

0123aii Common Core State Standards

- 1 Which expression is equivalent to $(x + 2)^2 - 5(x + 2) + 6$?
- 1) $x(x - 1)$ 3) $(x - 4)(x + 3)$
 2) $(x - 3)(x - 2)$ 4) $(x - 6)(x + 1)$
- 2 To the *nearest tenth*, the solution to the equation $4300e^{0.07x} - 123 = 5000$ is
- 1) 1.1 3) 6.3
 2) 2.5 4) 68.5
- 3 The value of an automobile t years after it was purchased is given by the function $V = 38,000(0.84)^t$. Which statement is true?
- 1) The value of the car increases 84% each year. 3) The value of the car increases 16% each year.
 2) The value of the car decreases 84% each year. 4) The value of the car decreases 16% each year.
- 4 Which function represents exponential decay?
- 1) $p(x) = \left(\frac{1}{4}\right)^{-x}$ 3) $r(x) = 2.3^{2x}$
 2) $q(x) = 1.8^{-x}$ 4) $s(x) = 4^{\frac{x}{2}}$
- 5 The expression $\frac{x^4 - 5x^2 + 4x + 14}{x + 2}$ is equivalent to
- 1) $x^3 - 2x^2 - x + 6 + \frac{2}{x + 2}$ 3) $x^3 + 2x^2 - x + 2 + \frac{18}{x + 2}$
 2) $x^3 - 5x + 4 - \frac{14}{x + 2}$ 4) $x^3 + 2x^2 - 9x + 22 - \frac{30}{x + 2}$
- 6 The sum of the first 20 terms of the series $-2 + 6 - 18 + 54 - \dots$ is
- 1) -610 3) $1,743,392,200$
 2) -59 4) $2,324,522,934$
- 7 If $f(x) = 2x^4 - x^3 - 16x + 8$, then $f\left(\frac{1}{2}\right)$
- 1) equals 0 and $2x + 1$ is a factor of $f(x)$ 3) does not equal 0 and $2x + 1$ is not a factor of $f(x)$
 2) equals 0 and $2x - 1$ is a factor of $f(x)$ 4) does not equal 0 and $2x - 1$ is a factor of $f(x)$
- 8 If $(6 - ki)^2 = 27 - 36i$, the value of k is
- 1) -36 3) 3
 2) -3 4) 6

9 What is the solution set of the equation $\frac{x+2}{x} + \frac{x}{3} = \frac{2x^2+6}{3x}$?

- | | |
|---------------|--------------|
| 1) $\{-3\}$ | 3) $\{3\}$ |
| 2) $\{-3,0\}$ | 4) $\{0,3\}$ |

10 How many real solutions exist for the system of equations below?

$$y = \frac{1}{4}x - 8$$

$$y = \frac{1}{2}x^2 + 2x$$

- | | |
|------|------|
| 1) 1 | 3) 3 |
| 2) 2 | 4) 0 |

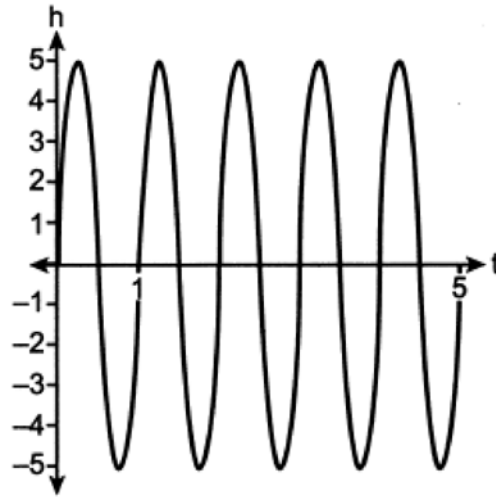
11 Which equation represents a polynomial identity?

- | | |
|--|--|
| 1) $x^3 + y^3 = (x+y)^3$ | 3) $x^3 + y^3 = (x+y)(x^2 - xy - y^2)$ |
| 2) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$ | 4) $x^3 + y^3 = (x-y)(x^2 + xy + y^2)$ |

12 Given $x > 0$, the expression $\frac{x^{\frac{1}{5}}}{x^{\frac{1}{2}}}$ can be rewritten as

- | | |
|----------------------|-------------------------------|
| 1) $\sqrt[3]{x}$ | 3) $\frac{1}{\sqrt[10]{x^3}}$ |
| 2) $-\sqrt[10]{x^3}$ | 4) $\sqrt[3]{x^{10}}$ |

- 13 A cyclist pedals a bike at a rate of 60 revolutions per minute. The height, h , of a pedal at time t , in seconds, is plotted below.



