The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Thursday, January 23, 2025 — 1:15 to 4:15 p.m., only

Student Name _Mr. Sobol_

School Name _SMAP, Inc_

> **The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

> **Notice ...**
>
> **A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.**

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The exact value of $\sin\left(\dfrac{8\pi}{3}\right)$ is

(1) $\dfrac{1}{2}$

(3) $-\dfrac{\sqrt{3}}{2}$

(2) $-\dfrac{1}{2}$

(4) $\dfrac{\sqrt{3}}{2}$

2 A teacher randomly divides all of her students into two groups. She grades the homework for one group but does not grade the homework for the other group. All homework is returned to the students. She then compares test scores for each of the groups to see if grading homework has an effect on the test scores.

This method of data collection is best described as

(1) an experiment

(3) a simulation

(2) an unbiased survey

(4) an observational study

3 Which expression is equivalent to $(x - 2)^2 + 27(x - 2) - 90$?

(1) $(x + 30)(x - 3)$

(3) $(x - 30)(x + 3)$

(2) $(x + 28)(x - 5)$

(4) $(x - 2)(x + 25)(x - 90)$

$$u = x - 2$$

$$u^2 + 27u - 90$$

$$(u + 30)(u - 3)$$

$$(x - 2 + 30)(x - 2 - 3)$$

4 Given the functions $f(x) = 2x + \dfrac{5}{2}$ and $g(x) = \dfrac{3}{x}$, what are the solutions to $f(x) = g(x)$?

(1) $(0.75, 4)$ or $(-2, -1.5)$ (3) $y = -1.5$ or $y = 4$

(2) $x = 0.75$ or $x = -2$ (4) $(-2, 0.75)$

$2x\left(2x + \dfrac{5}{2} = \dfrac{3}{x}\right)$

$4x^2 + 5x = 6$

$4x^2 + 5x - 6 = 0$

$(4x - 3)(x + 2) = 0$

$x = \dfrac{3}{4}, -2$

5 Given $f(x) = 2x^3 - 3x^2 - 5x - 12$ and $g(x) = x - 3$, the quotient of $\dfrac{f(x)}{g(x)}$ is

(1) $2x^2 + 3x + 4$ (3) $2x^2 - 9x + 22 - \dfrac{78}{x-3}$

(2) $2x^3 + 3x^2 + 4x$ (4) $2x^3 - 9x^2 + 22x - 78$

$3\,|\;\begin{array}{cccc} 2 & -3 & -5 & -12 \\ & 6 & 9 & 12 \\ \hline 2 & 3 & 4 & 0 \end{array}$

6 Abby is told that each day there is a 50% chance it will rain. Which simulation can Abby perform to determine the likelihood of it raining for the next seven days?

(1) Flip a coin seven times, count how many heads, and repeat 50 times.

(2) Roll a die seven times, count how many twos, and repeat 50 times.

(3) Roll a pair of dice, count totals of seven, and repeat 50 times.

(4) Flip a coin 50 times and count how many heads.

7 What are the solutions to $4x^2 - 7x - 2 = -10$?

(1) $-\dfrac{1}{4}, 2$

(3) $\dfrac{7}{8} \pm \dfrac{\sqrt{241}}{8}$

(2) $\dfrac{7}{8} \pm \dfrac{\sqrt{79}}{8}i$

(4) $\dfrac{7}{8} \pm \dfrac{\sqrt{143}}{8}i$

$$4x^2 - 7x + 8 = 0$$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(4)(8)}}{2(4)} = \frac{7 \pm \sqrt{-79}}{8}$$

8 If $x - 5$ is a factor of $p(x) = ax^4 + bx^3 + cx^2 + dx + e$, then which statement must be true?

(1) $p(-5) = 0$

(3) $p(5) = 0$

(2) $p(-5) \neq 0$

(4) $p(5) \neq 0$

9 In a small city, there are 22 gas stations. The mean price for a gallon of regular gas was \$2.12 with a standard deviation of \$0.05. The distribution of the data was approximately normal. Given this information, the middle 95% of the gas stations in this small city likely charge

