ALGEBRA

II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II
(Common Core)

Friday, June 16, 2017 — 1:15 to 4:15 p.m., only

Student Name _My $Sibol$_____

School Name _$JMAP$_____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II**, **III**, and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

**A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.**

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The graph of the function $p(x)$ is sketched below.
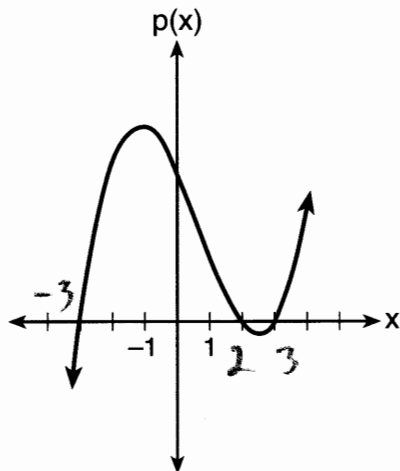
Which equation could represent $p(x)$?

(1) $p(x) = (x^2 - 9)(x - 2)$

(2) $p(x) = x^3 - 2x^2 + 9x + 18$

(3) $p(x) = (x^2 + 9)(x - 2)$

(4) $p(x) = x^3 + 2x^2 - 9x - 18$

$(x+3)(x-3)(x-2)$

$x = -3 \quad 3 \quad 2$

2 What is the solution to $8(2^{x+3}) = 48$?

(1) $x = \dfrac{\ln 6}{\ln 2} - 3$

(2) $x = 0$

(3) $x = \dfrac{\ln 48}{\ln 16} - 3$

(4) $x = \ln 4 - 3$

$\ln 2^{x+3} = \ln 6$

$(x+3)\ln 2 = \ln 6$

$x = \dfrac{\ln 6}{\ln 2} - 3$

3 Cheap and Fast gas station is conducting a consumer satisfaction survey. Which method of collecting data would most likely lead to a biased sample?

   (1) interviewing every 5th customer to come into the station

   (2) interviewing customers chosen at random by a computer at the checkout

   (3) interviewing customers who call an 800 number posted on the customers' receipts   *self selection*

   (4) interviewing every customer who comes into the station on a day of the week chosen at random out of a hat

4 The expression $6xi^3(-4xi + 5)$ is equivalent to

   (1) $2x - 5i$ (3) $-24x^2 + 30x - i$

   (2) $-24x^2 - 30xi$ (4) $26x - 24x^2i - 5i$

$$-24x^2 i^4 + 30xi^3$$
$$-24x^2(1) + 30x(-1)$$

5 If $f(x) = 3|x| - 1$ and $g(x) = 0.03x^3 - x + 1$, an approximate solution for the equation $f(x) = g(x)$ is

   (1) 1.96   (3) $(-0.99, 1.96)$

   (2) 11.29   (4) $(11.29, 32.87)$

6 Given the parent function $p(x) = \cos x$, which phrase best describes the transformation used to obtain the graph of $g(x) = \cos(x + a) - b$, if a and b are positive constants?

   (1) right a units, up b units

   (2) right a units, down b units

   (3) left a units, up b units

   (4) left a units, down b units

7 The solution to the equation $4x^2 + 98 = 0$ is

(1) ± 7

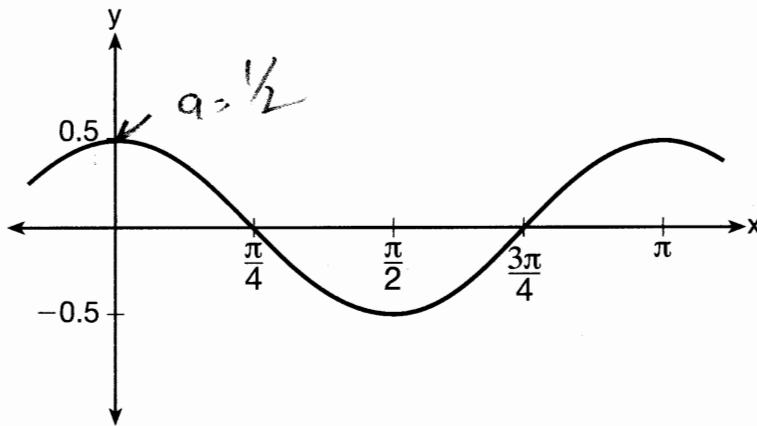(3) $\pm \dfrac{7\sqrt{2}}{2}$

(2) $\pm 7i$

(4) $\pm \dfrac{7i\sqrt{2}}{2}$

[handwritten in right margin:]

