16x^2} \cdot x^{\frac{2}{3}} + \sqrt[3]{8x^5}?$$

- (1)  $6\sqrt[5]{x^3}$                         (3)  $4\sqrt[3]{x^2} + 2\sqrt[3]{x^5}$   
(2)  $6\sqrt[3]{x^5}$                         (4)  $4\sqrt{x^3} + 2\sqrt[5]{x^3}$

$$\begin{aligned} 4x \cdot x^{\frac{2}{3}} + 2x^{\frac{5}{3}} \\ 4x^{\frac{5}{3}} + 2x^{\frac{5}{3}} \\ 6x^{\frac{5}{3}} \\ 6\sqrt[3]{x^5} \end{aligned}$$



Use this space for computations.

21 Which equation represents a parabola with a focus of  $(-2, 5)$  and a directrix of  $y = 9$ ?

- (1)  $(y - 7)^2 = 8(x + 2)$
- (2)  $(y - 7)^2 = -8(x + 2)$
- (3)  $(x + 2)^2 = 8(y - 7)$
- (4)  $(x + 2)^2 = -8(y - 7)$

$$(x+2)^2 = -8(y-7)$$

$$y = -\frac{1}{8}(x+2)^2 + 7 \quad p=2$$

vertex  $(-2, 7)$

directrix  $y = 7 + 2 = 9$

y value of focus  $y = 7 - 2 = 5$

22 Given the following polynomials

$$\begin{aligned}x &= (a + b + c)^2 \\y &= a^2 + b^2 + c^2 \\z &= ab + bc + ac\end{aligned}$$

Which identity is true?

- (1)  $x = y - z$
- (2)  $x = y + z$
- (3)  $x = y - 2z$
- (4)  $x = y + 2z$

$$x = y + 2z$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ac)$$

$$a^2 + ab + ac + ab + b^2 + bc + ac + ab + c^2 = a^2 + b^2 + c^2 + 2z$$

$$a^2 + b^2 + c^2 + 2ab + 2bc + 2ac = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$$

Use this space for  
computations.

- 23 On average, college seniors graduating in 2012 could compute their growing student loan debt using the function  $D(t) = 29,400(1.068)^t$ , where  $t$  is time in years. Which expression is equivalent to  $29,400(1.068)^t$  and could be used by students to identify an approximate daily interest rate on their loans?

(1)  $29,400\left(1.068^{\frac{1}{365}}\right)^t$

(3)  $29,400\left(1 + \frac{0.068}{365}\right)^t$

(2)  $29,400\left(\frac{1.068}{365}\right)^{365t}$

(4)  $29,400\left(1.068^{\frac{1}{365}}\right)^{365t}$

- 24 A manufacturing plant produces two different-sized containers of peanuts. One container weighs  $x$  ounces and the other weighs  $y$  pounds. If a gift set can hold one of each size container, which expression represents the number of gift sets needed to hold 124 ounces?

1 lb = 16 oz.

(1)  $\frac{124}{16x + y}$

(3)  $\frac{124}{x + 16y}$

(2)  $\frac{x + 16y}{124}$

(4)  $\frac{16x + y}{124}$

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Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 A survey about television-viewing preferences was given to randomly selected freshmen and seniors at Fairport High School. The results are shown in the table below.