(1) \$1.90 to \$2.34 for a gallon of gas

(2) \$1.97 to \$2.27 for a gallon of gas

(3) \$2.02 to \$2.22 for a gallon of gas

(4) \$2.07 to \$2.17 for a gallon of gas

$$2.12 \pm 2(.05)$$

10 The expression $\dfrac{4x^2 - 5}{x^2 - 1}$ is equivalent to

(1) $4 - \dfrac{1}{x^2 - 1}$

(3) $4 - \dfrac{9}{x^2 - 1}$

(2) $4 + \dfrac{1}{x^2 - 1}$

(4) $4 - \dfrac{4}{x^2 - 1}$

$$\frac{4x^2 - 5}{x^2 - 1} \cdot \frac{4(x^2 - 1)}{(x^2 - 1)} - \frac{1}{x^2 - 1}$$

11 For all positive values of x, which expression is equivalent to $\sqrt{x} \cdot \sqrt[4]{x^{11}}$?

(1) $x^{\frac{19}{22}}$

(3) $x^{\frac{13}{4}}$

(2) $x^{\frac{11}{8}}$

(4) $x^{\frac{2}{11}}$

$$x^{\frac{1}{2}} \cdot x^{\frac{11}{4}}$$

$$\frac{2}{4} + \frac{11}{4} = \frac{13}{4}$$

12 The expression $i^2(5x - 2i)^2$ is equivalent to

(1) $-25x^2 + 20xi - 4$

(3) $25x^2 + 20xi + 4$

(2) $-25x^2 + 20xi + 4$

(4) $25x^2 + 4$

$$-(25x^2 - 20xi + 4i^2)$$

$$-25x^2 + 20xi + 4$$

13 Functions f and g are given below.

$$f(x) = \frac{7}{2}x^2 - 5x + 11$$
$$g(x) = 3x^2 - 7x + 25$$

When $2f(x)$ is subtracted from $g(x)$, the result is

(1) $4x^2 - 3x - 3$ (3) $4x^2 - 17x - 47$

(2) $-4x^2 + 3x + 3$ (4) $-4x^2 - 17x + 47$

(handwritten: $3x^2 - 7x + 25 - (7x^2 - 10x + 22)$ *)*

14 A manufacturer claims that the number of ounces of a beverage dispensed by one of its automatic dispensers is normally distributed with a mean of 8.0 ounces and a standard deviation of 0.04 ounces. To the *nearest tenth of a percent,* what percent of the cups filled by this company's dispenser will contain between 7.9 and 8.11 ounces?

(1) 99.5   (3) 99.1

(2) 99.4   (4) 97.6

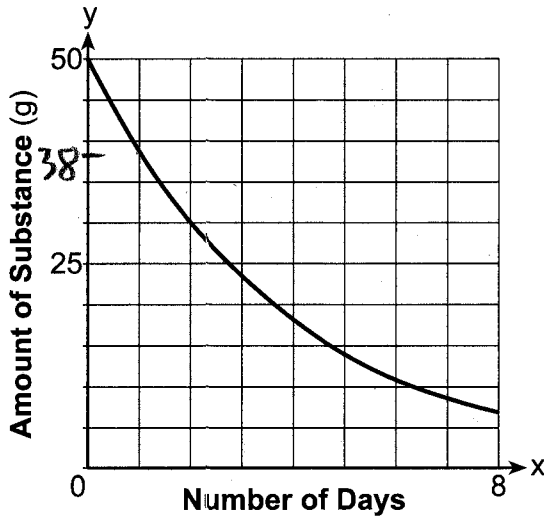**15** What is the value of x in the solution of the system of equations below?

$$5x + 2y - z = -14$$
$$7y - z = 31$$
$$5y + 4z - 5x = -23$$

(1) -17 (3) $-\dfrac{1}{5}$

(2) 2 (4) -7

(handwritten computations:

$z = 7y - 31$

$5x + 2y - 7y + 31 = -14$

$5y + 4(7y - 31) - 5x = -23$

$5x - 5y = -45$

$x - y = -9$

$33y - 5x = 101$

$33(x + 9) - 5x = 101$

$\dfrac{28x}{28} = \dfrac{-196}{28} = -7$

)

16 The graph below shows the amount of a radioactive substance left over time.

The daily rate of decay over an 8-day interval is approximately

(1) 23%

(2) 95%

(3) 5%

(4) 77%

Graph $y = 50 \cdot .77^x$ & use table to compare points

Estimate $\frac{38}{50} = 76\%$ from Graph

17 If $4(10^{5x-2}) = 12$, then x equals

(1) $\dfrac{2.3}{5}$

(2) $\dfrac{1}{3}\left(\dfrac{\log 12}{\log 40} + 5\right)$

(3) $\dfrac{\log(3) + 2}{5}$

(4) $\dfrac{1}{5}\left(\dfrac{\log 12}{\log 4} + 2\right)$

$\log 10^{5x-2} = \log 3$

$(5x-2)\log 10 = \log 3$

$\dfrac{5x}{5} = \dfrac{\log 3 + 2}{5}$

18 A random sample of 152 students was surveyed on a particular day about how they got to school. The survey results are summarized in the table below.

