The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II (Common Core)

Wednesday, June 1, 2016 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. Sibol

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 When $b > 0$ and d is a positive integer, the expression $(3b)^{\frac{2}{d}}$ is equivalent to

(1) $\frac{1}{\left(\sqrt[d]{3b}\right)^2}$

(2) $\left(\sqrt{3b}\right)^d$

(3) $\frac{1}{\sqrt{3b^d}}$

(4) $\left(\sqrt[d]{3b}\right)^2$

2 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, T, are left in the semester? 85 x 3

(1) $\frac{255 + 93T}{3T} = 90$

(2) $\frac{255 + 90T}{3T} = 93$

(3) $\frac{255 + 93T}{T+3} = 90$

(4) $\frac{255 + 90T}{T+3} = 93$

3 Given i is the imaginary unit, $(2 - yi)^2$ in simplest form is

(1) $y^2 - 4yi + 4$

(2) $-y^2 - 4yi + 4$

(3) $-y^2 + 4$

(4) $y^2 + 4$

$(2-yi)(2-yi)$

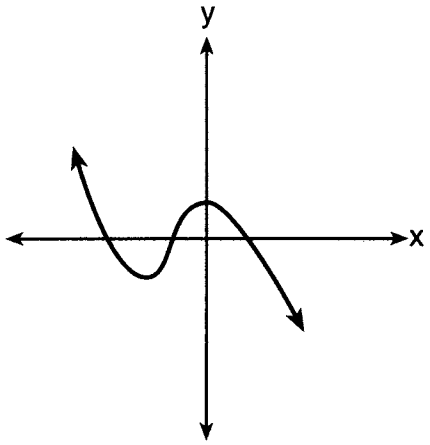
$4 - 4yi + y^2 i^2$

$-y^2 - 4yi + 4$

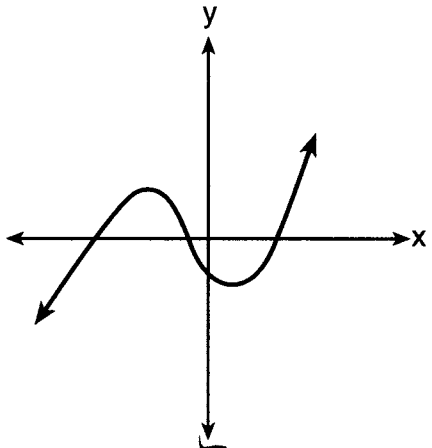
Use this space for computations.

4 Which graph has the following characteristics?

- three real zeros
- as $x \to -\infty$, $f(x) \to -\infty$
- as $x \to \infty$, $f(x) \to \infty$



(1)



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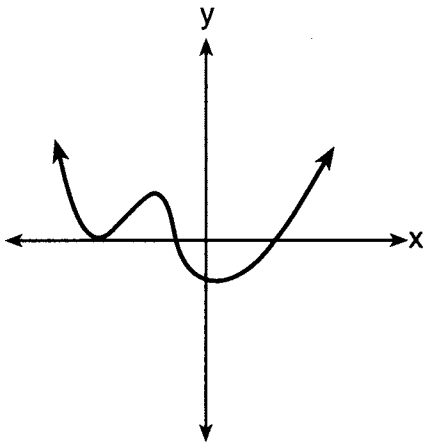
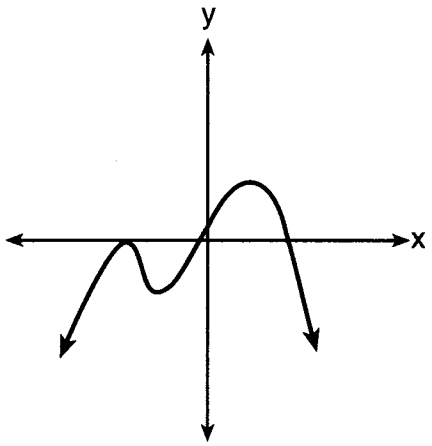


(2)



(4)

5 The solution set for the equation $\sqrt{56 - x} = x$ is

(1) $\{-8,7\}$

(2) $\{-7,8\}$

(3) $\{7\}$

(4) $\{\ \}$

$56 - x = x^2$

$0 = x^2 + x - 56$

$0 = (x+8)(x-7)$

$x = -8, 7$

extraneous

Use this space for computations.

