The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Thursday, January 25, 2024 — 1:15 to 4:15 p.m., only

Student Name *Mr. S. bol*

School Name *JMAP*

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

## DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 A cafeteria food manager studied the lunchtime eating habits of a group of employees in their office building. The purpose of the study was to determine the proportion of employees who purchased lunch in the cafeteria, brought their lunch from home, or purchased lunch from an outside vendor. This collection of data would best be classified as

(1) a census

(3) an observational study

(2) an experiment

(4) a simulation

2 Which graph has imaginary roots?

(1)

(3)

(2)

(4)
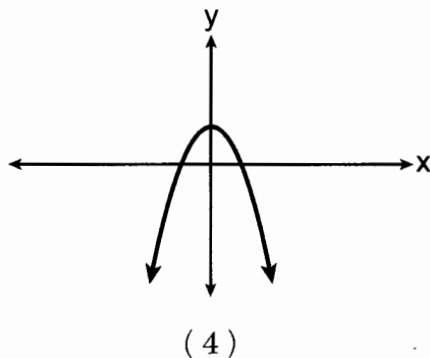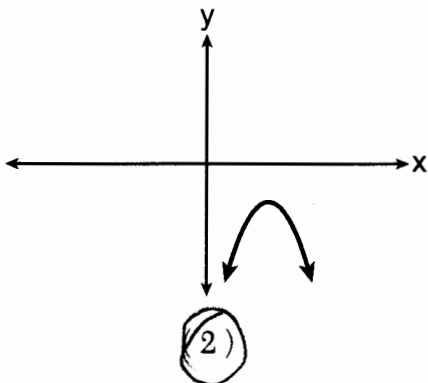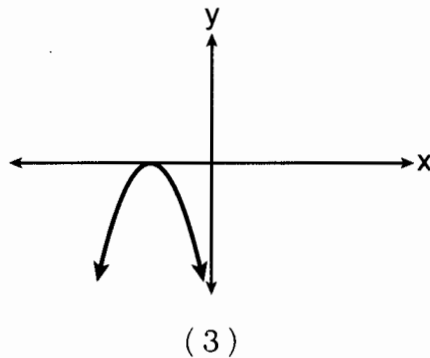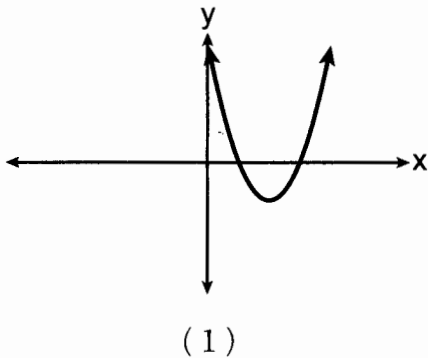
3 Given 3 is a root of $f(x) = x^4 - x^3 - 21x^2 + 45x$, what are the other unique roots of $f(x)$?

(1) −5, only   (3) −3, 1 and 5

(2) −5 and 0   (4) −5, −3 and 0

(handwritten: (2) circled)

(handwritten computations:
$$3 | 1 \quad -1 \quad -21 \quad 45 \quad 0$$
$$\quad 3 \quad 6 \quad -45 \quad 0$$
$$\overline{ 1 \quad 2 \quad -15 \quad 0 \quad 0}$$
$$x^3 + 2x^2 - 15x$$
$$x(x^2 + 2x - 15)$$
$$x(x+5)(x-3)$$
$$0 \quad -5 \quad 3$$
)

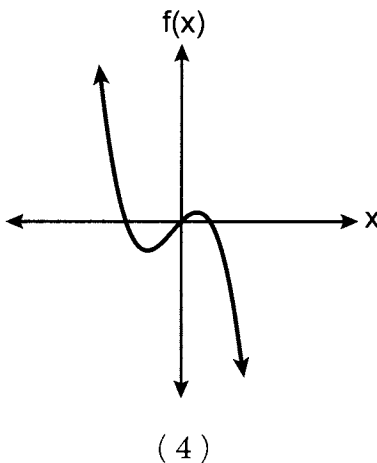**4** Given $p \ne q$, $p = \left(\frac{1}{2}\right)^q$, expressed in logarithmic form, is equivalent to