		Attendance Status	
		Late	On-Time
Method of Transportation	Car	6	24
	Bus	20	80
	Walk	4	18

Which statement is best supported by the data? 4/22

(1) The probability of being late given that a student walked is greater than the probability that a student walked given that the student was late. 4/30

(2) The probability of being late given that a student walked is less than the probability that a student walked given that the student was late.

(3) The probability of being late given that a student walked is equal to the probability that a student walked given that the student was late.

(4) The probability of being late given that a student walked cannot be determined.

19 If $f(x) = \sqrt[3]{x} + 4$, then $f^{-1}(x)$ equals

(1) $\sqrt[3]{x - 4}$

(2) $(x - 4)^3$

(3) $x^3 + \dfrac{1}{4}$

(4) $-\sqrt[3]{x} - 4$

$y = \sqrt[3]{x} + 4$

$x = \sqrt[3]{y} + 4$

$x - 4 = \sqrt[3]{y}$

$(x-4)^3 = y$

20 Given the equation $S(x) = 1.7\sin(bx) + 12$, where the period of $S(x)$ is 12, what is the value of b?

(1) $\dfrac{\pi}{6}$ (3) $\dfrac{\pi}{12}$

(2) 24π (4) 6π

$$period = \frac{2\pi}{b}$$

$$12 = \frac{2\pi}{b}$$

$$b = \frac{\pi}{6}$$

21 Jin solved the equation $\sqrt{4 - x} = x + 8$ by squaring both sides. What extraneous solution did he find?

(1) -5 (3) 3

(2) -12 (4) 4

$$4 - x = x^2 + 16x + 64$$
$$0 = x^2 + 17x + 60$$
$$0 = (x+12)(x+5)$$
$$x = -12 \quad -5$$
$$-12 + 8 = -4 \quad -5 + 8 = 3$$

$$x^4 + 2x^2y^2 + y^4$$

22 The expression $(x^2 + y^2)^2$ is *not* equivalent to

(1) $(x^2 - y^2)^2 + (2xy)^2$

(2) $(x + y)^4 + 2(xy)^2$

(3) $x^2(x^2 + 2y^2) + (y^2)^2$

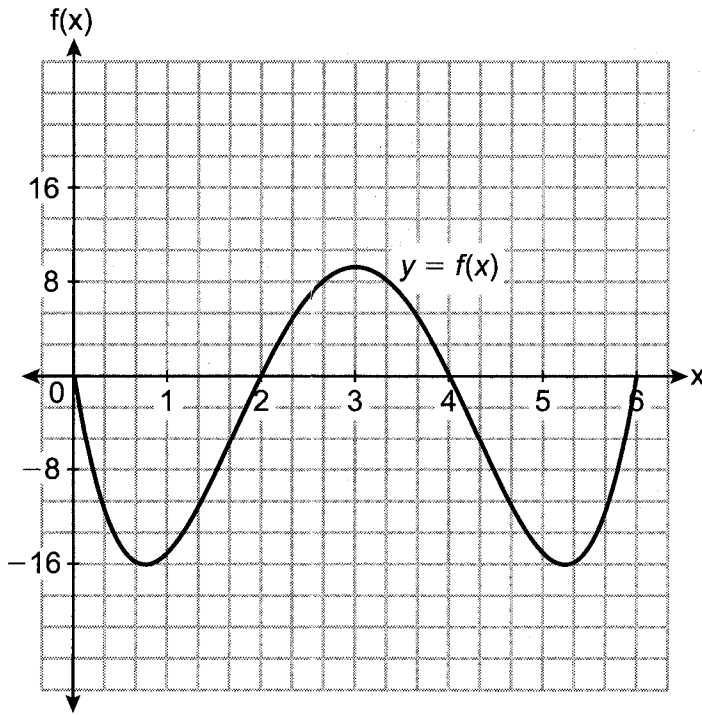(4) $(2x^2 + y^2)^2 - (3x^4 + 2x^2y^2)$

1) $x^4 - 2x^2y^2 + y^4 + 4x^2y^2$

3) $x^4 + 2x^2y^2 + y^4$

4) $4x^4 + 4x^2y^2 + y^4 - 3x^4 - 2x^2y^2$

23 The height of a running trail is modeled by the quartic function $y = f(x)$ shown below, where x is the distance in miles from the start of the trail and y is the height in feet relative to sea level.