6 The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are

(1) $\{0,\pm 3,4\}$

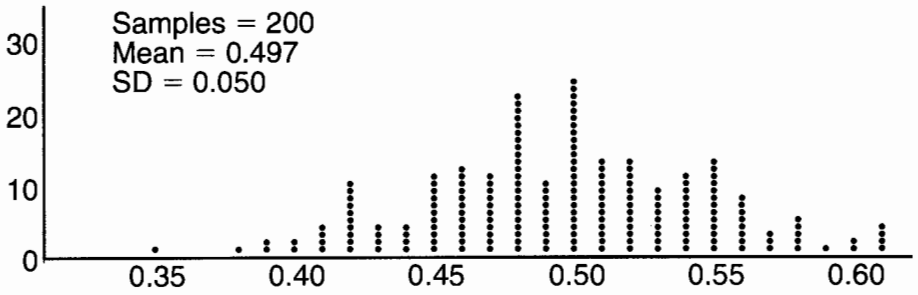
(2) $\{0,3,4\}$

(3) $\{0,\pm 3,-4\}$

(4) $\{0,3,-4\}$

$x(x^3-4x^2-9x+36)=0$
$x(x^2(x-4)-9(x-4))=0$
$x(x^2-9)(x-4)=0$
$x(x+3)(x-3)(x-4)=0$

7 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Given the results of her coin flips and of her computer simulation, which statement is most accurate?

(1) 73 of the computer's next 100 coin flips will be heads.

(2) 50 of her next 100 coin flips will be heads.

(3) Her coin is not fair.

(4) Her coin is fair.

8 If $g(c) = 1 - c^2$ and $m(c) = c + 1$, then which statement is *not* true?

(1) $g(c) \cdot m(c) = 1 + c - c^2 - c^3$

(2) $g(c) + m(c) = 2 + c - c^2$

(3) $m(c) - g(c) = c + c^2$

(4) $\frac{m(c)}{g(c)} = \frac{-1}{1-c}$

$\frac{m(c)}{g(c)} = \frac{c+1}{1-c^2} = \frac{c+1}{(1-c)(1+c)} = \frac{1}{1-c}$

Use this space for computations.

9 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the *nearest whole percent*, is

(1) 6
(2) 48
(3) 68
(4) 95

$z = \frac{69.5 - 64}{2.75} = 2$

10 The formula below can be used to model which scenario?

$$a_1 = 3000$$
$$a_n = 0.80a_{n-1}$$

(1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.

(2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.

(3) A bank account starts with a deposit of \$3000, and each year it grows by 80%.

(4) The initial value of a specialty toy is \$3000, and its value each of the following years is 20% less.

11 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

(1) independent
(2) dependent
(3) mutually exclusive
(4) complements

$P(R) = P(R \mid S)$

Use this space for computations.

12 A solution of the equation $2x^2 + 3x + 2 = 0$ is

(1) $-\frac{3}{4} + \frac{1}{4}i\sqrt{7}$

(2) $-\frac{3}{4} + \frac{7}{4}i$

(3) $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$

(4) $\frac{1}{2}$

$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(2)}}{2(2)}$

$\frac{-3 \pm \sqrt{-7}}{4}$

$-\frac{3}{4} \pm \frac{i\sqrt{7}}{4}$

13 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, H, in feet, above the ground of one of the six-person cars can be modeled by $H(t) = 70 \sin\left(\frac{2\pi}{7}(t - 1.75)\right) + 80$, where t is time, in minutes. Using $H(t)$ for one full rotation, this car's minimum height, in feet, is

(1) 150

(2) 70

(3) 10

(4) 0

$70(-1) + 80$

$-70 + 80 = 10$

14 The expression $\frac{4x^3 + 5x + 10}{2x + 3}$ is equivalent to

(1) $2x^2 + 3x - 7 + \frac{31}{2x + 3}$

(2) $2x^2 - 3x + 7 - \frac{11}{2x + 3}$

(3) $2x^2 + 2.5x + 5 + \frac{15}{2x + 3}$

(4) $2x^2 - 2.5x - 5 - \frac{20}{2x + 3}$

$2x+3 \overline{)4x^3 + 0x^2 + 5x + 10}$

$2x^2 - 3x + 7$

$4x^3 + 6x^2$

$-6x^2 + 5x$

$-6x^2 - 9x$

$14x + 10$

$14x + 21$

-11

15 Which function represents exponential decay?

(1) $y = 2^{0.3t}$

(2) $y = 1.2^{3t}$

(3) $y = \left(\frac{1}{2}\right)^{-t}$

(4) $y = 5^{-t}$

$\left(\frac{1}{5}\right)^t$

Use this space for computations.

