The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II (Common Core)

Friday, January 27, 2017 — 9:15 a.m. to 12:15 p.m., only.

Student Name: _My. Sibol_

School Name: _JMAP_

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 Relative to the graph of $y = 3\sin x$, what is the shift of the graph of $y = 3\sin\left(x + \frac{\pi}{3}\right)$?

(1) $\frac{\pi}{3}$ right

(3) $\frac{\pi}{3}$ up

(2) $\frac{\pi}{3}$ left

(4) $\frac{\pi}{3}$ down

2 A rabbit population doubles every 4 weeks. There are currently five rabbits in a restricted area. If t represents the time, in weeks, and $P(t)$ is the population of rabbits with respect to time, about how many rabbits will there be in 98 days?

(1) 56

(3) 3688

(2) 152

(4) 81,920

3 When factored completely, $m^5 + m^3 - 6m$ is equivalent to

(1) $(m + 3)(m - 2)$

(3) $m(m^4 + m^2 - 6)$

(2) $(m^3 + 3m)(m^2 - 2)$

(4) $m(m^2 + 3)(m^2 - 2)$

4 If $\sin^2(32°) + \cos^2(M) = 1$, then M equals

(1) 32°
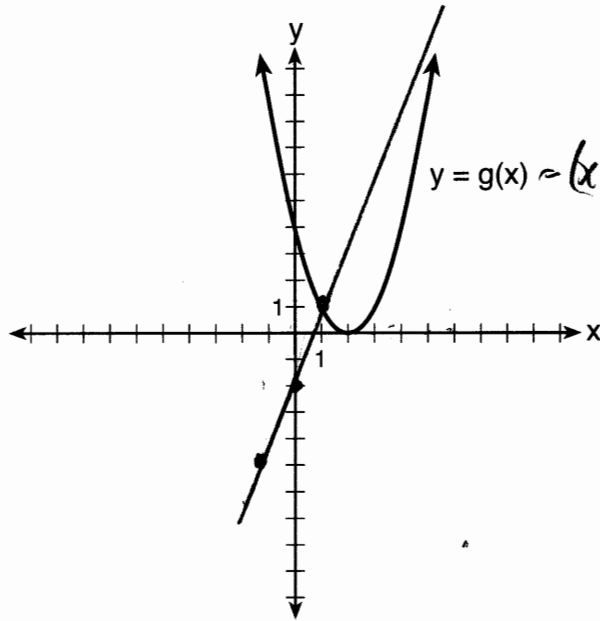
(3) 68°

(2) 58°

(4) 72°

5 What is the solution to the system of equations $y = 3x - 2$ and $y = g(x)$ where $g(x)$ is defined by the function below?

$y = g(x) = (x - 2)^2$

$x^2 - 4x + 4 = 3x - 2$
$x^2 - 7x + 6 = 0$
$(x - 6)(x - 1) = 0$
$x = 6, 1$

$y = 3(6) - 2 = 16$
$y = 3(1) - 2 = 1$

(1) $\{(0,-2)\}$

(3) $\{(1,6)\}$

(2) $\{(0,-2),(1,6)\}$

(4) $\{(1,1),(6,16)\}$

6 Which statement about statistical analysis is *false*?

(1) Experiments can suggest patterns and relationships in data.

(2) Experiments can determine cause and effect relationships.

(3) Observational studies can determine cause and effect relationships.

(4) Observational studies can suggest patterns and relationships in data.

7 The expression $\left(\dfrac{m^2}{m^{\frac{1}{3}}}\right)^{-\frac{1}{2}}$ is equivalent to

$\left(m^{5/3}\right)^{-1/2} = m^{-5/6} = \dfrac{1}{m^{5/6}}$

(1) $-\sqrt[6]{m^5}$

(3) $-m\sqrt[5]{m}$

(2) $\dfrac{1}{\sqrt[6]{m^5}}$

(4) $\dfrac{1}{m\sqrt[5]{m}}$

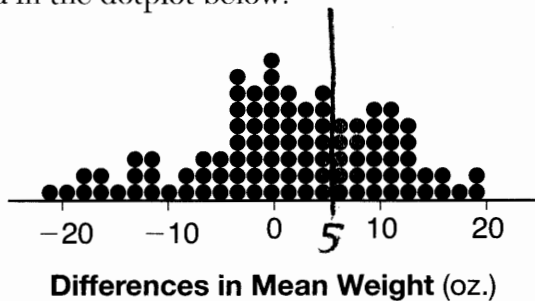8 What is the inverse of the function $y = \log_3 x$?