If this trail has a minimum height of 16 feet below sea level, which function(s) could represent a running trail whose minimum height is half of the minimum height of the original trail?

I. $y = f\left(\frac{1}{2}x\right)$ II. $y = f(x) + 8$ III. $y = \frac{1}{2}f(x)$

(1) I, only   (3) I and III

(2) II, only   (4) II and III

I Minimum does not change

II −16 + 8 = −8

III $\frac{1}{2}$ · −16 = −8

24 The crew aboard a small fishing boat caught 350 pounds of fish on Monday. From that Monday through the end of the week on Friday, the weight of the fish caught increased 15% per day. The total weight, in pounds, of fish caught is approximately

(1) 411   (3) 1748

(2) 612   (4) 2360

$S_5 = \dfrac{350 - 350 \cdot 1.15^5}{1 - 1.15} \approx 2360$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Describe the translations that map $f(x) = \log x$ to $g(x) = \log(x + 3) - 5$.

left 3, down 5

26 Solve algebraically for x: $\dfrac{1}{2x} - \dfrac{5}{6} = \dfrac{3}{x}$

$$\frac{6-10x}{12x} = \frac{3}{x}$$

$$6x - 10x^2 = 36x$$

$$0 = 10x^2 + 30x$$

$$0 = 10x(x+3)$$

$$x = \cancel{0} \quad -3$$

27 Given $\cos \theta = -\dfrac{2}{7}$ with θ in Quadrant II, find the exact value of $\sin \theta$.

$$\left(-\frac{2}{7}\right)^2 + \sin^2\theta = 1$$

$$\frac{4}{49} + \sin^2\theta = \frac{49}{49}$$

$$\sin^2\theta = \frac{45}{49}$$

$$\sin\theta = \pm\frac{3\sqrt{5}}{7}$$

$$\frac{3\sqrt{5}}{7}$$

28 Given $a > 1$, use the properties of rational exponents to determine the value of x for the equation below.

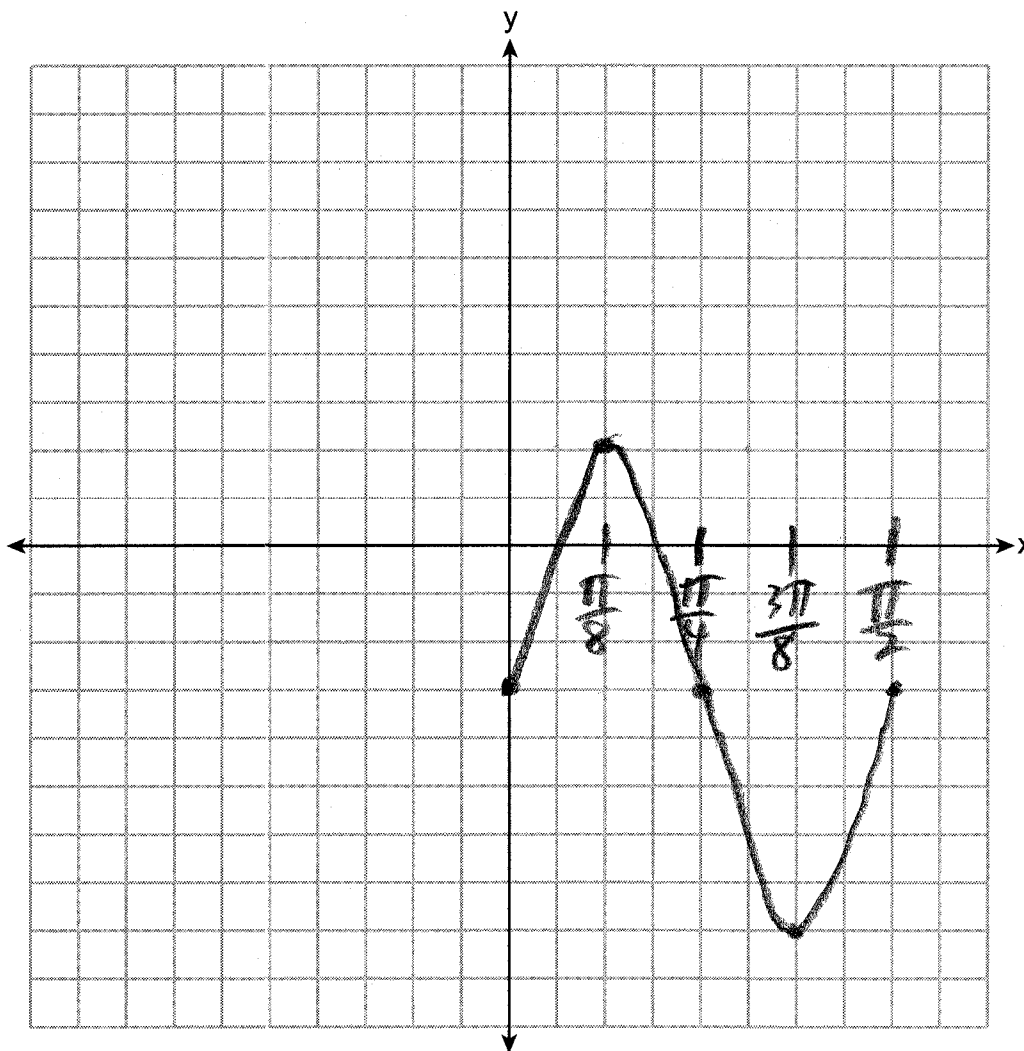$$\frac{\sqrt[5]{a^{10}}}{(a^3)^{\frac{1}{2}}} = a^x.$$