16 Given $f^{-1}(x) = -\frac{3}{4}x + 2$, which equation represents $f(x)$?

(1) $f(x) = \frac{4}{3}x - \frac{8}{3}$

(3) $f(x) = \frac{3}{4}x - 2$

(2) $f(x) = -\frac{4}{3}x + \frac{8}{3}$

(4) $f(x) = -\frac{3}{4}x + 2$

$x = -\frac{3}{4}y + 2$

$-4x = 3y - 8$

$-4x + 8 = 3y$

$-\frac{4}{3}x + \frac{8}{3} = y$

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of $\cos\theta$?

(1) $-\frac{3}{5}$

(3) $\frac{3}{5}$

(2) $-\frac{3}{4}$

(4) $\frac{4}{5}$

Since θ terminates in Quadrant II, $\cos\theta$ is negative.

$\sqrt{10^2 - 8^2} = 6$ $\quad \frac{-6}{10} = -\frac{3}{5}$

18 Which statement about the graph of $c(x) = \log_6 x$ is *false*?

(1) The asymptote has equation $y = 0$.

(2) The graph has no y-intercept.

(3) The domain is the set of positive reals.

(4) The range is the set of all real numbers.

The asymptote has equation $x = 0$

19 The equation $4x^2 - 24x + 4y^2 + 72y = 76$ is equivalent to

(1) $4(x - 3)^2 + 4(y + 9)^2 = 76$

(2) $4(x - 3)^2 + 4(y + 9)^2 = 121$

(3) $4(x - 3)^2 + 4(y + 9)^2 = 166$

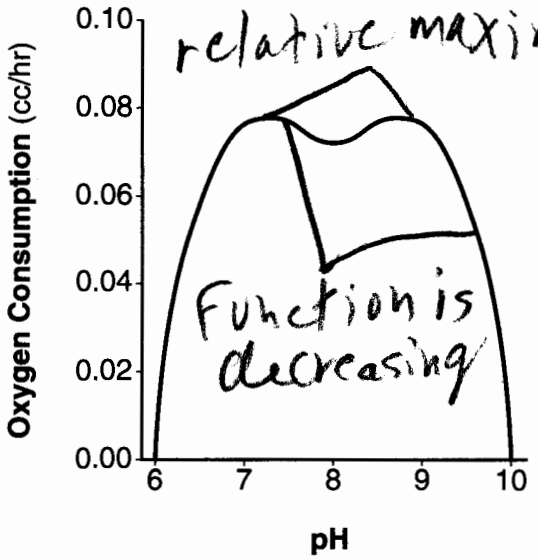
(4) $4(x - 3)^2 + 4(y + 9)^2 = 436$

$4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 436$

$4(x - 3)^2 + 4(y + 9)^2 = 436$

Use this space for computations.

20 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.



Which statement about this function is *incorrect*?

(1) The degree of the polynomial is even. degree is 4

(2) There is a positive leading coefficient.

(3) At two pH values, there is a relative maximum value.

(4) There are two intervals where the function is decreasing.

21 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let m represent months.]

(1) $(1.0525)^m$

(2) $(1.0525)^{\frac{12}{m}}$

(3) $(1.00427)^m$

(4) $(1.00427)^{\frac{m}{12}}$

$1.0525^{1/12} = 1.00427$

Use this space for computations.

22 Which value, to the *nearest tenth,* is *not* a solution of $p(x) = q(x)$ if $p(x) = x^3 + 3x^2 - 3x - 1$ and $q(x) = 3x + 8$?

(1) -3.9
(2) -1.1
(3) 2.1
(4) 4.7

Use graphing calculator

23 The population of Jamesburg for the years 2010 – 2013, respectively, was reported as follows:

250,000 250,937 251,878 252,822

How can this sequence be recursively modeled?