(1) $\log_p\left(\frac{1}{2}\right) = q$ (3) $\log_{\frac{1}{2}}(p) = q$

(2) $\log_q(p) = \frac{1}{2}$ (4) $\log_{\frac{1}{2}}(q) = p$

(handwritten: (3) circled)

5 Which graph best represents the graph of $f(x) = (x + a)^2(x - b)$, where a and b are positive real numbers?

(handwritten: "leading coefficient is positive", "repeated root at −a")

(1)

(3)

(2)

(4)

(handwritten: (1) circled)
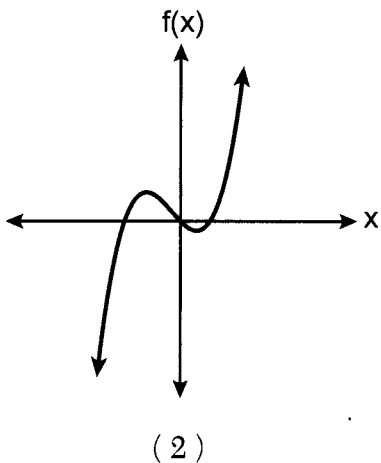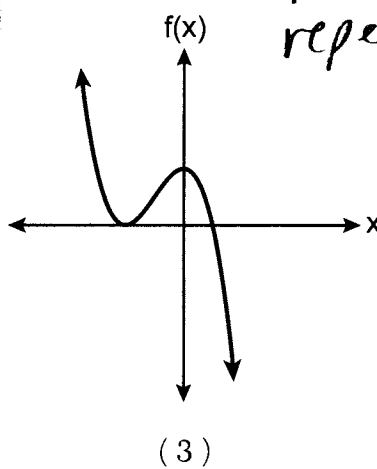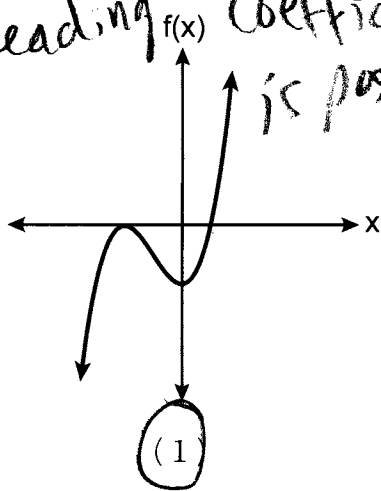
6 The equations $y = 3t + 6$ and $y = (1.82)^t$ approximately model the growth of two separate populations where $t > 0$. What is the best approximation of the time, t, at which the populations are the same?

(1) -1.9 (3) 5.1

(2) 0.3   (4) 21.3

7 Given $y = -2x$ and $x^2 + y^2 = 5$, the point of intersection in Quadrant II is

(1) $(1, -2)$ (3) $(-1, 1)$

(2) $(-2, 1)$ (4) $(-1, 2)$

$$x^2 + (-2x)^2 = 5$$
$$5x^2 = 5$$
$$x^2 = 1$$
$$x = \pm 1$$
$$y = -2(-1) = 2$$
$$(-1, 2)$$

8 The rational expression $\dfrac{2x^4 - 5x^2 + 3x - 2}{x - 3}$ is equivalent to

(1) $2x^3 - 5x - 12 - \dfrac{38}{x - 3}$

(2) $2x^3 + 6x^2 + 13x + 42 + \dfrac{124}{x - 3}$

(3) $2x^3 - 5x + 18 - \dfrac{56}{x - 3}$

(4) $2x^3 - 6x^2 + 13x - 36 + \dfrac{106}{x - 3}$

$$\begin{array}{r|rrrrr} 3 & 2 & 0 & -5 & 3 & -2 \\ & & 6 & 18 & 39 & 126 \\ \hline & 2 & 6 & 13 & 42 & 124 \end{array}$$

9 The equation of the parabola that has its focus at the point $(-3, 2)$ and directrix at $y = 0$ is

(1) $y = \frac{1}{4}(x + 3)^2 + 1$ (3) $y = \frac{1}{8}(x + 3)^2 + 1$

(2) $y = \frac{1}{4}(x - 3)^2 + 1$ (4) $y = \frac{1}{8}(x - 3)^2 + 1$

(handwritten) distance from focus to directrix is 2
p = 1   Vertex (-3, 1)
$y = \frac{1}{4(1)}(x+3)^2 + 1$