The graph can be modeled by the function $h(t) = 5 \sin(kt)$, where k is equal to

- | | |
|-----------|---------------------|
| 1) 1 | 3) 60 |
| 2) 2π | 4) $\frac{\pi}{30}$ |
- 14 Which statement about data collection is most accurate?
- | | |
|---|--|
| 1) A survey about parenting styles given to every tenth student entering the library will provide unbiased results. | 3) Margin of error increases as sample size increases. |
| 2) An observational study allows a researcher to determine the cause of an outcome. | 4) A survey collected from a random sample of students in a school can be used to represent the opinions of the school population. |
- 15 If $f(x) = \frac{1}{2}x + 2$, then the inverse function is
- | | |
|------------------------------------|-------------------------|
| 1) $f^{-1}(x) = -\frac{1}{2}x - 2$ | 3) $f^{-1}(x) = 2x - 4$ |
| 2) $f^{-1}(x) = \frac{1}{2}x - 1$ | 4) $f^{-1}(x) = 2x + 2$ |
- 16 Given $f(x) = x^4 - x^3 - 6x^2$, for what values of x will $f(x) > 0$?
- | | |
|------------------------|----------------------------------|
| 1) $x < -2$, only | 3) $x < -2$ or $0 \leq x \leq 3$ |
| 2) $x < -2$ or $x > 3$ | 4) $x > 3$, only |
- 17 For which approximate value(s) of x will $\log(x + 5) = |x - 1| - 3$?
- | | |
|----------------|-------------|
| 1) 5, 1 | 3) -2.41, 5 |
| 2) -2.41, 0.41 | 4) 5, only |

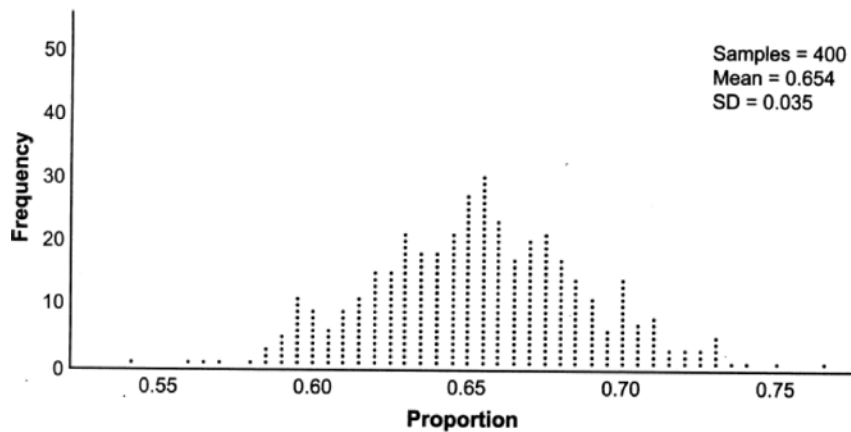
18 Consider a cubic polynomial with the characteristics below.

- exactly one real root
- as $x \rightarrow \infty, f(x) \rightarrow -\infty$

Given $a > 0$ and $b > 0$, which equation represents a cubic polynomial with these characteristics?

- 1) $f(x) = (x - a)(x^2 + b)$ 3) $f(x) = (a - x^2)(x^2 + b)$
2) $f(x) = (a - x)(x^2 + b)$ 4) $f(x) = (x - a)(b - x^2)$

19 Betty conducted a survey of her class to see if they like pizza. She gathered 200 responses and 65% of the voters said they did like pizza. Betty then ran a simulation of 400 more surveys, each with 200 responses, assuming that 65% of the voters would like pizza. The output of the simulation is shown below.



Considering the middle 95% of the data, what is the margin of error for the simulation?