$4x^2 = -98$

$x^2 = \dfrac{-49}{2}$

$x = \pm\sqrt{\dfrac{-49}{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

8 Which equation is represented by the graph shown below?

[handwritten: a = ½]

[handwritten in right margin:]

period $\cdot \dfrac{2\pi}{b}$

$\pi \cdot \dfrac{2\pi}{b}$

$b = 2$

(1) $y = \dfrac{1}{2} \cos 2x$

(3) $y = \dfrac{1}{2} \cos x$

(2) $y = \cos x$

(4) $y = 2 \cos \dfrac{1}{2}x$

9 A manufacturing company has developed a cost model, $C(x) = 0.15x^3 + 0.01x^2 + 2x + 120$, where x is the number of items sold, in thousands. The sales price can be modeled by $S(x) = 30 - 0.01x$. Therefore, revenue is modeled by $R(x) = x \cdot S(x)$.

The company's profit, $P(x) = R(x) - C(x)$, could be modeled by

(1) $0.15x^3 + 0.02x^2 - 28x + 120$

(2) $-0.15x^3 - 0.02x^2 + 28x - 120$

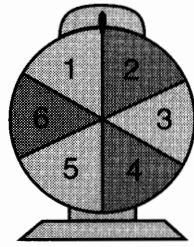(3) $-0.15x^3 + 0.01x^2 - 2.01x - 120$

(4) $-0.15x^3 + 32x + 120$

[handwritten:]

$x \cdot S(x) - C(x)$

$x(30 - 0.01x) - (0.15x^3 + 0.01x^2 + 2x + 120)$

$30x - 0.01x^2 - 0.15x^3 - 0.01x^2 - 2x - 120$

10 A game spinner is divided into 6 equally sized regions, as shown in the diagram below.

For Miles to win, the spinner must land on the number 6. After spinning the spinner 10 times, and losing all 10 times, Miles complained that the spinner is unfair. At home, his dad ran 100 simulations of spinning the spinner 10 times, assuming the probability of winning each spin is $\frac{1}{6}$. The output of the simulation is shown in the diagram below.
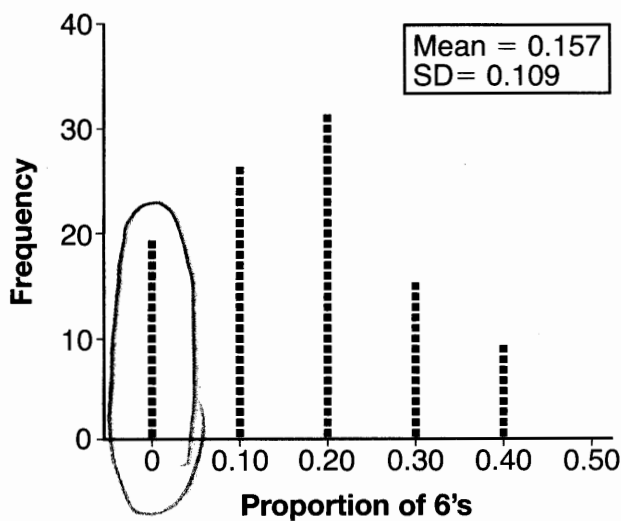
$$\frac{1}{6} = .166$$

0 falls within
2 standard deviations
of the mean

Which explanation is appropriate for Miles and his dad to make?

(1) The spinner was likely unfair, since the number 6 failed to occur in about 20% of the simulations.

(2) The spinner was likely unfair, since the spinner should have landed on the number 6 by the sixth spin.

(3) The spinner was likely not unfair, since the number 6 failed to occur in about 20% of the simulations.

(4) The spinner was likely not unfair, since in the output the player wins once or twice in the majority of the simulations.