10 The seventh term of the geometric sequence $\sqrt{6}, -2\sqrt{3}, 2\sqrt{6}, -4\sqrt{3} \ldots$ is

(1) $6\sqrt{6}$ (3) $8\sqrt{6}$

(2) $-6\sqrt{3}$ (4) $-8\sqrt{3}$

(handwritten) $r = \frac{-2\sqrt{3}}{\sqrt{6}} = \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-2\sqrt{2}}{2} = -\sqrt{2}$

$a_7 = \sqrt{6}(\sqrt{2})^{7-1}$
$= \sqrt{6}((-2)^{\frac{1}{2}})^6$
$8\sqrt{6}$

11 A company wishes to determine the cooking time for one pound of spaghetti. The company's technicians cooked one pound of spaghetti and recorded the time needed for the spaghetti to be ready to eat. Repeating this process 35 times resulted in an approximately normal distribution, with a mean of 9.82 minutes and a standard deviation of 1.4 minutes. In which interval should the middle 95% of cooking times fall?

(1) (8.42, 11.22)   (3) (9.35, 10.29)

(2) (7.02, 12.62)   (4) (6.82, 11.32)

(handwritten) $9.82 \pm 2(1.4)$

Use this space for computations.

12 Given $f(x) = 2x^2 + 7x - 15$ and $g(x) = 3 - 2x$, what is $\dfrac{f(x)}{g(x)}$ for all defined values?

(1) $-x - 5$ (3) $x - 5$

(2) $-x + 5$ (4) $x + 5$

$$\frac{2x^2 + 7x - 15}{-(2x-3)} \quad = \quad \frac{(2x-3)(x+5)}{-(2x-3)}$$

$$\frac{(x+5)}{-1}$$

13 Which equation is equivalent to $P = 210x^{\frac{4}{3}}y^{\frac{7}{3}}$?

(1) $P = \sqrt[3]{210x^4y^7}$ (3) $P = 210xy^2\sqrt[3]{xy}$

(2) $P = 70xy^2\sqrt[3]{xy}$ (4) $P = 210xy^2\sqrt[3]{x^3y^5}$

$$210x^{\frac{3}{3}} \cdot x^{\frac{1}{3}} \cdot y^{\frac{6}{3}} \cdot y^{\frac{1}{3}}$$

14 The average cost of a gallon of milk in the United States between the years of 1995 and 2018 can be modeled by the equation $P(t) = -0.0004t^3 + 0.0114t^2 - 0.0150t + 2.6602$, where $P(t)$ represents the cost, in dollars, and t is time in years since January 1995. During this time period, in what year did $P(t)$ reach its maximum?

(1) 1995   (3) 2014

(2) 2013   (4) 2018

15 The temperature, F, in degrees Fahrenheit, after t hours of a roast put into an oven is given by the equation $F = 325 - 185e^{-0.4t}$. What was the temperature of the roast when it was put into the oven?