(1) $y = x^3$

(3) $y = 3^x$

(2) $y = \log_x 3$

(4) $x = 3^y$

9 Gabriel performed an experiment to see if planting 13 tomato plants in black plastic mulch leads to larger tomatoes than if 13 plants are planted without mulch. He observed that the average weight of the tomatoes from tomato plants grown in black plastic mulch was 5 ounces greater than those from the plants planted without mulch. To determine if the observed difference is statistically significant, he rerandomized the tomato groups 100 times to study these random differences in the mean weights. The output of his simulation is summarized in the dotplot below.

Differences in Mean Weight (oz.)

Given these results, what is an appropriate inference that can be drawn?

(1) There was no effect observed between the two groups.

(2) There was an effect observed that could be due to the random assignment of plants to the groups.

(3) There is strong evidence to support the hypothesis that tomatoes from plants planted in black plastic mulch are larger than those planted without mulch.

(4) There is strong evidence to support the hypothesis that tomatoes from plants planted without mulch are larger than those planted in black plastic mulch.

10 If $p(x) = ab^x$ and $r(x) = cd^x$, then $p(x) \cdot r(x)$ equals

(1) $ac(b + d)^x$

(3) $ac(bd)^x$

(2) $ac(b + d)^{2x}$

(4) $ac(bd)^{x^2}$

$$6x^2 - 8x + 29 = 0$$

11 The solution to the equation $18x^2 - 24x + 87 = 0$ is

(1) $-\frac{2}{3} \pm 6i\sqrt{158}$

(3) $\frac{2}{3} \pm 6i\sqrt{158}$

(2) $-\frac{2}{3} \pm \frac{1}{6}i\sqrt{158}$

(4) $\frac{2}{3} \pm \frac{1}{6}i\sqrt{158}$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(6)(29)}}{2(6)}$$

$$x = \frac{8 \pm \sqrt{-632}}{12} = \frac{8 \pm i\sqrt{4}\sqrt{158}}{12}$$

12 When $g(x) = \dfrac{2}{x+2}$ and $h(x) = \log(x + 1) + 3$ are graphed on the same set of axes, which coordinates best approximate their point of intersection?
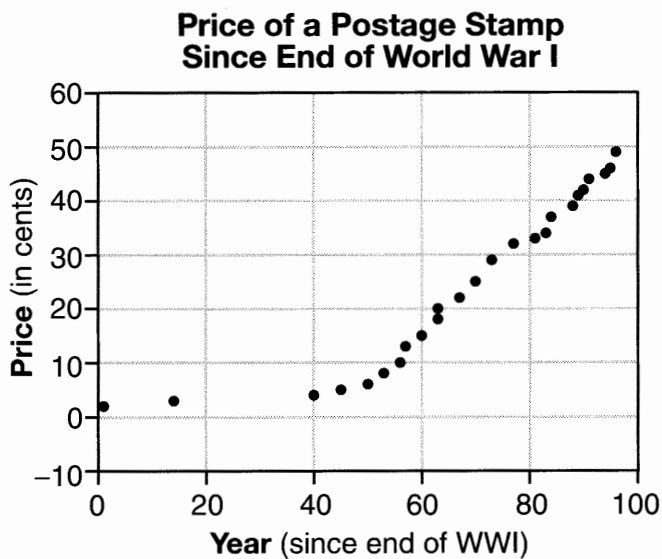
(1) $(-0.9, 1.8)$