$$\frac{a^{\frac{10}{5}}}{a^{\frac{3}{2}}} = a^x$$

$$\frac{a^2}{a^{\frac{3}{2}}} = a^x$$

$$a^{\frac{1}{2}} = a^x$$

$$x = \frac{1}{2}$$

29 Graph *at least one* cycle of $y = 5\sin(4x) - 3$ on the set of axes below.

$$\text{period} = \frac{2\pi}{4} = \frac{\pi}{2}$$

30 The cost of a brand-new electric-hybrid vehicle is listed at \$33,400, and the average annual depreciation for the vehicle is 15%. The car's value can be modeled by the function $V(x) = 33,400(0.85)^x$, where x represents the years since purchase.

Julia and Jacob have each written a function that is equivalent to the original.

Jacob's function: $V(x) = 33,400(0.1422)^{\frac{1}{12}x}$

Julia's function: $V(x) = 33,400(0.9865)^{12x}$

Whose function is correctly rewritten to reveal the approximate monthly depreciation rate? Justify your answer.

$$Julia$$

$$V(x) = 33,400\left(.85^{\frac{1}{12}}\right)^{12x}$$

$$V(x) = 33,400\left(.9865\right)^{12x}$$

31 Write a recursive formula for the sequence 8, 20, 50, 125, 312.5,...

$$a_1 = 8$$

$$a_n = 2.5 a_{n-1}$$

32 A grocery store orders 50 bags of oranges from a company's distribution center. The bags have a mean weight of 3.85 pounds per bag. The company claims that their bags of oranges have a mean weight of 4 pounds. The grocery store ran a simulation of 50 bags, 2500 times, assuming a mean of 4 pounds. The results are shown below.