(1) 325

(3) 185

(2) 200

(4) 140

$F(0) = 325 - 185e^{-0.4(0)}$

$= 325 - 185$

16 The roots of the equation $0 = x^2 + 6x + 10$ in simplest $a + bi$ form are

(1) $-3 \pm 2i$

(3) $-3 \pm i$

(2) $-6 \pm i$

(4) $-3 \pm i\sqrt{2}$

$x^2 + 6x + 9 = -10 + 9$

$(x+3)^2 = -1$

$x+3 = \pm i$

$x = -3 \pm i$

17 Which equation does *not* represent an identity?

(1) $x^2 - y^2 = (x + y)(x - y)$

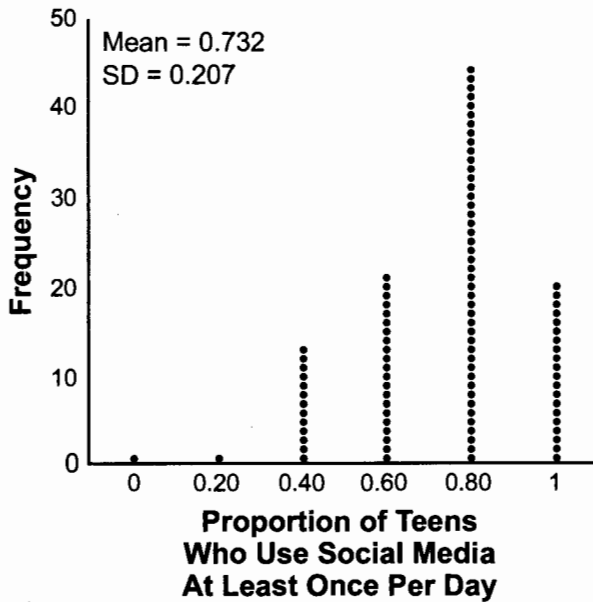(2) $(x - y)^2 = (x - y)(x - y)$

(3) $(x + y)^2 = x^2 + 2xy + y^2$

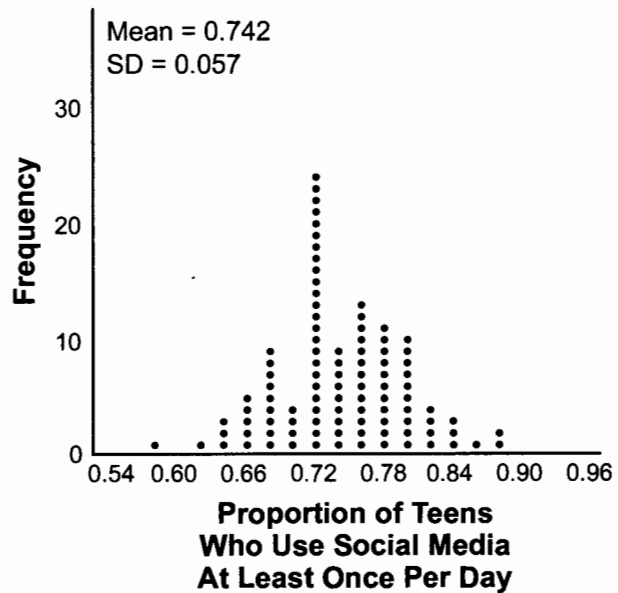(4) $(x + y)^3 = x^3 + 3xy + y^3$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

18 Two surveys were conducted to estimate the proportion of teens who use social media at least once per day.

100 Samples of 5 Teens

Mean = 0.732
SD = 0.207

(Frequency graph with axis: 0, 10, 20, 30, 40, 50; x-axis: 0, 0.20, 0.40, 0.60, 0.80, 1)

Proportion of Teens Who Use Social Media At Least Once Per Day

100 Samples of 50 Teens

Mean = 0.742
SD = 0.057

(Frequency graph with axis: 0, 10, 20, 30; x-axis: 0.54, 0.60, 0.66, 0.72, 0.78, 0.84, 0.90, 0.96)

Proportion of Teens Who Use Social Media At Least Once Per Day

Based on these results, it was determined that approximately 75% of teens use social media at least once per day. What is the best explanation of the difference in the results between the two surveys?

(1) The smaller sample size of five teens resulted in a smaller margin of error and should provide a more accurate estimate.

(2) The smaller sample size of five teens resulted in a bigger margin of error and should provide a more accurate estimate.

(3) The larger sample size of 50 teens resulted in a smaller margin of error and should provide a more accurate estimate.

(4) The larger sample size of 50 teens resulted in a bigger margin of error and should provide a more accurate estimate.

19 Given $f(x) = x^3 - 3$ and $f^{-1}(x) = \sqrt[3]{x - 3b}$, the value of b is

(1) 1   (3) 3

(2) −1   (4) −3

$X = y^3 - 3$

$X + 3 = y^3$

$\sqrt[3]{X+3} = y$

20 Robert is buying a car that costs $22,000. After a down payment of $4000, he borrows the remainder from a bank, a six year loan at 6.24% annual interest rate. The following formula can be used to calculate his monthly loan payment.