(3) $(1.4, 3.3)$

(2) $(-0.9, 1.9)$

(4) $(1.4, 3.4)$

13 The price of a postage stamp in the years since the end of World War I is shown in the scatterplot below.

Price of a Postage Stamp Since End of World War I

• Price of a postage stamp

The equation that best models the price, in cents, of a postage stamp based on these data is

not linear

(1) $y = 0.59x - 14.82$

(3) $y = 1.43(1.04)^x$

(2) $y = 1.04(1.43)^x$

(4) $y = 24\sin(14x) + 25$

growth rate too high

not periodic

[5]

[OVER]

14 The eighth and tenth terms of a sequence are 64 and 100. If the sequence is either arithmetic or geometric, the ninth term can *not* be

(1) −82   (3) 80

(2) −80   (4) 82

$64, 82, 100 \quad d=18$

$64, 80, 100 \quad r=\frac{5}{4}$

$64, -80, 100 \quad r=-\frac{5}{4}$

15 The loudness of sound is measured in units called decibels (dB). These units are measured by first assigning an intensity I_0 to a very soft sound that is called the threshold sound. The sound to be measured is assigned an intensity, I, and the decibel rating, d, of this sound is found using $d = 10 \log \frac{I}{I_0}$. The threshold sound audible to the average person is 1.0×10^{-12} W/m² (watts per square meter).

Consider the following sound level classifications:

Moderate	45-69 dB
Loud	70-89 dB
Very loud	90-109 dB
Deafening	>110 dB

$d = 10 \log \dfrac{6.3 \times 10^{-3}}{1 \times 10^{-12}}$

≈ 98

How would a sound with intensity 6.3×10^{-3} W/m² be classified?

(1) moderate   (3) very loud

(2) loud   (4) deafening

16 Pedro and Bobby each own an ant farm. Pedro starts with 100 ants and says his farm is growing exponentially at a rate of 15% per month. Bobby starts with 350 ants and says his farm is steadily *decreasing* by 5 ants per month.

$P(x) = 100(1.15)^x$

$B(x) = 350 - 5x$

Assuming both boys are accurate in describing the population of their ant farms, after how many months will they both have approximately the same number of ants?

$100(1.15)^x = 350 - 5x$

$x \approx 8$

(1) 7   (3) 13

(2) 8   (4) 36

17 What is the solution, if any, of the equation

$$\frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12}?$$

(1) −1

(2) −5

(3) all real numbers

(4) no real solution

Handwritten work:

$$\frac{2(x-4)}{(x+3)(x-4)} + \frac{3(x+3)}{(x+3)(x-4)} = \frac{2x-2}{(x+3)(x-4)}$$

$$2x - 8 + 3x + 9 = 2x - 2$$

$$3x = -3$$

$$x = -1$$

18 In 2013, approximately 1.6 million students took the Critical Reading portion of the SAT exam. The mean score, the modal score, and the standard deviation were calculated to be 496, 430, and 115, respectively. Which interval reflects 95% of the Critical Reading scores?

(1) 430 ± 115

(2) 430 ± 230

(3) 496 ± 115

(4) 496 ± 230

Handwritten work: $496 \pm 2(115)$

19 Which statement regarding the graphs of the functions below is *untrue*?

$f(x) = 3 \sin 2x$, from $-\pi < x < \pi$ $\qquad h(x) = \log_2 x$

$g(x) = (x - 0.5)(x + 4)(x - 2)$ $\qquad j(x) = -|4x - 2| + 3$

(1) $f(x)$ and $j(x)$ have a maximum y-value of 3.

(2) $f(x)$, $h(x)$, and $j(x)$ have one y-intercept.

(3) $g(x)$ and $j(x)$ have the same end behavior as $x \to -\infty$.

(4) $g(x)$, $h(x)$, and $j(x)$ have rational zeros.

Handwritten note: $h(x)$ does not have a y-intercept

20 When $g(x)$ is divided by $x + 4$, the remainder is 0. Given $g(x) = x^4 + 3x^3 - 6x^2 - 6x + 8$, which conclusion about $g(x)$ is true?