11 Which binomial is a factor of $x^4 - 4x^2 - 4x + 8$?

(1) $x - 2$ (3) $x - 4$

(2) $x + 2$ (4) $x + 4$

$$2\mid 1 \quad 0 \quad -4 \quad -4 \quad 8$$
$$ \quad 2 \quad 4 \quad 0 \quad -8$$
$$ 1 \quad 2 \quad 0 \quad -4 \quad 0$$

12 Given that $\sin^2 \theta + \cos^2 \theta = 1$ and $\sin \theta = -\dfrac{\sqrt{2}}{5}$, what is a possible value of $\cos \theta$?

(1) $\dfrac{5 + \sqrt{2}}{5}$ (3) $\dfrac{3\sqrt{3}}{5}$

(2) $\dfrac{\sqrt{23}}{5}$ (4) $\dfrac{\sqrt{35}}{5}$

$$\cos\theta = \pm\sqrt{1 - \left(\frac{-\sqrt{2}}{5}\right)^2}$$
$$= \pm\sqrt{\frac{25}{25} - \frac{2}{25}} = \pm\frac{\sqrt{23}}{5}$$

13 A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model $P = 714(0.75)^d$, where P is the population, in thousands, d decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after y years. Suzanne's model is best represented by

(1) $P = 714(0.6500)^y$ (3) $P = 714(0.9716)^y$

(2) $P = 714(0.8500)^y$ (4) $P = 714(0.9750)^y$

$$(.75)^{1/10} \approx .9716$$

14 The probability that Gary and Jane have a child with blue eyes is 0.25, and the probability that they have a child with blond hair is 0.5. The probability that they have a child with both blue eyes and blond hair is 0.125. Given this information, the events blue eyes and blond hair are

     I: dependent
     II: independent
     III: mutually exclusive

(1) I, only   (3) I and III

(2) II, only   (4) II and III

If independent,

$$P(A \& B) = P(A) \cdot P(B)$$
$$.125 = .5 \cdot .25 \checkmark$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \& B)$$
$$P(A \text{ or } B) = 0.25 + .5 - .125$$
$$= .625$$

If mutually exclusive

$$P(A \text{ or } B) = P(A) + P(B)$$
$$.625 = .5 + .25 \quad No$$

15 Based on climate data that have been collected in Bar Harbor, Maine, the average monthly temperature, in degrees F, can be modeled by the equation $B(x) = 23.914\sin(0.508x - 2.116) + 55.300$. The same governmental agency collected average monthly temperature data for Phoenix, Arizona, and found the temperatures could be modeled by the equation $P(x) = 20.238\sin(0.525x - 2.148) + 86.729$.

Which statement can *not* be concluded based on the average monthly temperature models x months after starting data collection?

(1) The average monthly temperature variation is more in Bar Harbor than in Phoenix.

(2) The midline average monthly temperature for Bar Harbor is lower than the midline temperature for Phoenix.

(3) The maximum average monthly temperature for Bar Harbor is 79° F, to the nearest degree.

(4) The minimum average monthly temperature for Phoenix is 20° F, to the nearest degree.

Handwritten: RANGE

Handwritten:
	BH	PH
MIN	31.386	66.491
MID	55.3	86.729
MAX	79.214	106.967
RANGE	47.828	40.476

16 For $x \neq 0$, which expressions are equivalent to one divided by the sixth root of x?
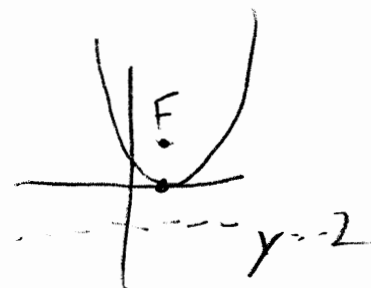
Handwritten: $\frac{1}{\sqrt[6]{x}}$

I. $\dfrac{\sqrt[6]{x}}{\sqrt[3]{x}}$ II. $\dfrac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}}$ III. $x^{-\frac{1}{6}}$