Favorite Type of Program

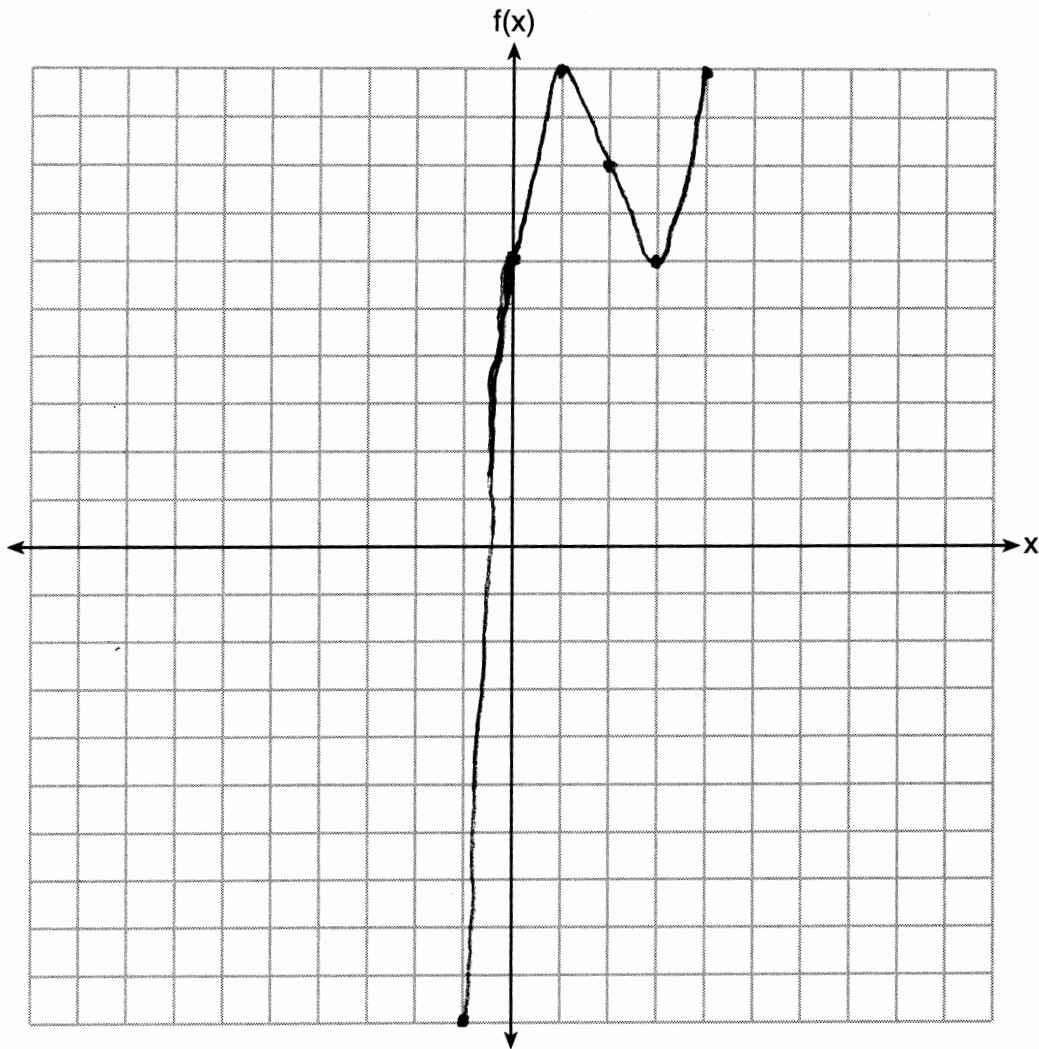
	Sports	Reality Show	Comedy Series
Senior	83	110	67
Freshman	119	103	54

213

A student response is selected at random from the results. State the *exact* probability the student response is from a freshman, given the student prefers to watch reality shows on television.

$$\frac{103}{213}$$

26 On the grid below, graph the function  $f(x) = x^3 - 6x^2 + 9x + 6$  on the domain  $-1 \leq x \leq 4$ .



27 Solve the equation  $2x^2 + 5x + 8 = 0$ . Express the answer in  $a + bi$  form.

$$X = \frac{-5 \pm \sqrt{5^2 - 4(2)(8)}}{2(2)}$$

$$= -\frac{5}{4} \pm \frac{i\sqrt{39}}{4}$$

28 Chuck's Trucking Company has decided to initiate an Employee of the Month program. To determine the recipient, they put the following sign on the back of each truck.



The driver who receives the highest number of positive comments will win the recognition. Explain *one* statistical bias in this data collection method.

Self selection is a cause of bias in this case because people with more free time or like surveys are more likely to respond.

29 Determine the quotient and remainder when  $(6a^3 + 11a^2 - 4a - 9)$  is divided by  $(3a - 2)$ .

Express your answer in the form  $q(a) + \frac{r(a)}{d(a)}$ .

$$\begin{array}{r} 2a^2 + 5a + 7 - \frac{5}{3a-2} \\ 3a-2 \overline{) 6a^3 + 11a^2 - 4a - 9} \\ \underline{6a^3 - 4a^2} \phantom{- 9} \\ 15a^2 - 4a \phantom{- 9} \\ \underline{15a^2 - 10a} \phantom{- 9} \\ 6a - 9 \phantom{- 9} \\ \underline{6a - 4} \phantom{- 9} \\ -5 \end{array}$$

30 The recursive formula to describe a sequence is shown below.

$$a_1 = 3$$

$$a_n = 1 + 2a_{n-1}$$

State the first four terms of this sequence.

$$a_1 = 3$$

$$a_2 = 7$$

$$a_3 = 15$$

$$a_4 = 31$$

Can this sequence be represented using an explicit geometric formula? Justify your answer.