Mean = 4.001
SD = 0.026
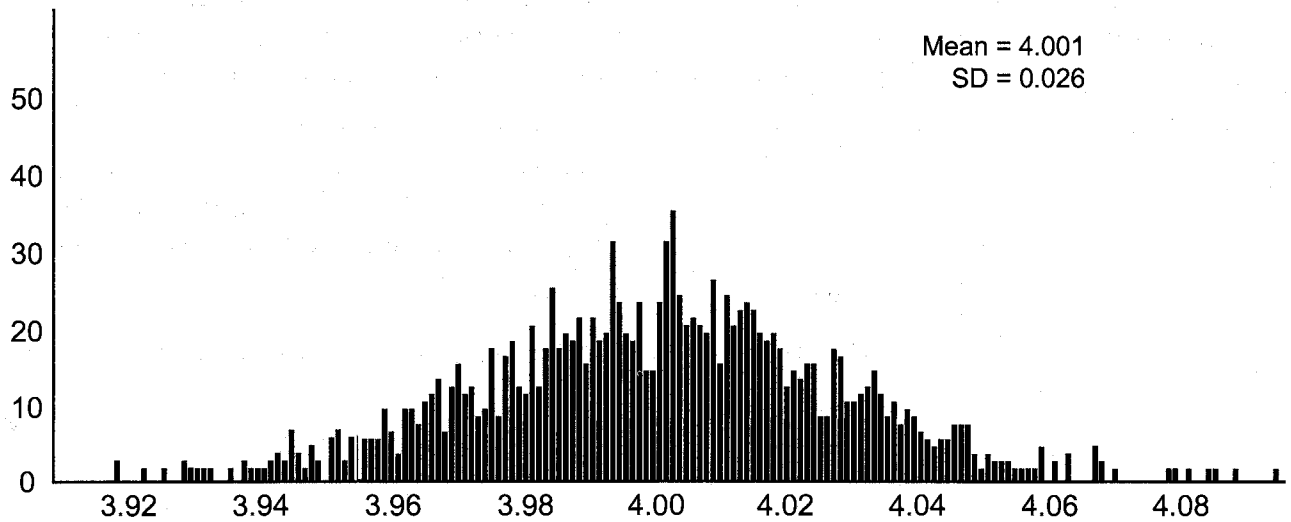
Is the mean weight of the grocery store's sample unusual? Explain using the results of the simulation.

Yes; Using a 95% confidence interval, values outside the interval 3.95 - 4.05 are unusual

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 At the Lakeside Resort, the probability that a guest room has a view of the lake is 0.24. The probability that a guest room has a queen-size bed is 0.74. Let A be the event that the guest room has a view of the lake, and let B be the event that the guest room has a queen-size bed. Events A and B are found to be independent of each other.

Determine the exact probability that a randomly selected guest room has a view of the lake and a queen-size bed.

$$.74 \cdot .24 = .1776$$

Determine the exact probability that a randomly selected guest room has a view of the lake or a queen-size bed.