(1) $g(4) = 0$

(2) $g(-4) = 0$

(3) $x - 4$ is a factor of $g(x)$.

(4) No conclusion can be made regarding $g(x)$.

21 Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of B dollars after m months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after m months.

m	B
0	1000.00
10	1172.00
19	1352.00
36	1770.80
60	2591.90
69	2990.00
72	3135.80
73	3186.00

(1) $\dfrac{2591.90 - 1172}{60 - 10} \approx 28.4\%$
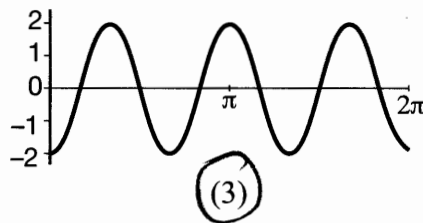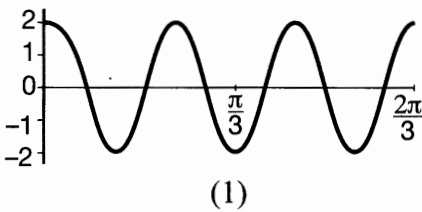
(2) $\dfrac{2990 - 1352}{69 - 19} \approx 32.8\%$

(3) $\dfrac{3135.8 - 1770.8}{72 - 36} \approx 37.9\%$
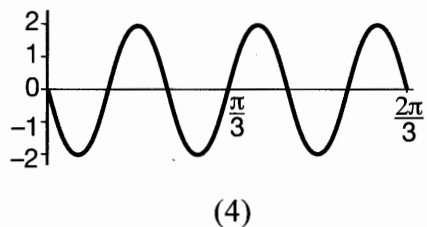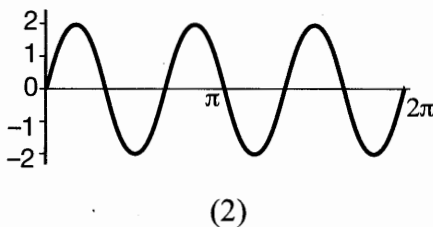
(4) $\dfrac{3186 - 2591.90}{73 - 60} \approx 45.7\%$

Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

(1) month 10 to month 60   (3) month 36 to month 72

(2) month 19 to month 69   (4) month 60 to month 73

22 Which graph represents a cosine function with no horizontal shift, an amplitude of 2, and a period of $\dfrac{2\pi}{3}$?

(1)

(3)

repeats 3 times over 2π

(2)

(4)

23 According to a pricing website, Indroid phones lose 58% of their cash value over 1.5 years. Which expression can be used to estimate the value of a $300 Indroid phone in 1.5 years?

(1) $300e^{-0.87}$

(2) $300e^{-0.63}$

(3) $300e^{-0.58}$

(4) $300e^{-0.42}$

$$\frac{A}{P} = \frac{\cancel{P}e^{rt}}{\cancel{P}}$$

$$\ln.42 = \ln e^{rt}$$

$$-0.87 \approx rt$$

24 A cardboard box manufacturing company is building boxes with length represented by $x + 1$, width by $5 - x$, and height by $x - 1$. The volume of the box is modeled by the function below.

Over which interval is the volume of the box changing at the fastest average rate?

(1) [1,2]

(2) [1,3.5]

(3) [1,5]

(4) [0,3.5]

(1) $\frac{9-0}{2-1} = 9$

(2) $\frac{17-0}{3.5-1} = 6.8$

(3) $\frac{0-0}{5-1} = 0$

(4) $\frac{17-(5)}{3.5-0} \approx 6.3$

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Express $(1 - i)^3$ in $a + bi$ form.