No, because there is no common ratio

$$\frac{7}{3} \neq \frac{15}{7}$$



- 31 The Wells family is looking to purchase a home in a suburb of Rochester with a 30-year mortgage that has an annual interest rate of 3.6%. The house the family wants to purchase is \$152,500 and they will make a \$15,250 down payment and borrow the remainder. Use the formula below to determine their monthly payment, to the *nearest dollar*.

$$M = \frac{P\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1}$$

$M$  = monthly payment

$P$  = amount borrowed

$r$  = annual interest rate

$n$  = total number of monthly payments

$$M = \frac{(152,500 - 15,250) \left(\frac{.036}{12}\right) \left(1 + \frac{.036}{12}\right)^{360}}{\left(1 + \frac{.036}{12}\right)^{360} - 1} \approx 624$$

- 32 An angle,  $\theta$ , is in standard position and its terminal side passes through the point  $(2, -1)$ . Find the *exact* value of  $\sin \theta$ .

$$\frac{-1}{\sqrt{2^2 + (-1)^2}} = \frac{-1}{\sqrt{5}}$$

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve algebraically for all values of  $x$ :

$$\sqrt{6-2x} + x = 2(x+15) - 9$$

$$\sqrt{6-2x} + x = 2x + 30 - 9$$

$$\sqrt{6-2x} = x + 21$$

$$6-2x = x^2 + 42x + 441$$

$$x^2 + 44x + 435 = 0$$

$$(x+29)(x+15) = 0$$

$$\cancel{-29} \quad -15$$

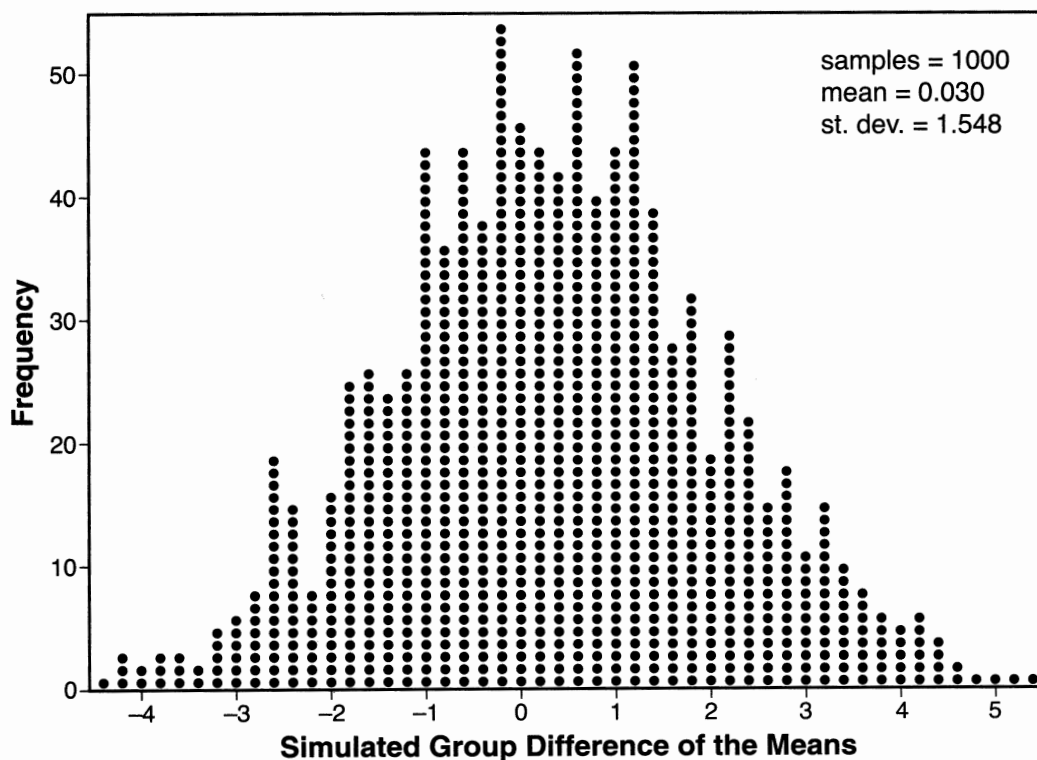
34 Joseph was curious to determine if scent improves memory. A test was created where better memory is indicated by higher test scores. A controlled experiment was performed where one group was given the test on scented paper and the other group was given the test on unscented paper. The summary statistics from the experiment are given below.

	Scented Paper	Unscented Paper
$\bar{x}$	23	18
$s_x$	2.898	2.408

Calculate the difference in means in the experimental test grades (scented – unscented).

$$23 - 18 = 5$$

A simulation was conducted in which the subjects' scores were rerandomized into two groups 1000 times. The differences of the group means were calculated each time. The results are shown below.



Question 34 is continued on the next page.

**Question 34 continued**

Use the simulation results to determine the interval representing the middle 95% of the difference in means, to the *nearest hundredth*.

$$\bar{x} \pm 2\sigma$$
$$-3.07 - 3.13$$

Is the difference in means in Joseph's experiment statistically significant based on the simulation? Explain.