(1) I and II, only

(2) I and III, only

(3) II and III, only

(4) I, II, and III

17 A parabola has its focus at (1,2) and its directrix is $y = -2$. The equation of this parabola could be

(1) $y = 8(x + 1)^2$

(2) $y = \frac{1}{8}(x + 1)^2$

(3) $y = 8(x - 1)^2$

(4) $y = \frac{1}{8}(x - 1)^2$

Handwritten:
$y = \frac{1}{4p}(x-h)^2 + k$
$y = \frac{1}{8}(x-1)^2 + 0$
Vertex (1,0)
$p = 2$

18 The function $p(t) = 110e^{0.03922t}$ models the population of a city, in millions, t years after 2010. As of today, consider the following two statements:

2010

    I.   The ~~current~~ population is 110 million.

    II. The population increases continuously by approximately 3.9% per year.

This model supports

(1) I, only   (3) both I and II

(2) II, only   (4) neither I nor II

19 To solve $\dfrac{2x}{x-2} - \dfrac{11}{x^2-2x} = \dfrac{8}{x^2-2x}$, Ren multiplied both sides by the least common denominator. Which statement is true?

(1) 2 is an extraneous solution.

(2) $\frac{7}{2}$ is an extraneous solution.

(3) 0 and 2 are extraneous solutions.

(4) This equation does not contain any extraneous solutions.

2x² − 11x + 22 = 8

2x² − 11x + 14 = 0

(2x − 7)(x − 2) = 0

x = 7/2   x = 2

20 Given $f(9) = -2$, which function can be used to generate the sequence $-8, -7.25, -6.5, -5.75, \ldots$?

(1) $f(n) = -8 + 0.75n$

(2) $f(n) = -8 - 0.75(n-1)$

(3) $f(n) = -8.75 + 0.75n$

(4) $f(n) = -0.75 + 8(n-1)$

21 The function $f(x) = 2^{-0.25x} \cdot \sin\left(\dfrac{\pi}{2}x\right)$ represents a damped sound wave function. What is the average rate of change for this function on the interval $[-7, 7]$, to the *nearest hundredth*?

(1) -3.66 (3) -0.26

(2) -0.30 (4) 3.36

$\dfrac{2^{-0.25(7)} \cdot \sin\frac{7\pi}{2} - 2^{-0.25(-7)} \cdot \sin\frac{-7\pi}{2}}{7--7}$

≈ -0.26

22 Mallory wants to buy a new window air conditioning unit. The cost for the unit is $329.99. If she plans to run the unit three months out of the year for an annual operating cost of $108.78, which function models the cost per year over the lifetime of the unit, $C(n)$, in terms of the number of years, n, that she owns the air conditioner.

(1) $C(n) = 329.99 + 108.78n$

(2) $C(n) = 329.99 + 326.34n$

(3) $C(n) = \dfrac{329.99 + 108.78n}{n}$

(4) $C(n) = \dfrac{329.99 + 326.34n}{n}$

23 The expression $\dfrac{-3x^2 - 5x + 2}{x^3 + 2x^2}$ can be rewritten as

(1) $\dfrac{-3x - 3}{x^2 + 2x}$

(2) $\dfrac{-3x - 1}{x^2}$

(3) $-3x^{-1} + 1$

(4) $-3x^{-1} + x^{-2}$

$$\dfrac{(-3x+1)(x+2)}{x^2(x+2)}$$

$$\dfrac{-3x}{x^2} + \dfrac{1}{x^2}$$

24 Jasmine decides to put $100 in a savings account each month. The account pays 3% annual interest, compounded monthly. How much money, S, will Jasmine have after one year?

(1) $S = 100(1.03)^{12}$

(2) $S = \dfrac{100 - 100(1.0025)^{12}}{1 - 1.0025}$

(3) $S = 100(1.0025)^{12}$

(4) $S = \dfrac{100 - 100(1.03)^{12}}{1 - 1.03}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.   [16]

25 Given $r(x) = x^3 - 4x^2 + 4x - 6$, find the value of $r(2)$.

$$\begin{array}{r|rrrr} 2 & 1 & -4 & 4 & -6 \\ & & 2 & -4 & 0 \\ \hline & 1 & -2 & 0 & -6 \end{array}$$

$$r(2) = -6$$

What does your answer tell you about $x - 2$ as a factor of $r(x)$? Explain.

x-2 is not a factor since there is a remainder

26 The weight of a bag of pears at the local market averages 8 pounds with a standard deviation of 0.5 pound. The weights of all the bags of pears at the market closely follow a normal distribution. Determine what percentage of bags, to the *nearest integer*, weighed *less* than 8.25 pounds.