Use this space for computations.

$$R = \frac{(P)(i)}{1 - (1 + i)^{-t}}$$

$\frac{18600 \times .52\%}{1 - (1 + .52\%)^{-72}} \approx 300.36$

R = monthly payment
P = loan amount
i = monthly interest rate
t = time, in months

$.52\% = \frac{6.24\%}{12}$

$12 \cdot 6$

Robert's monthly payment will be

(1) $298.31   (3) $307.35

(②) $300.36   (4) $367.10

21 Given $\tan \theta = -\frac{4}{3}$ where $\frac{\pi}{2} < \theta < \pi$, what is the value of $\sec \theta$?

(①) $-\frac{5}{3}$ $\cos \theta = \frac{-3}{5}$ (3) $\frac{4}{5}$

(2) $-\frac{3}{5}$ (4) $\frac{5}{3}$

$\sec \theta = -\frac{5}{3}$

22 To solve the equation $\frac{7}{x + 7} + \frac{4x}{x - 7} = \frac{3x + 7}{x - 7}$, Joan's first step is to multiply both sides by the least common denominator. Which statement is true?

(1) -14 is an extraneous solution.

(2) 7 and -7 are extraneous solutions.

(③) 7 is an extraneous solution.

(4) There are no extraneous solutions.

$7(x-7) + 4x(x+7) = (3x+7)(x+7)$

$7x - 49 + 4x^2 + 28x = 3x^2 + 21x + 7x + 49$

$4x^2 + 35x - 49 = 3x^2 + 28x + 49$

$x^2 + 7x - 98 = 0$

$(x + 14)(x - 7) = 0$

$-14 \quad 7$

Algebra II – Jan. '24   [9]   [OVER]

23 Beginning July 1, 2019, Michelle deposited $250 into an account that yields 0.15% each month. She continued to make $250 deposits into this account on the first of each month for 3 years. Which expression represents the amount of money that was in the account after her last deposit was made on June 1, 2022?

(1) $250(1.0015)^3$

(3) $\dfrac{250 - 250(1.0015)^3}{1 - 1.0015}$

(2) $250(1.0015)^{36}$

(4) $\dfrac{250 - 250(1.0015)^{36}}{1 - 1.0015}$

24 A study of the red tailed hawk population in a given area shows the population, $H(t)$, can be represented by the function $H(t) = 50(1.19)^t$ where t represents the number of years since the study began. In terms of the monthly rate of growth, the population can be best approximated by the function

(1) $H(t) = 50(1.015)^{12t}$

(3) $H(t) = 50(1.19)^{12t}$

(2) $H(t) = 50(1.15)^{\frac{t}{12}}$

(4) $H(t) = 50(1.19)^{\frac{t}{12}}$

$$50\left(1.19^{\frac{1}{12}}\right)^{12t} \approx 50(1.015)^{12t}$$

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.   [16]

25 Factor $x^3 + 4x^2 - 9x - 36$, completely.

$$x^2(x+4) - 9(x+4)$$
$$(x^2-9)(x+4)$$
$$(x+3)(x-3)(x+4)$$

26 Determine if $x + 4$ is a factor of $2x^3 + 10x^2 + 4x - 16$. Explain your answer.

$$\begin{array}{r|rrrr} -4 & 2 & 10 & 4 & -16 \\ & & -8 & -8 & 16 \\ \hline & 2 & 2 & -4 & 0 \end{array}$$

Yes, because there is no remainder

27 An initial investment of $1000 reaches a value, $V(t)$, according to the model $V(t) = 1000(1.01)^{4t}$, where t is the time in years.

Determine the average rate of change, to the *nearest dollar per year*, of this investment from year 2 to year 7.

$$\frac{V(7) - V(2)}{7 - 2} \approx 48$$

28 When $\left(\dfrac{1}{\sqrt[3]{y^2}}\right) y^4$ is written in the form y^n, what is the value of n? Justify your answer.

$$\dfrac{y^{\frac{12}{3}}}{y^{\frac{2}{3}}} = y^{\frac{10}{3}}$$

$$\dfrac{10}{3}$$

29 The heights of the members of a ski club are normally distributed. The average height is 64.7 inches with a standard deviation of 4.3 inches. Determine the percentage of club members, to the *nearest percent*, who are between 67 inches and 72 inches tall.

normal cdf(67,72,64.7,4.3) ≈ 25

30 The explicit formula $a_n = 6 + 6n$ represents the number of seats in each row in a movie theater, where n represents the row number. Rewrite this formula in recursive form.