- 1) 0.01 3) 0.05
2) 0.02 4) 0.07

20 If $\cos A = \frac{\sqrt{5}}{3}$ and $\tan A < 0$, what is the value of $\sin A$?

- 1) $\frac{2}{3}$ 3) $-\frac{2}{3}$
2) $-\frac{\sqrt{5}}{3}$ 4) $\frac{3}{\sqrt{5}}$

21 A tree farm initially has 150 trees. Each year, 20% of the trees are cut down and 80 seedlings are planted. Which recursive formula models the number of trees, a_n , after n years?

- 1) $a_1 = 150$ 3) $a_n = 150(0.2)^n + 80$
 $a_n = a_{n-1}(0.2) + 80$
2) $a_1 = 150$ 4) $a_n = 150(0.8)^n + 80$
 $a_n = a_{n-1}(0.8) + 80$

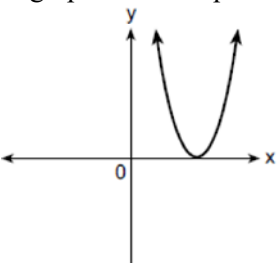
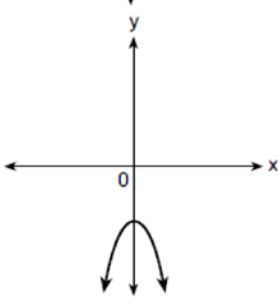
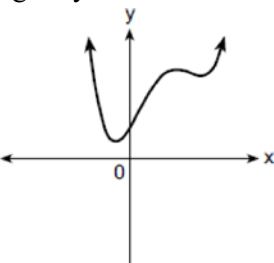
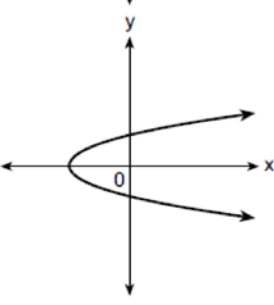
22 Which equation represents a parabola with a focus of $(4, -3)$ and directrix of $y = 1$?

- 1) $(x - 1)^2 = 4(y + 3)$ 3) $(x + 4)^2 = 4(y - 3)$
 2) $(x - 1)^2 = -8(y - 3)$ 4) $(x - 4)^2 = -8(y + 1)$

23 Mia has a student loan that is in deferment, meaning that she does not need to make payments right now. The balance of her loan account during her deferment can be represented by the function $f(x) = 35,000(1.0325)^x$, where x is the number of years since the deferment began. If the bank decides to calculate her balance showing a monthly growth rate, an approximately equivalent function would be

- 1) $f(x) = 35,000(1.0027)^{12x}$ 3) $f(x) = 35,000(1.0325)^{12x}$
 2) $f(x) = 35,000(1.0027)^{\frac{x}{12}}$ 4) $f(x) = 35,000(1.0325)^{\frac{x}{12}}$

24 Which graph shows a quadratic function with two imaginary zeros?

- 1) 
- 2) 
- 3) 
- 4) 

25 Algebraically determine the zeros of the function below.

$$r(x) = 3x^3 + 12x^2 - 3x - 12$$

26 Given $a > 0$, solve the equation $a^{x+1} = \sqrt[3]{a^2}$ for x algebraically.

27 Given $P(A) = \frac{1}{3}$ and $P(B) = \frac{5}{12}$, where A and B are independent events, determine $P(A \cap B)$.

28 The scores on a collegiate mathematics readiness assessment are approximately normally distributed with a mean of 680 and a standard deviation of 120. Determine the percentage of scores between 690 and 900, to the *nearest percent*.

29 Consider the data in the table below.

x	1	2	3	4	5	6
y	3.9	6	11	18.1	28	40.3

State an exponential regression equation to model these data, rounding all values to the *nearest thousandth*.