69%, using a graphing calculator

27 Over the set of integers, factor the expression $4x^3 - x^2 + 16x - 4$ completely.

$x^2(4x-1) + 4(4x-1)$

$(x^2+4)(4x-1)$

28 The graph below represents the height above the ground, h, in inches, of a point on a triathlete's bike wheel during a training ride in terms of time, t, in seconds.
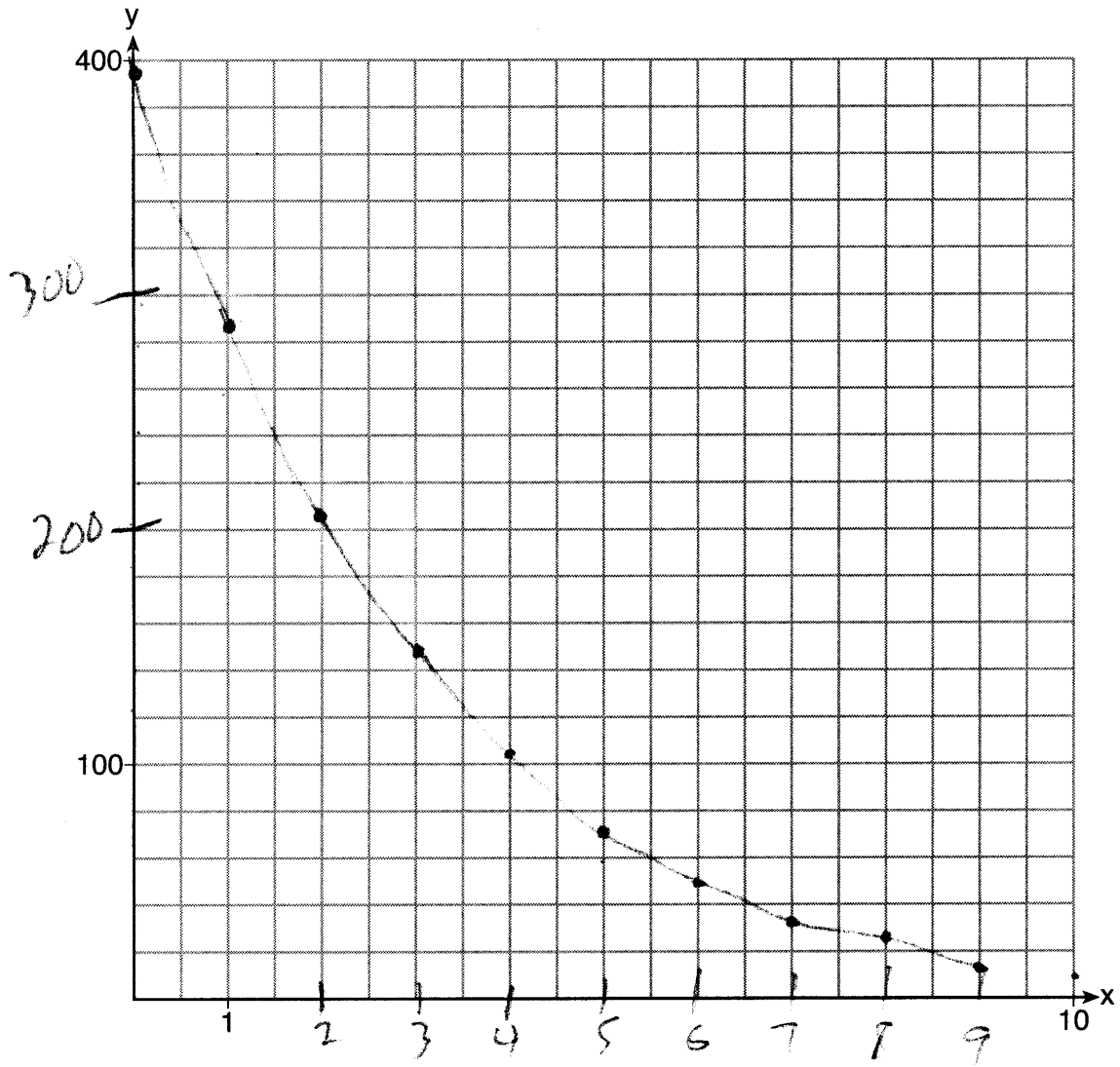
$$\frac{2}{3}$$