$$.74 + .24 - .1776 = .8624$$

34 Which function has a greater average rate of change on the interval $[-1,4]$? Justify your answer.

x	m(x)
-2	-3
-1	1
0	1
1	3
2	13
3	37
4	81
5	151

$p(x) = 3^x + 1$

$$\frac{m(4) - m(-1)}{4 - -1}$$

$$\frac{81 - 1}{5}$$

$$\frac{80}{5}$$

$$\frac{p(4) - p(-1)}{4 - -1}$$

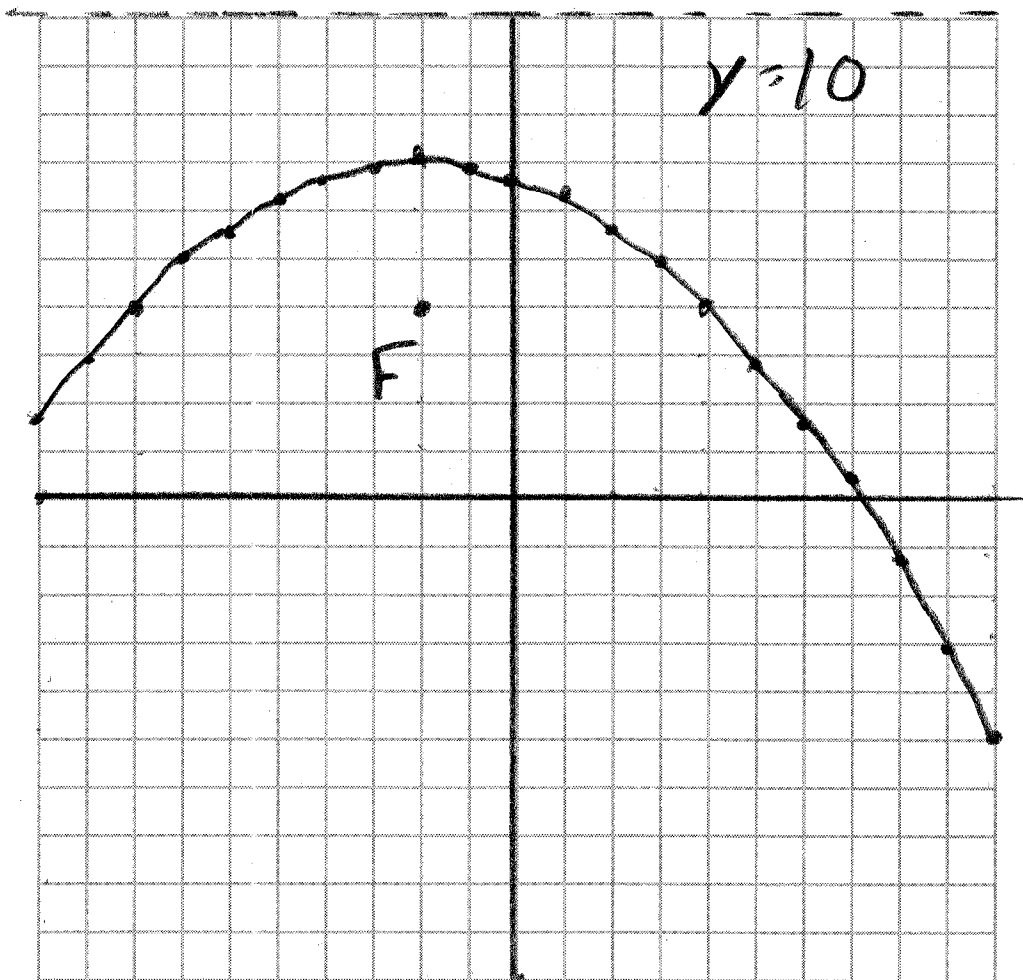$$\frac{81 + 1 - (\frac{1}{3} + 1)}{5}$$

$$\frac{82 - \frac{5}{3}}{5}$$

$p(x)$ has a greater rate of change

35 Determine an equation for the parabola with focus $(-2, 4)$ and directrix $y = 10$.
   (The use of the grid below is optional.)

$$\frac{4+10}{2} = 7 \quad p = 3$$

$$y = \frac{-1}{4(3)}(x+2)^2 + 7 \quad \text{Vertex } (-2, 7)$$

$$y = \frac{-1}{12}(x+2)^2 + 7$$

$y = 10$

F

36 Algebraically find the zeros of $c(x) = x^3 + 2x^2 - 16x - 32$.
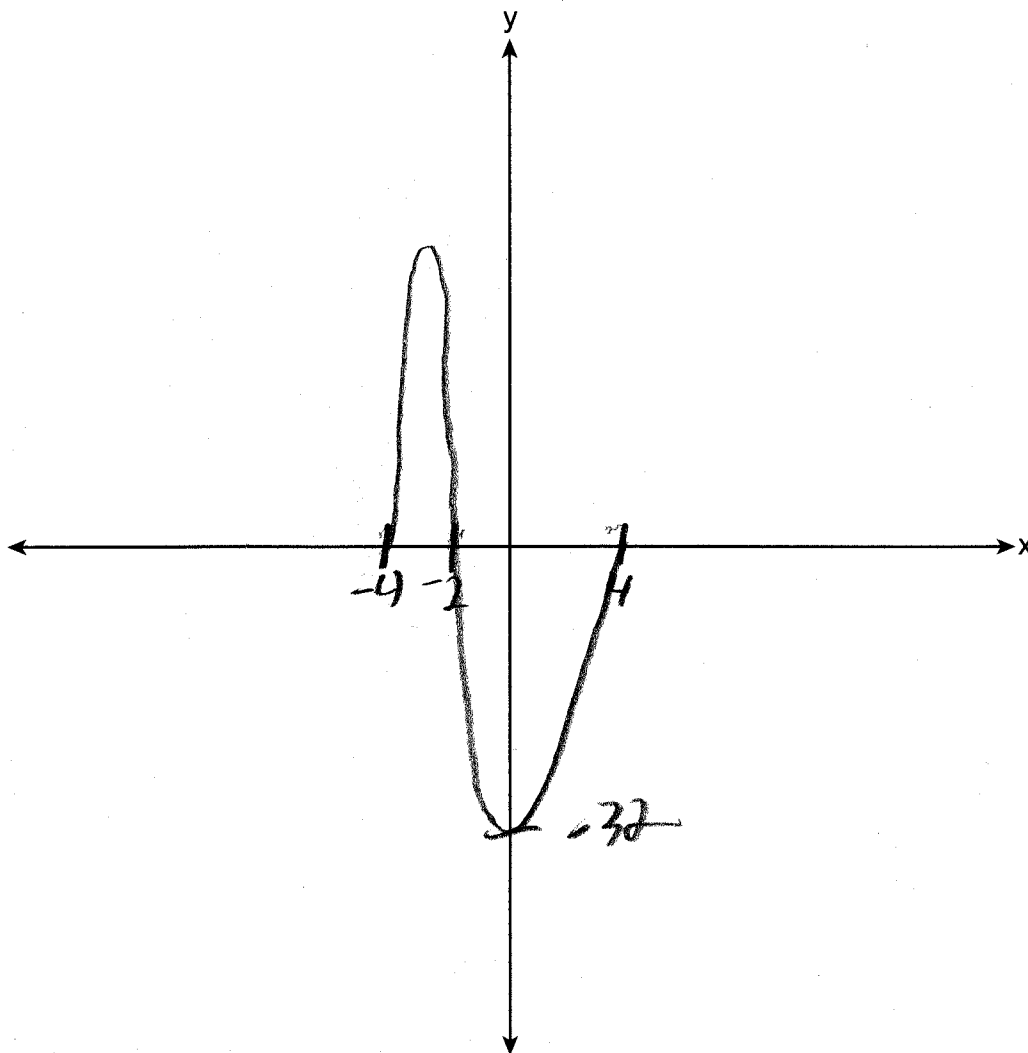
$$x^2(x+2) - 16(x+2) = 0$$
$$(x^2 - 16)(x+2) = 0$$
$$(x+4)(x-4)(x+2) = 0$$