30 Write the expression $A(x) \cdot B(x) - 3C(x)$ as a polynomial in standard form.

$$A(x) = x^3 + 2x - 1$$

$$B(x) = x^2 + 7$$

$$C(x) = x^4 - 5x$$

31 Over the set of integers, completely factor $x^4 - 5x^2 + 4$.

32 Natalia's teacher has given her the following information about angle θ .

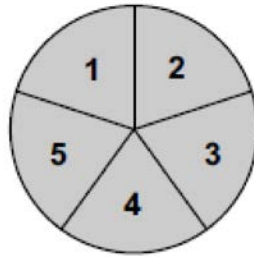
$$\bullet \pi < \theta < 2\pi$$

$$\bullet \cos \theta = \frac{\sqrt{3}}{4}$$

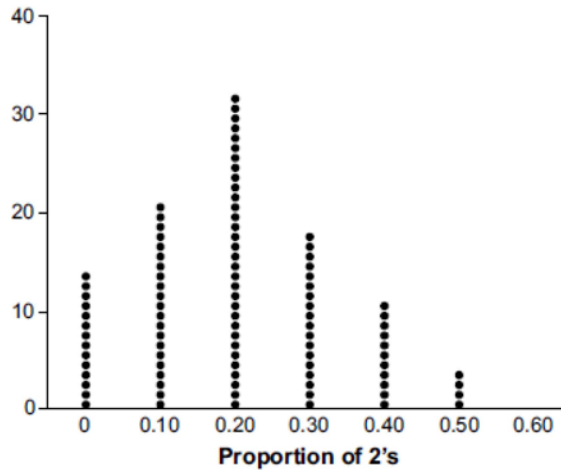
Explain how Natalia can determine if the value of $\tan \theta$ is positive or negative.

33 Solve the equation $\sqrt{49 - 10x} + 5 = 2x$ algebraically.

- 34 Joette is playing a carnival game. To win a prize, one has to correctly guess which of five equally sized regions a spinner will land on, as shown in the diagram below.

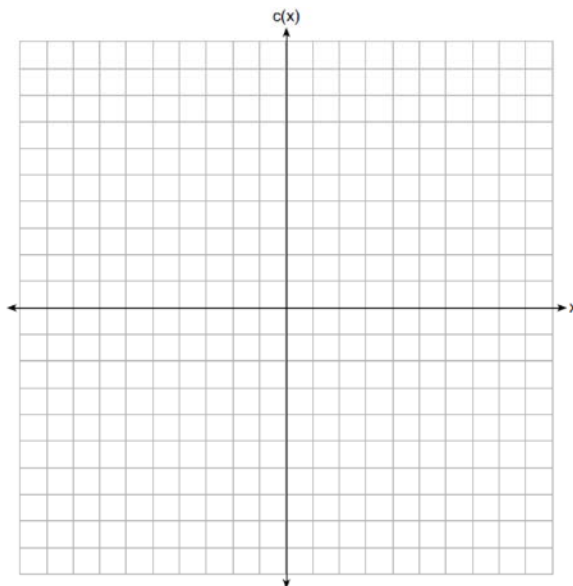


She complains that the game is unfair because her favorite number, 2, has only been spun once in ten times she played the game. State the proportion of 2's that were spun. State the theoretical probability of spinning a 2. The simulation output below shows the results of simulating ten spins of a fair spinner, repeated 100 times.



Does the output indicate that the carnival game was unfair? Explain your answer.

- 35 Graph $c(x) = -9(3)^{x-4} + 2$ on the axes below.



Describe the end behavior of $c(x)$ as x approaches positive infinity. Describe the end behavior of $c(x)$ as x approaches negative infinity.

- 36 The monthly high temperature ($^{\circ}\text{F}$) in Buffalo, New York can be modeled by $B(m) = 24.9 \sin(0.5m - 2.05) + 55.25$, where m is the number of the month and January = 1. Find the average rate of change in the monthly high temperature between June and October, to the *nearest hundredth*. Explain what this value represents in the given context.
- 37 Objects cool at different rates based on the formula below.

$$T = (T_0 - T_R)e^{-rt} + T_R$$

T_0 : initial temperature

T_R : room temperature

r : rate of cooling of the object

t : time in minutes that the object cools to a temperature, T

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F . The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F . Using this information, write an equation for the temperature of the shirt, T , after t minutes. Use the equation to find the temperature of the shirt, to the *nearest degree*, after five minutes. At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to 450°F . After eight minutes, the hoodie measured 270°F . The room temperature is still 75°F . Determine the rate of cooling of the hoodie, to the *nearest ten thousandth*. The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest minute*.