26

Height Above the Ground (in)

$$\frac{2}{3} \qquad \frac{4}{3} \qquad \frac{6}{3}$$

Time (sec)
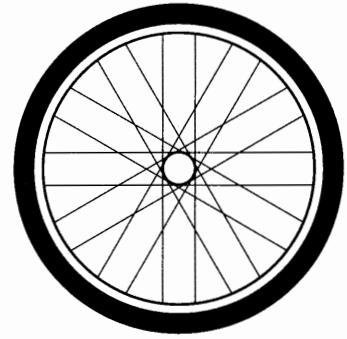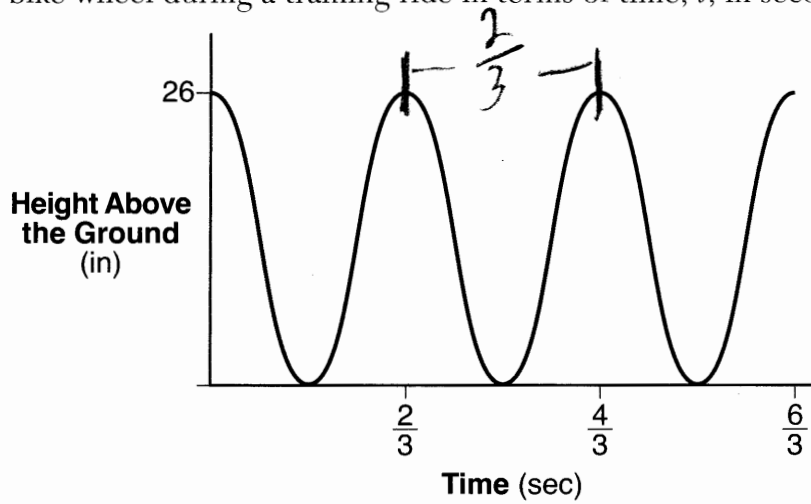
Identify the period of the graph and describe what the period represents in this context.

$$\frac{2}{3} \, sec$$

The wheel rotates once every

$$\frac{2}{3} \, sec.$$

29 Graph $y = 400(.85)^{2x} - 6$ on the set of axes below.

30 Solve algebraically for all values of x:

$$\sqrt{x-4} + x = 6$$

$$\sqrt{x-4} = 6-x$$
$$x-4 = 36 -12x + x^2$$
$$0 = x^2 -13x + 40$$
$$0 = (x-8)(x-5)$$
$$x = 5, \cancel{8}$$

$$\sqrt{8-4} + 8 \neq 6$$

31 Write $\sqrt[3]{x} \cdot \sqrt{x}$ as a single term with a rational exponent.

$$x^{1/3} \cdot x^{1/2} = x^{5/6}$$

32 Data collected about jogging from students with two older siblings are shown in the table below.

	Neither Sibling Jogs	One Sibling Jogs	Both Siblings Jog
Student Does Not Jog	1168	1823	1380
Student Jogs	188	416	400

Using these data, determine whether a student with two older siblings is more likely to jog if one sibling jogs or if both siblings jog. Justify your answer.

1 jogs

$$\frac{416}{2239}$$

.19

both jog

$$\frac{400}{1780}$$

.22

$.19 < .22$

more likely

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the following system of equations algebraically for all values of x, y, and z:

$$x + y + z = 1$$
$$2x + 4y + 6z = 2$$
$$-x + 3y - 5z = 11$$

$x + y + z = 1$

$-x + 3y - 5z = 11$

$4y - 4z = 12$

$y - z = 3$

$2x + 2y + 2z = 2$

$2x + 4y + 6z = 2$

$2y + 4z = 0$

$y + 2z = 0$

$y = -2z$

$-2z - z = 3$

$-3z = 3$

$z = -1$

$y - (-1) = 3$

$y = 2$

$x + 2 - 1 = 1$

$x = 0$

$(0, 2, -1)$

34 Jim is looking to buy a vacation home for $172,600 near his favorite southern beach. The formula to compute a mortgage payment, M, is $M = P \cdot \dfrac{r(1+r)^N}{(1+r)^N - 1}$ where P is the principal amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15-year mortgage.