$$a_1 = 12$$
$$a_n = a_{n-1} + 6$$

31 Express $(2xi^3 - 3y)^2$ in simplest form.

$$4x^2 i^6 - 12xyi^3 + 9y^2$$
$$-4x^2 + 12xyi + 9y^2$$

32 A survey was given to 1250 randomly selected high school students at the end of their junior year. The survey offered four post-graduation options: two-year college, four-year college, military, or work. Of the 1250 responses, 475 chose a four-year college. State *one* possible conclusion that can be made about the population of high school juniors, based on this survey.

$$\frac{475}{1250} = 38\%$$

About 38% of high school juniors in the population will choose a four-year college.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 A researcher wants to determine if nut allergies and milk allergies are related to each other. The researcher surveyed 1500 people and asked them if they are allergic to nuts or milk. The survey results are summarized in the table below.

	Allergic to Nuts	Not Allergic to Nuts	
Allergic to Milk	3	42	45
Not Allergic to Milk	12	1443	1455
	15	1485 1500	

Determine the probability that a randomly selected survey respondent is allergic to milk.

$$\frac{45}{1500}$$

Determine the probability that a randomly selected survey respondent is allergic to milk, given that the person is allergic to nuts.

$$\frac{3}{15}$$

Based on the survey data, determine whether nut allergies and milk allergies are independent events. Justify your answer. No, because someone is more likely to be allergic to milk if they are also allegic to nuts,

34 Algebraically solve for x: $2x = 6 + 2\sqrt{x - 1}$

$$2x - 6 = 2\sqrt{x-1}$$

$$4x^2 - 24x + 36 = 4(x-1)$$

$$x^2 - 6x + 9 = x - 1$$

$$x^2 - 7x + 10 = 0$$

$$(x - 5)(x - 2) = 0$$

$$5 \qquad \cancel{2} \qquad 2(2) - 6 < 0$$

35 During the summer, Adam saved $4000 and Betty saved $3500. Adam deposited his money in Bank A at an annual rate of 2.4% compounded monthly. Betty deposited her money in Bank B at an annual rate of 4% compounded quarterly. Write two functions that represent the value of each account after t years if no other deposits or withdrawals are made, where Adam's account value is represented by $A(t)$, and Betty's by $B(t)$.

$$A(t) = 4000\left(1 + \frac{2.4\%}{12}\right)^{12t}$$

$$B(t) = 3500\left(1 + \frac{4\%}{4}\right)^{4t}$$

Using technology, determine, to the *nearest tenth of a year*, how long it will take for the two accounts to have the same amount of money in them. Justify your answer.

8.4, the value of t

For which $A(t) = B(t)$.

36 On the graph below, draw *at least one* complete cycle of a sine graph passing through point (0,2) that has an amplitude of 3, a period of π, and a midline at $y = 2$.

$$y = 3\sin 2x + 2$$
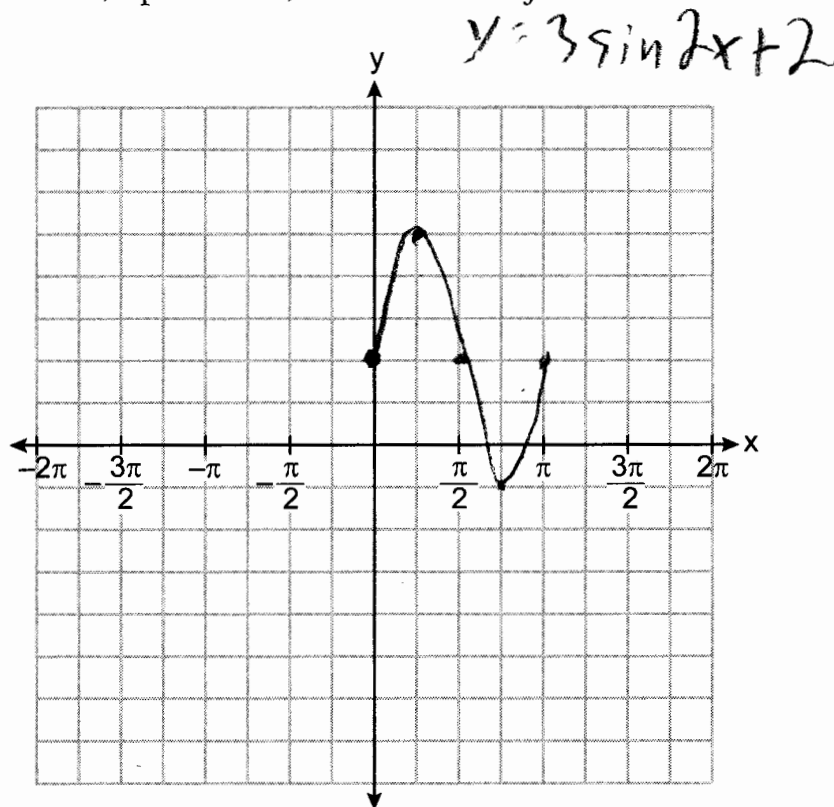
Based on your graph, state an interval in which the graph is increasing.