Yes, a difference of 5 or more occurred 3 times out of 1000, which is statistically significant.

35 Carla wants to start a college fund for her daughter Lila. She puts \$63,000 into an account that grows at a rate of 2.55% per year, compounded monthly. Write a function,  $C(t)$ , that represents the amount of money in the account  $t$  years after the account is opened, given that no more money is deposited into or withdrawn from the account.

$$C(t) = 63000 \left(1 + \frac{.0255}{12}\right)^{12t}$$

Calculate algebraically the number of years it will take for the account to reach \$100,000, to the nearest hundredth of a year.

$$100000 = 63000 (1.002125)^{12t}$$

$$\log \frac{100}{63} = 12t \log(1.002125)^{12t}$$

$$\log \frac{100}{63} = t$$

$$\frac{\log \frac{100}{63}}{12 \log(1.002125)}$$

$$18.14 \approx t$$

- 36 The height,  $h(t)$  in cm, of a piston, is given by the equation  $h(t) = 12\cos\left(\frac{\pi}{3}t\right) + 8$ , where  $t$  represents the number of seconds since the measurements began.

Determine the average rate of change, in cm/sec, of the piston's height on the interval  $1 \leq t \leq 2$ .

$$\frac{h(2) - h(1)}{2 - 1} = -12$$

At what value(s) of  $t$ , to the *nearest tenth of a second*, does  $h(t) = 0$  in the interval  $1 \leq t \leq 5$ ? Justify your answer.

$$h(t) = 0 \text{ at } t \approx 2.2, 3.8$$

Using graphing calculator  
to find where  $h(t) = 0$

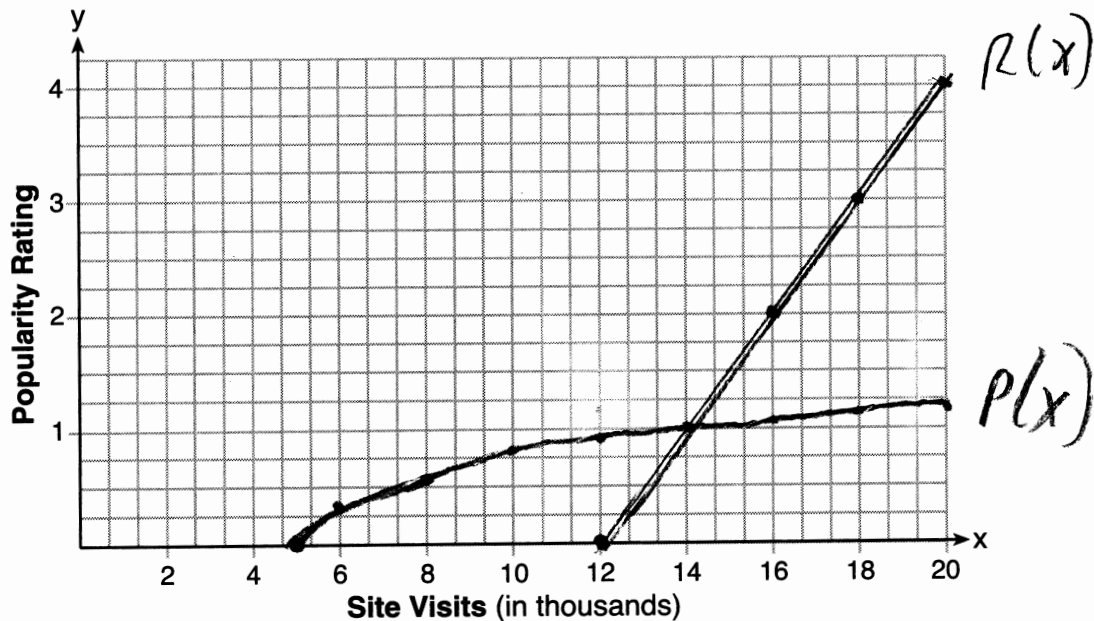
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

- 37 Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is  $P(x) = \log(x - 4)$ , where  $x$  is the number of visits per week in thousands and  $P(x)$  is the website's popularity rating. According to this model, if a website is visited 16,000 times in one week, what is its popularity rating, rounded to the nearest tenth?

$$P(16) = \log(16 - 4) \approx 1.1$$

Graph  $y = P(x)$  on the axes below.



An alternative rating model is represented by  $R(x) = \frac{1}{2}x - 6$ , where  $x$  is the number of visits per week in thousands. Graph  $R(x)$  on the same set of axes. For what number of weekly visits will the two models provide the same rating?

14,000