$$(1-i)(1-i)(1-i)$$
$$(1-2i+i^2)(1-i)$$
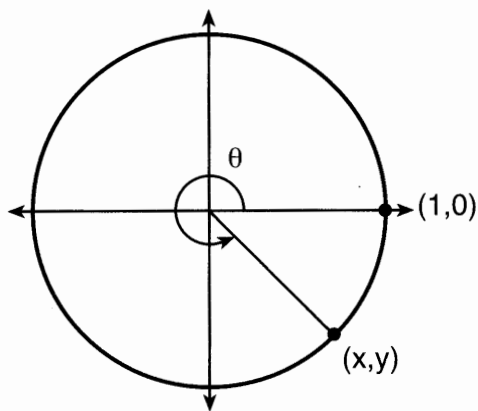$$-2i(1-i)$$
$$-2i+2i^2$$
$$-2-2i$$

26 An orange-juice processing plant receives a truckload of oranges. The quality control team randomly chooses three pails of oranges, each containing 50 oranges, from the truckload. Identify the sample and the population in the given scenario.

Sample - pails of oranges

population - truckload of oranges

State *one* conclusion that the quality control team could make about the population if 5% of the sample was found to be unsatisfactory.

It is likely that about 5% of all the oranges are unsatisfactory

27 Using the unit circle below, explain why $\csc\theta = \dfrac{1}{y}$.

$\csc\theta = \dfrac{1}{\sin\theta}$, & $\sin\theta$ on a unit circle represents the y value of a point on the unit circle.

Since $y = \sin\theta$ & $\csc\theta = \dfrac{1}{\sin\theta}$,
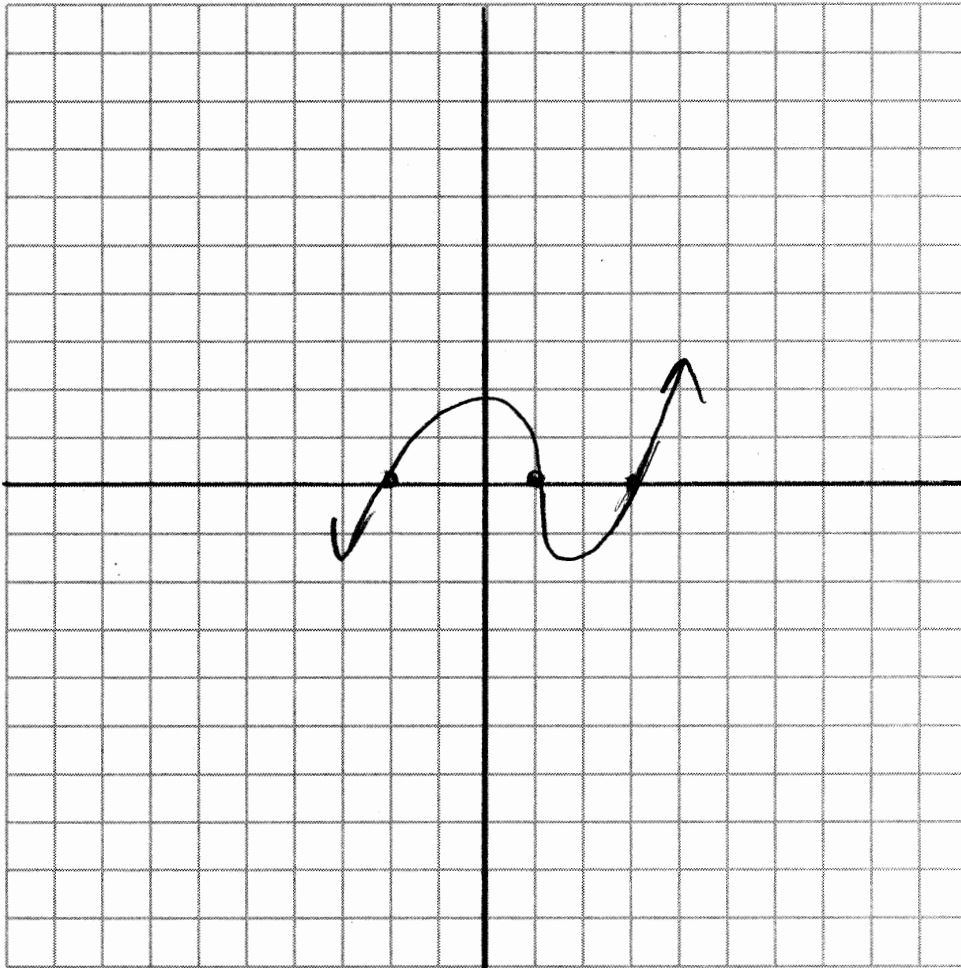
$\csc\theta = \dfrac{1}{y}$