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Answer Section**

1 ANS: 1

$$u = x + 2 \quad u^2 - 5u + 6$$

$$(u - 3)(u - 2)$$

$$(x + 2 - 3)(x + 2 - 2)$$

$$(x - 1)x$$

PTS: 2 REF: 012301aii NAT: A.SSE.A.2 TOP: Factoring Polynomials
KEY: higher power

2 ANS: 2

$$4300e^{0.07x} = 5123$$

$$\ln e^{0.07x} = \ln \frac{5123}{4300}$$

$$0.07x = \ln \frac{5123}{4300}$$

$$x = \frac{\ln \frac{5123}{4300}}{0.07}$$

$$x \approx 2.5$$

PTS: 2 REF: 012302aii NAT: F.LE.A.4 TOP: Exponential Equations
KEY: without common base

3 ANS: 4 PTS: 2 REF: 012303aii NAT: F.LE.B.5
TOP: Modeling Exponential Functions

4 ANS: 2

$$p(x) = 4^x, q(x) = \left(\frac{5}{9}\right)^x, r(x) = 5.29^x, s(x) = 2^x$$

PTS: 2 REF: 012304aii NAT: F.IF.C.7 TOP: Graphing Exponential Functions

5 ANS: 1

$$\begin{array}{r}
 x^3 - 2x^2 - x + 6 \\
 x + 2 \overline{) x^4 + 0x^3 - 5x^2 + 4x + 14} \\
 \underline{x^4 + 2x^3} \\
 -2x^3 - 5x^2 \\
 \underline{-2x^3 - 4x^2} \\
 -x^2 + 4x + 14 \\
 \underline{-x^2 - 2x} \\
 6x + 14 \\
 \underline{6x + 12} \\
 2
 \end{array}$$

PTS: 2 REF: 012305aia NAT: A.APR.D.6 TOP: Rational Expressions
 KEY: division

6 ANS: 3

$$S_{20} = \frac{-2 - (-2)(-3)^{20}}{1 - (-3)} = 1,743,392,200$$

PTS: 2 REF: 012306aia NAT: A.SSE.B.4 TOP: Series
 KEY: geometric

7 ANS: 2

$$\begin{aligned}
 2x^4 - x^3 - 16x + 8 &= 0 \\
 x^3(2x - 1) - 8(2x - 1) &= 0 \\
 (x^3 - 8)(2x - 1) &= 0 \\
 x &= 2, \frac{1}{2}
 \end{aligned}$$

PTS: 2 REF: 012307aia NAT: A.APR.B.2 TOP: Remainder and Factor Theorems

8 ANS: 3

$$(6 - ki)^2 = 27 - 36i$$

$$36 - 12ki + k^2 i^2 = 27 - 36i$$

$$9 - k^2 - 12ki = -36i$$

Set real part equal to real part: $9 - k^2 = 0$ Set imaginary part equal to imaginary part: $-12ki = -36i$

$$k = \pm 3$$

$$\frac{-12ki}{-12i} = \frac{-36i}{-12i}$$

$$k = 3$$

PTS: 2

REF: 012308aai

NAT: N.CN.A.2

TOP: Operations with Complex Numbers

9 ANS: 3

$$\frac{x+2}{x} + \frac{x}{3} = \frac{2x^2+6}{3x} \quad 0 \text{ is extraneous.}$$

$$\frac{x^2+3x+6}{3x} = \frac{2x^2+6}{3x}$$

$$x^2+3x+6 = 2x^2+6$$

$$x^2-3x = 0$$

$$x(x-3) = 0$$

$$x = 0, 3$$

PTS: 2

REF: 012309aai

NAT: A.REI.A.2

TOP: Solving Rationals

10 ANS: 4

$$\frac{1}{2}x^2 + 2x = \frac{1}{4}x - 8 \quad b^2 - 4ac$$

$$2x^2 + 8x = x - 32 \quad 7^2 - 4(2)(32) < 0$$

$$2x^2 + 7x + 32 = 0$$

PTS: 2

REF: 012310aai

NAT: A.REI.C.7

TOP: Quadratic-Linear Systems

11 ANS: 2

PTS: 2

REF: 012311aai

NAT: A.APR.C.4

TOP: Polynomial Identities

12 ANS: 3

$$\frac{x^{\frac{1}{5}}}{x^{\frac{1}{2}}} = x^{\frac{1}{5} - \frac{1}{2}} = x^{-\frac{3}{10}} = \frac{1}{x^{\frac{3}{10}}} = \frac{1}{\sqrt[10]{x^3}}$$