(1) $j_n = 250{,}000(1.00375)^{n-1}$

(2) $j_n = 250{,}000 + 937^{(n-1)}$

(3) $j_1 = 250{,}000$
$j_n = 1.00375\, j_{n-1}$

(4) $j_1 = 250{,}000$
$j_n = j_{n-1} + 937$

24 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles *every second*. Which equation best represents the value of the voltage as it flows through the electric wires, where t is time in seconds?

(1) $V = 120 \sin(t)$
(2) $V = 120 \sin(60t)$
(3) $V = 120 \sin(60\pi t)$
(4) $V = 120 \sin(120\pi t)$

$\text{period} = \frac{2\pi}{B}$

$\frac{1}{60} = \frac{2\pi}{B}$

$B = 120\pi$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$\frac{3-x}{3x} = \frac{-1}{3x}$

$3 - x = -1$

$4 = x$

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

Randomly assign participants to two groups, one using toothpaste with ingredient X, one without

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$x - 5 = 0$

$x = 5$

$2(5)^3 - 4(5)^2 - 7(5) - 10$

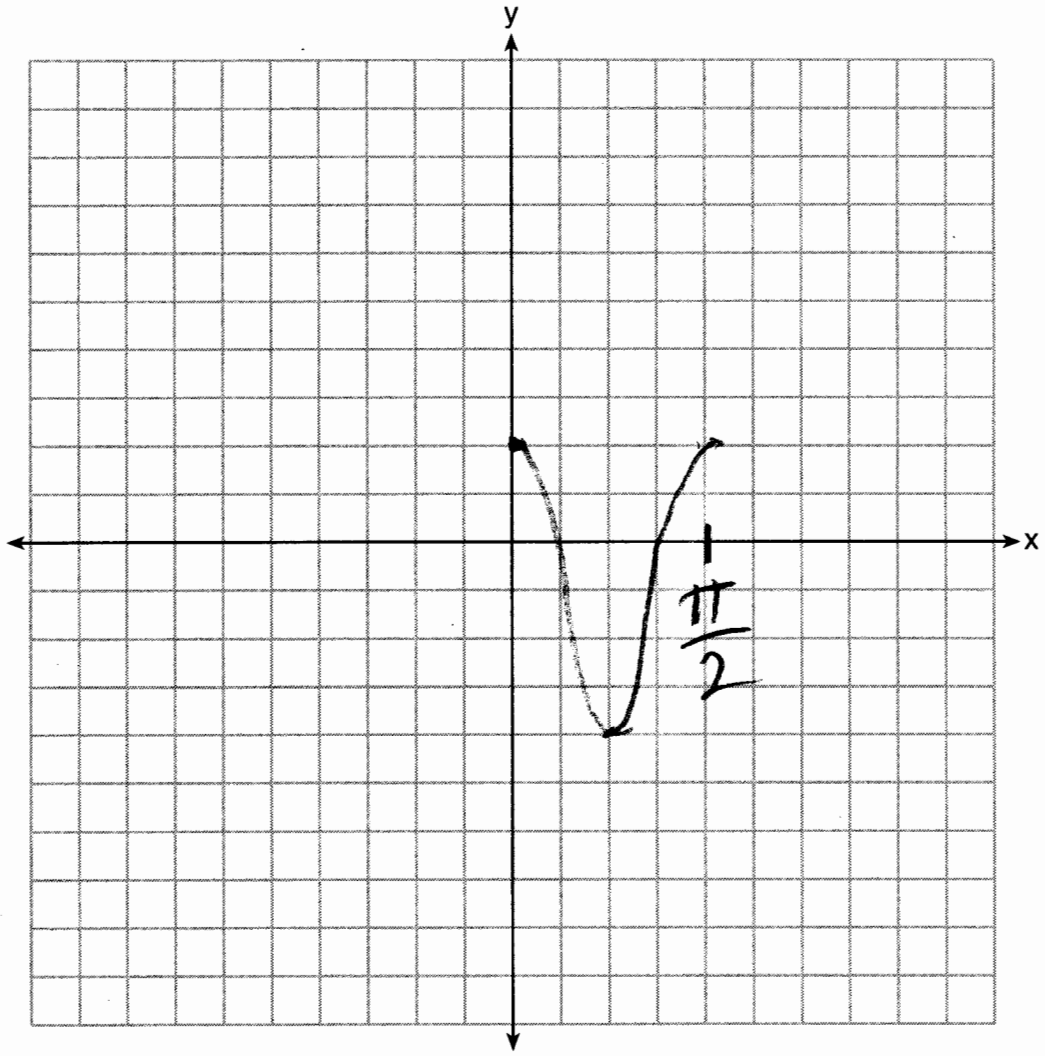
$250 - 100 - 35 - 10$

$105 \neq 0$

Since 5 is not a zero,

$x - 5$ is not a factor

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