28 The function $M(t)$ represents the mass of radium over time, t, in years.

$$M(t) = 100e^{\frac{\left(\ln\frac{1}{2}\right)t}{1590}}$$

Determine if the function $M(t)$ represents growth or decay. Explain your reasoning.

$\dfrac{\ln\frac{1}{2}}{1590}$ is negative, so $M(t)$ represents decay.

29 On the grid below, sketch a cubic polynomial whose zeros are 1, 3, and −2.

30 Given the equal terms $\sqrt[3]{x^5}$ and $y^{\frac{5}{6}}$, determine and state y, in terms of x.

$$\left(x^{\frac{5}{3}}\right)^{\frac{6}{5}} = \left(y^{5/6}\right)^{\frac{6}{5}}$$

$$x^2 = y$$

31 The results of a survey of the student body at Central High School about television viewing preferences are shown below.

	Comedy Series	Drama Series	Reality Series	Total
Males	95	65	70	230
Females	80	70	110	260
Total	175	135	180	490

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.

No

The events are independent if

$$P(M/R) = P(M)$$

$$\frac{70}{180} \neq \frac{230}{490}$$

$$.38 \neq .47$$

32 Given $f(x) = 3x^2 + 7x - 20$ and $g(x) = x - 2$, state the quotient and remainder of $\dfrac{f(x)}{g(x)}$, in the form $q(x) + \dfrac{r(x)}{g(x)}$.

$$
\begin{array}{r}
3x + 13 \\
x-2\overline{)3x^2 + 7x - 20} \\
\underline{3x^2 - 6x} \\
13x - 20 \\
\underline{13x - 26} \\
6
\end{array}
$$

$$3x + 13 + \frac{6}{x-2}$$

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

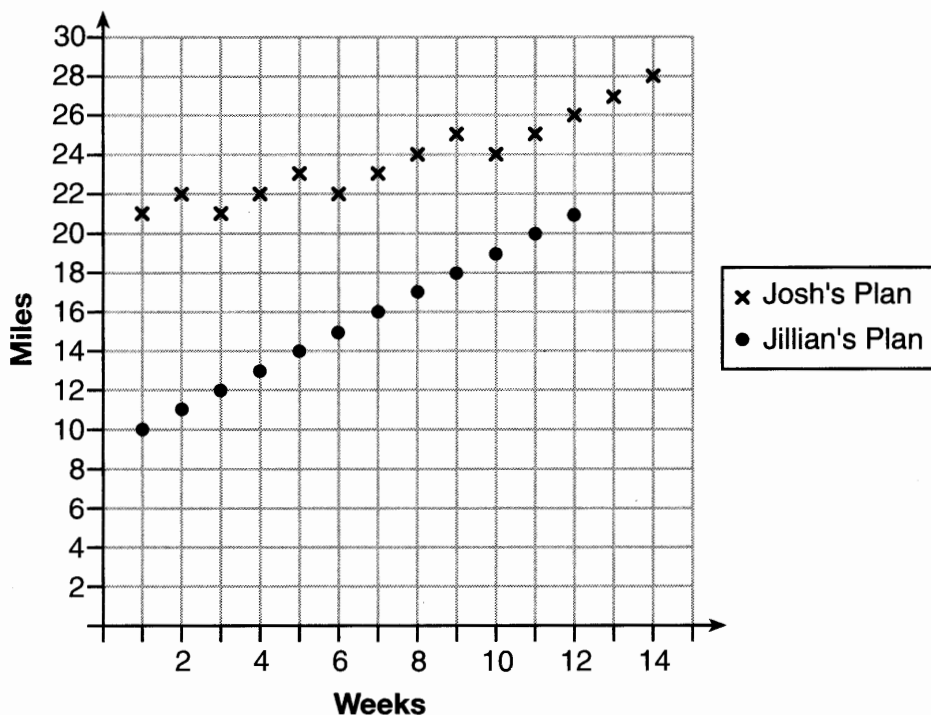**33** Algebraically determine the values of h and k to correctly complete the identity stated below.