PTS: 2

REF: 012312aai

NAT: N.RN.A.2

TOP: Radicals and Rational Exponents

13 ANS: 2

$$1 = \frac{2\pi}{k}$$

$$k = 2\pi$$

PTS: 2

REF: 012313aai

NAT: F.TF.B.5

TOP: Modeling Trigonometric Functions

14 ANS: 4

PTS: 2

REF: 012314aai

NAT: S.IC.B.3

TOP: Analysis of Data

15 ANS: 3

$$x = \frac{1}{2}y + 2$$

$$2x = y + 4$$

$$y = 2x - 4$$

PTS: 2

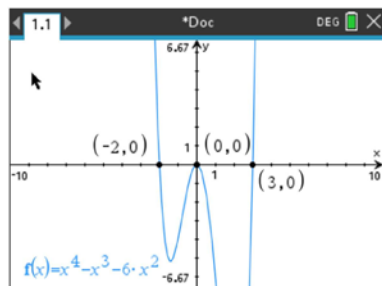
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NAT: F.BF.B.4

TOP: Inverse of Functions

KEY: linear

16 ANS: 2



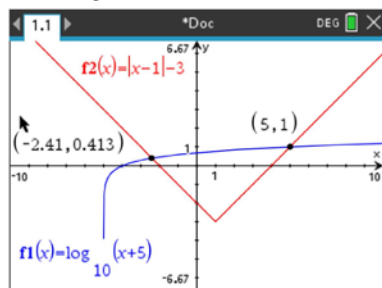
PTS: 2

REF: 012316aai

NAT: F.IF.B.4

TOP: Graphing Polynomial Functions

17 ANS: 3



PTS: 2

REF: 012317aai

NAT: A.REI.D.11

TOP: Other Systems

18 ANS: 2

1) $x \rightarrow \infty, f(x) \rightarrow \infty$; 3) quartic polynomial; 4) three real roots

PTS: 2

REF: 012318aai

NAT: A.APR.B.3

TOP: Graphing Polynomial Functions

19 ANS: 4

$$2 \times 0.035 = 0.07 \text{ or } ME = \left(z \sqrt{\frac{p(1-p)}{n}} \right) = \left(1.96 \sqrt{\frac{(0.65)(0.35)}{200}} \right) \approx 0.07$$

PTS: 2 REF: 012319aai NAT: S.IC.B.4 TOP: Analysis of Data

20 ANS: 3

$$\sin^2 A + \left(\frac{\sqrt{5}}{3} \right)^2 = 1 \quad \text{Since } \tan A < 0, \sin A = -\frac{2}{3}$$

$$\sin^2 A + \frac{5}{9} = \frac{9}{9}$$

$$\sin^2 A = \frac{4}{9}$$

$$\sin A = \pm \frac{2}{3}$$

PTS: 2 REF: 012320aai NAT: F.TF.C.8 TOP: Determining Trigonometric Functions

21 ANS: 2 PTS: 2 REF: 012321aai NAT: F.BF.A.2

TOP: Sequences

22 ANS: 4

The distance between the focus and directrix is $1 - -3 = 4$. p is half this distance, or 2. The vertex of the parabola is $(4, -1)$. Since the directrix is above the focus, the parabola faces downward. $y = -\frac{1}{4p}(x-h)^2 + k$

$$y = -\frac{1}{4(2)}(x-4)^2 - 1$$

$$y + 1 = -\frac{1}{8}(x-4)^2$$

PTS: 2 REF: 012322aai NAT: G.GPE.A.2 TOP: Graphing Quadratic Functions

23 ANS: 1

$$1.0325^{\frac{1}{12}} \approx 1.0027$$

PTS: 2 REF: 012323aai NAT: A.SSE.B.3 TOP: Modeling Exponential Functions

24 ANS: 2

1) 1 real, mult. 2; 3) not a quadratic; 4) not a function.