$$0 < x < \frac{\pi}{4}$$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 A manufacturer of sweatshirts finds that profits and costs fluctuate depending on the number of products created. Creating more products doesn't always increase profits because it requires additional costs, such as building a larger facility or hiring more workers. The manufacturer determines the profit, $p(x)$, in thousands of dollars, as a function of the number of sweatshirts sold, x, in thousands. This function, p, is given below.

$$p(x) = -x^3 + 11x^2 - 7x - 69$$

Graph $y = p(x)$, over the interval $0 \le x \le 9$, on the set of axes below.
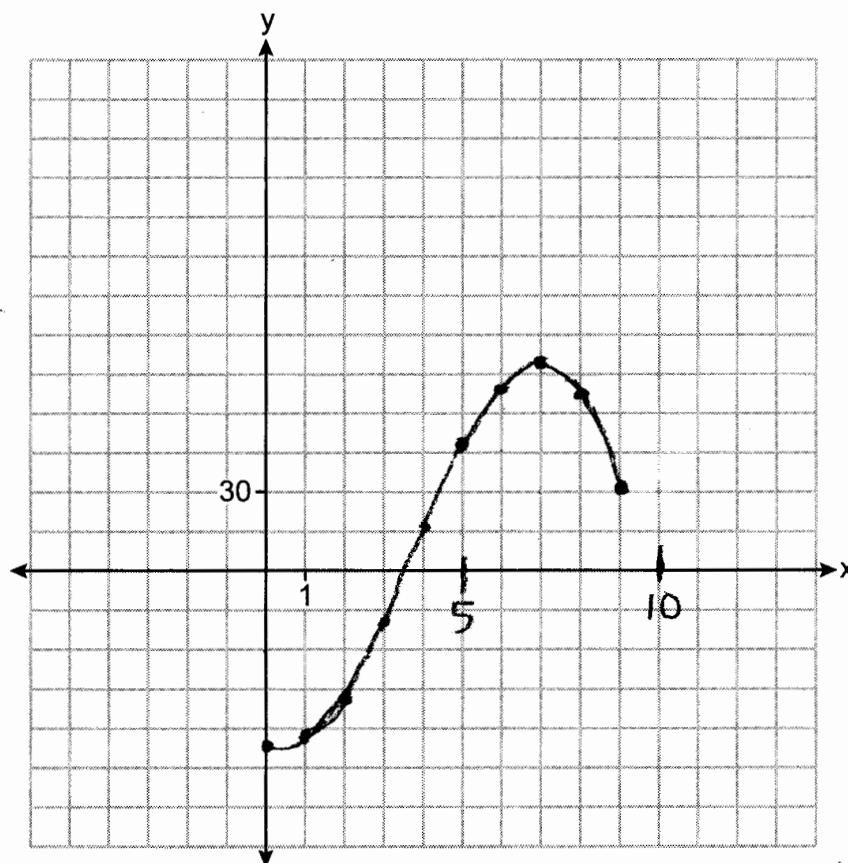
Over the given interval, state the coordinates of the maximum of p and round all values to the *nearest integer*. Explain what this point represents in terms of the number of sweatshirts sold and profit.

$(7, 78)$

IF 7000 sweatshirts are sold, the profit is $78,000

Determine how many sweatshirts, to the *nearest whole sweatshirt*, the manufacturer would need to produce in order to first make a positive profit. Justify your answer.

3,549, because that is when $p(x)$ is first greater than zero