With no down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*.

$$M = 172600 \cdot \frac{.00305(1+.00305)^{12 \cdot 15}}{(1+.00305)^{12 \cdot 15} - 1}$$

$$\approx 1247$$

Algebraically determine and state the down payment, rounded to the *nearest dollar*, that Jim needs to make in order for his mortgage payment to be $1100.
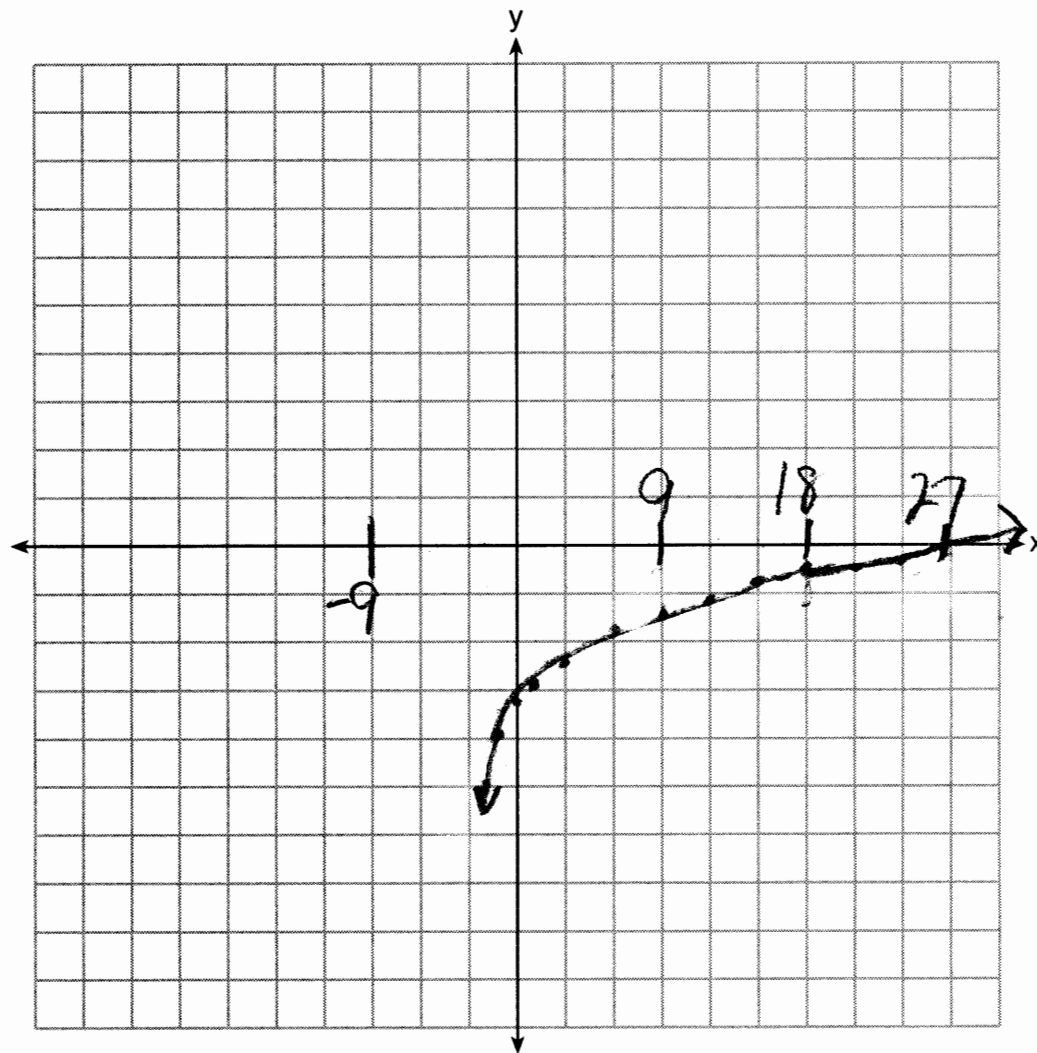
$$1100 = (172,600 - x)\left(\frac{.00305(1.00305)^{12 \cdot 15}}{(1.00305)^{12 \cdot 15} - 1} \right)$$

$$1100 \approx (172600 - x)(.007228)$$

$$152193 \approx 172600 - x$$

$$x \approx 20407$$

35 Graph $y = \log_2(x + 3) - 5$ on the set of axes below. Use an appropriate scale to include *both* intercepts.


Describe the behavior of the given function as x approaches -3 and as x approaches positive infinity.