649 + 433 − 974 = $\frac{108}{1376}$

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$12y + 36 = (x-4)^2$

$\frac{12y}{12} = \frac{(x-4)^2}{12} - \frac{36}{12}$

$y = \frac{1}{12}(x-4)^2 - 3$

the vertex is $(4, -3)$

$(x-4)^2 = 4(3)(y+3)$

$p = 3$

The focus is $(4, 0)$

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$$x^3+8 \overline{)\, x^3+0x^2+0x+9}$$

$$1 + \frac{1}{x^3+8}$$

$$x^3 + 8$$

$$1$$

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

$135{,}000 = 100{,}000(1+r)^5$

$\sqrt[5]{1.35} = \sqrt[5]{(1+r)^5}$

$.06 \approx r$

6%

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$

$$2x + 2y = 10$$

$$\frac{2x}{2} + \frac{2y}{2} = \frac{10}{2}$$

$$y = -x + 5$$

$$(x-3)^2 + (-x+5+2)^2 = 16$$

$$x^2 - 6x + 9 + x^2 - 14x + 49 = 16$$

$$2x^2 - 20x + 42 = 0$$

$$x^2 - 10x + 21 = 0$$

$$(x-7)(x-3) = 0$$

$$x = 7 \quad 3$$

$$y = -7 + 5 \qquad y = -3 + 5$$

$$= -2 \qquad y = 2$$

$$(7, -2) \qquad (3, 2)$$

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

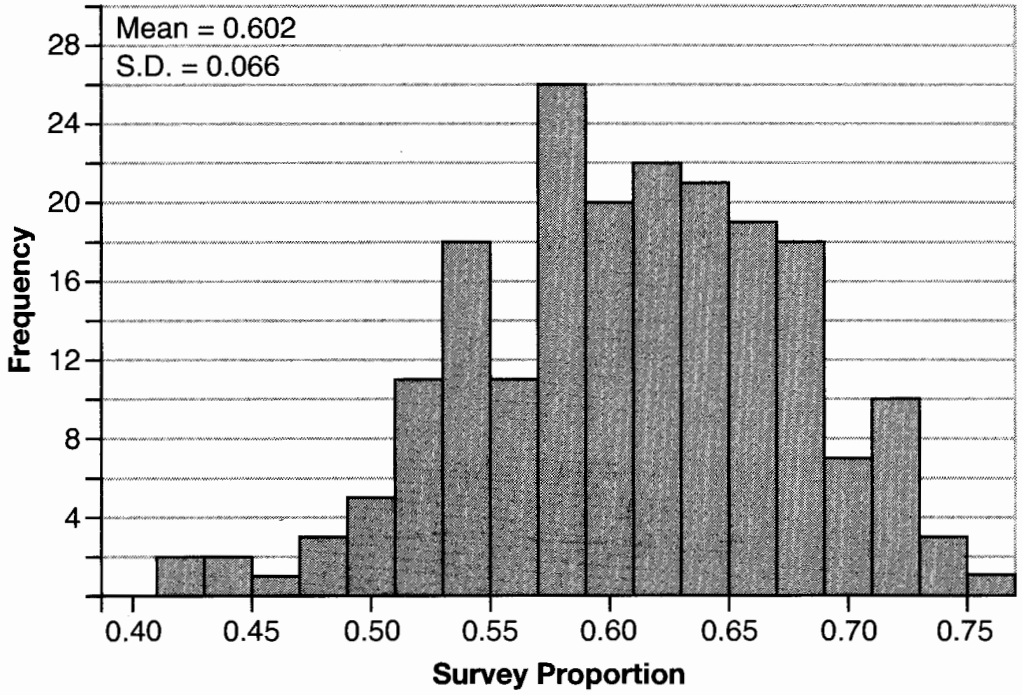
$$S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04} \approx 660,778.39$$