$$2x^3 - 10x^2 + 11x - 7 = (x - 4)(2x^2 + hx + 3) + k$$

$$2x^3 - 10x^2 + 11x - 7 = 2x^3 + hx^2 + 3x - 8x^2 - 4hx - 12 + k$$

$$+8x^2 - 3x + 12 \qquad -3x + 8x^2 \qquad +7$$

$$\overline{-2x^2 + 8x + 5 = hx^2 - 4hx + k}$$

$$h = -2$$

$$k = 5$$

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34 Elaina has decided to run the Buffalo half-marathon in May. She researched training plans on the Internet and is looking at two possible plans: Jillian's 12-week plan and Josh's 14-week plan. The number of miles run per week for each plan is plotted below.

Which one of the plans follows an arithmetic pattern? Explain how you arrived at your answer.

Jillian's Plan, because distance increases by one mile each week.

Write a recursive definition to represent the number of miles run each week for the duration of the plan you chose.

$$a_1 = 10$$
$$a_{n+1} = a_n + 1$$

Jillian's plan has an alternative if Elaina wanted to train instead for a full 26-mile marathon. Week one would start at 13 miles and follow the same pattern for the half-marathon, but it would continue for 14 weeks. Write an explicit formula, in *simplest form*, to represent the number of miles run each week for the full-marathon training plan.

$$a_n = n + 12$$

35 The guidance department has reported that of the senior class, 2.3% are members of key club, K, 8.6% are enrolled in AP Physics, P, and 1.9% are in both.

Determine the probability of P given K, to the *nearest tenth of a percent*.

$$P(P/K) = \frac{P(P \wedge K)}{P(K)} = \frac{1.9}{2.3} = 82.6\%$$

The principal would like a basic interpretation of these results. Write a statement relating your calculated probabilities to student enrollment in the given situation.

A key club member has an 82.6% probability of being enrolled in AP physics.

36 Using the formula below, determine the monthly payment on a 5-year car loan with a monthly percentage rate of 0.625% for a car with an original cost of $21,000 and a $1000 down payment, to the *nearest cent*.

$$P_n = PMT\left(\frac{1-(1+i)^{-n}}{i}\right)$$

P_n = present amount borrowed

n = number of monthly pay periods

PMT = monthly payment

i = interest rate per month

$$20000 = PMT\left(\frac{1-(1+.00625)^{-60}}{.00625}\right)$$

$$PMT \approx 400.76$$

The affordable monthly payment is $300 for the same time period. Determine an appropriate down payment, to the *nearest dollar*.

$$21,000 - x = 300\left(\frac{1-(1+.00625)^{-60}}{.00625}\right)$$

$$x \approx 6028$$

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 The speed of a tidal wave, s, in hundreds of miles per hour, can be modeled by the equation $s = \sqrt{t} - 2t + 6$, where t represents the time from its origin in hours. Algebraically determine the time when $s = 0$.

$$0 = \sqrt{t} - 2t + 6$$
$$2t - 6 = \sqrt{t}$$
$$4t^2 - 24t + 36 = t$$
$$4t^2 - 25t + 36 = 0$$
$$(4t - 9)(t - 4) = 0$$
$$t = \frac{9}{4} \qquad t = 4$$
$$2\left(\frac{9}{4}\right) - 6 < 0$$

How much faster was the tidal wave traveling after 1 hour than 3 hours, to the *nearest mile per hour*? Justify your answer.

$$\sqrt{1} - 2(1) + 6 - (\sqrt{3} - 2(3) + 6)$$
$$5 - \sqrt{3}$$
$$3.268$$

$$327 \text{ mph}$$