As $x \to -3$, $y \to -\infty$

As $x \to \infty$, $y \to \infty$

36 Charlie's Automotive Dealership is considering implementing a new check-in procedure for customers who are bringing their vehicles for routine maintenance. The dealership will launch the procedure if 50% or more of the customers give the new procedure a favorable rating when compared to the current procedure. The dealership devises a simulation based on the minimal requirement that 50% of the customers prefer the new procedure. Each dot on the graph below represents the proportion of the customers who preferred the new check-in procedure, each of sample size 40, simulated 100 times.
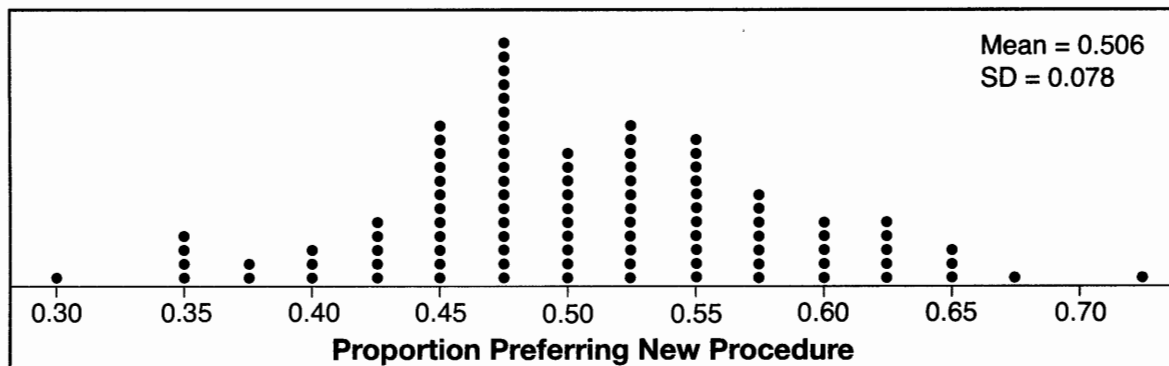
Assume the set of data is approximately normal and the dealership wants to be 95% confident of its results. Determine an interval containing the plausible sample values for which the dealership will launch the new procedure. Round your answer to the *nearest hundredth*.

$$.506 \pm 2(.078)$$

$$0.35 - 0.66$$

Forty customers are selected randomly to undergo the new check-in procedure and the proportion of customers who prefer the new procedure is 32.5%. The dealership decides *not* to implement the new check-in procedure based on the results of the study. Use statistical evidence to explain this decision.

The 32.5% value falls below the 95% confidence level

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 A radioactive substance has a mass of 140 g at 3 p.m. and 100 g at 8 p.m. Write an equation in the form $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$ that models this situation, where h is the constant representing the number of hours in the half-life, A_0 is the initial mass, and A is the mass t hours after 3 p.m.

$$100 = 140\left(\frac{1}{2}\right)^{5/h}$$

Using this equation, solve for h, to the *nearest ten thousandth.*

$$\log\frac{100}{140} = \log\frac{1}{2}^{5/h}$$

$$\log\frac{5}{7} = \frac{5}{h}\log\frac{1}{2}$$

$$h = \frac{5\log 1/2}{\log 5/7} \approx 10.3002$$

Determine when the mass of the radioactive substance will be 40 g. Round your answer to the *nearest tenth of an hour.*

$$40 = 140(.5)^{\frac{t}{10.3002}}$$

$$\log\frac{2}{7} = \log .5^{t/10.3002}$$

$$\frac{10.3002\log 2/7}{\log .5} = t$$

$$\approx 18.6$$