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

$0.602 \pm 2(0.066)$

$0.47 - 0.73$

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

.50 is within this interval, so its possible to get a split vote

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

$\frac{g(4) - g(-2)}{4 - -6} = \frac{179 - -49}{6}$

$= 38$

$\frac{f(4) - f(-2)}{4 - -2} = \frac{80 - 1.25}{6} = 13.125$

$g(x)$ has a greater rate of change.

Part IV

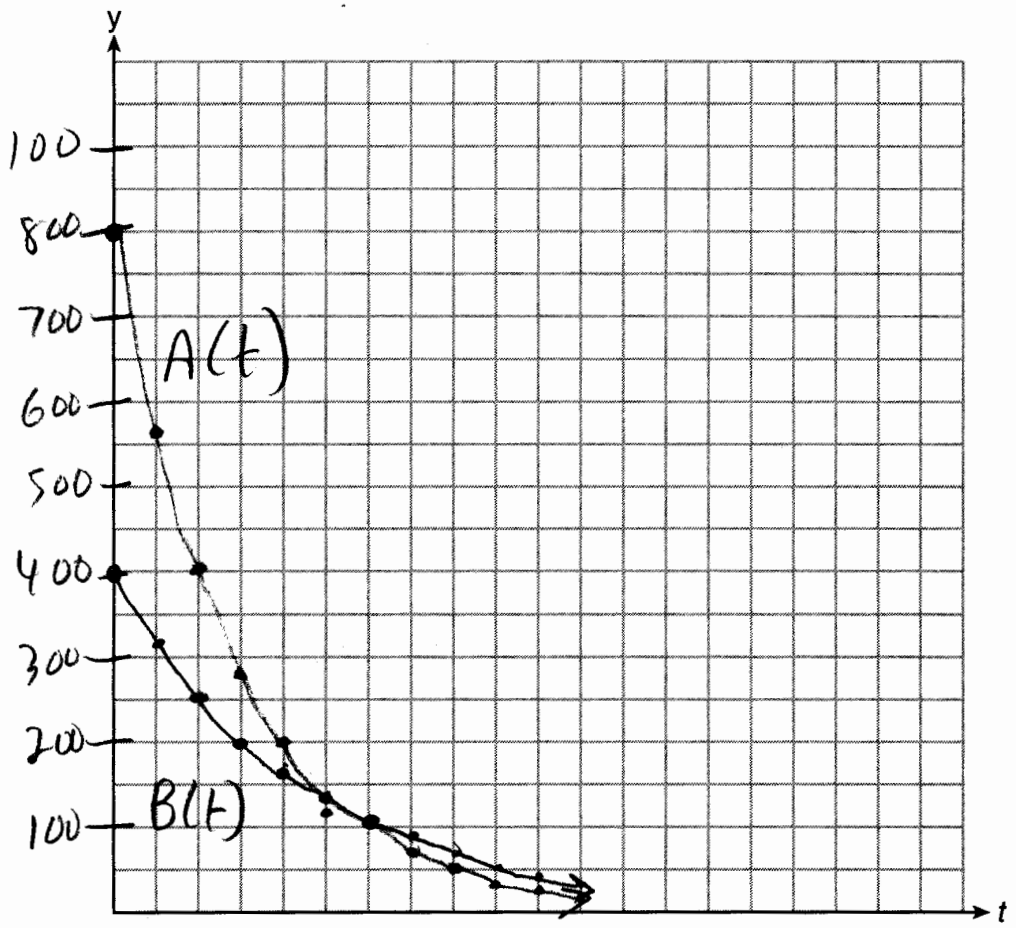
Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$A(t) = 800e^{-.347t}$ $B(t) = 400e^{-.231t}$

Graph each function on the set of axes below.



To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

6

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

$$\ln 0.15 = \ln e^{-.347t}$$

$$\frac{\ln 0.15}{-.347} = \frac{-.347t}{-.347}$$

$$5.5 \approx t$$