$$-4 \quad 4 \quad -2$$

On the axes below, sketch $y = c(x)$.

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 The populations of honeybees in two different colonies are studied for four months. During this time, the colony population can be approximated by $P(t) = P_0 e^{rt}$, where $P(t)$ is the colony population of bees at t months, P_0 is the initial population, and r is the growth rate.

Colony A has an initial population of 10,000 bees and a continuous growth rate of 0.25. Colony B has an initial population of 6000 bees and a continuous growth rate of 0.45. Write functions for both $A(t)$ and $B(t)$ that model the honeybee populations of the colonies after t months.

$$A(t) = 10000e^{.25t}$$

$$B(t) = 6000e^{.45t}$$

Graph $A(t)$ and $B(t)$ for $0 \le t \le 4$.
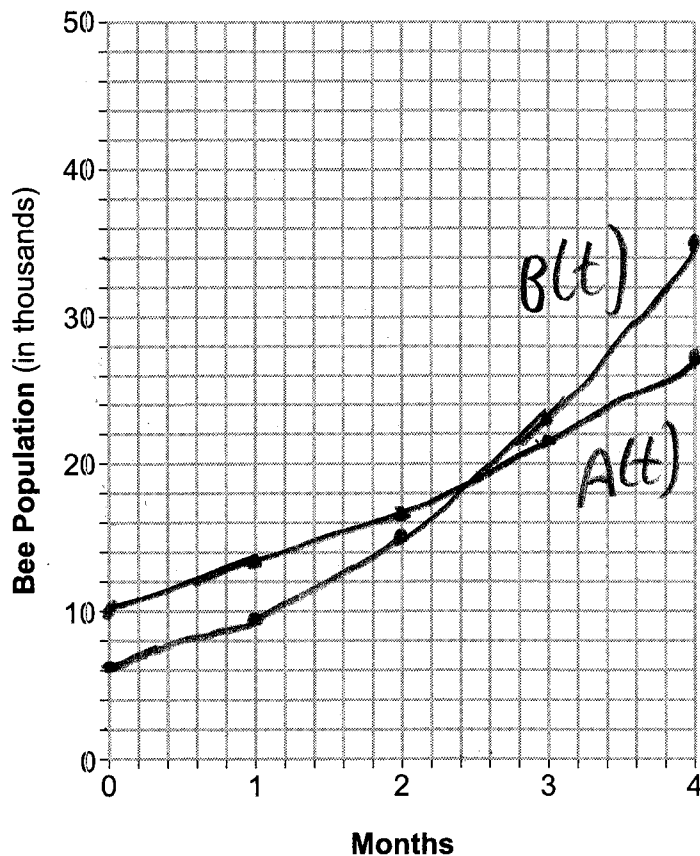
State, to the *nearest tenth of a month*, when the colonies will have the same population.

$$2.6$$

Determine algebraically how long it will take, to *the nearest tenth of a month*, for the population in Colony A to triple.

$$30,000 = 10000\, e^{.25t}$$

$$\ln 3 = \ln e^{.25t}$$

$$\frac{\ln 3}{.25} = \frac{.25t\, \ln e}{.25}$$

$$4.4 \approx t$$