PTS: 2 REF: 012324aai NAT: F.IF.C.7 TOP: Graphing Polynomial Functions

25 ANS:

$$3(x^3 + 4x^2 - x - 4) = 0$$

$$(x^2(x + 4) - (x + 4)) = 0$$

$$(x^2 - 1)(x + 4) = 0$$

$$x = \pm 1, -4$$

PTS: 2

REF: 012325aai

NAT: A.APR.B.3

TOP: Solving Polynomial Equations

26 ANS:

$$a^{x+1} = a^{\frac{2}{3}}$$

$$x + 1 = \frac{2}{3}$$

$$x = -\frac{1}{3}$$

PTS: 2

REF: 012326aai

NAT: A.CED.A.1

TOP: Exponential Equations

KEY: common base shown

27 ANS:

$$\frac{1}{3} \times \frac{5}{12} = \frac{5}{36}$$

PTS: 2

REF: 012327aai

NAT: S.CP.A.2

TOP: Probability of Compound Events

KEY: probability

28 ANS:



43

PTS: 2

REF: 012328aai

NAT: S.ID.A.4

TOP: Normal Distributions

KEY: percent

29 ANS:

$$y = 2.459(1.616)^x$$

PTS: 2

REF: 012329aai

NAT: S.ID.B.6

TOP: Regression

KEY: exponential

30 ANS:

$$\begin{aligned} & (x^3 + 2x - 1)(x^2 + 7) - 3(x^4 - 5x) \\ & x^5 + 7x^3 + 2x^3 + 14x - x^2 - 7 - 3x^4 + 15x \\ & x^5 - 3x^4 + 9x^3 - x^2 + 29x - 7 \end{aligned}$$

PTS: 2 REF: 012330aai NAT: F.BF.A.1 TOP: Operations with Functions

31 ANS:

$$\begin{aligned} & x^4 - 5x^2 + 4 \\ & (x^2 - 4)(x^2 - 1) \\ & (x + 2)(x - 2)(x + 1)(x - 1) \end{aligned}$$

PTS: 2 REF: 012331aai NAT: A.SSE.A.2 TOP: Factoring Polynomials

32 ANS:

$\pi < \theta < 2\pi \rightarrow$ Quadrant III or IV θ must be in Quadrant IV, where $\tan \theta$ is negative.

$$\cos \theta = \frac{\sqrt{3}}{4} \rightarrow \text{Quadrant I or IV}$$

PTS: 2 REF: 012332aai NAT: F.TF.A.2 TOP: Finding the Terminal Side of an Angle

33 ANS:

$$\sqrt{49 - 10x} = 2x - 5 \quad -\frac{3}{2} \text{ is extraneous.}$$

$$49 - 10x = 4x^2 - 20x + 25$$

$$0 = 4x^2 - 10x - 24$$

$$0 = 2x^2 - 5x - 12$$

$$0 = (2x + 3)(x - 4)$$

$$x = -\frac{3}{2}, 4$$

PTS: 4 REF: 012333aai NAT: A.REI.A.2 TOP: Solving Radicals

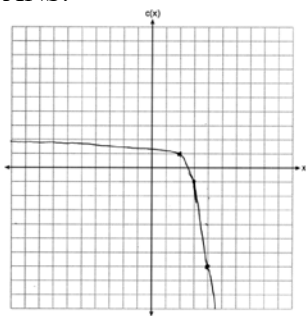
KEY: extraneous solutions

34 ANS:

$\frac{1}{10}, \frac{1}{5}$, and no, since 0.10 clearly falls within 95% of 0.20.

PTS: 4 REF: 012334aai NAT: S.IC.A.2 TOP: Analysis of Data

35 ANS:



As $x \rightarrow \infty, c(x) \rightarrow -\infty$. As $x \rightarrow -\infty, c(x) \rightarrow 2$.

PTS: 4

REF: 012335aai

NAT: F.IF.C.7

TOP: Graphing Exponential Functions

36 ANS:

$\frac{B(10) - B(6)}{10 - 6} \approx -3.88$. The average monthly high temperature decreases about 4° each month from June and October.

PTS: 4

REF: 012336aai

NAT: F.IF.B.6

TOP: Rate of Change

37 ANS:

$$T = (400 - 75)e^{-0.0735t} + 75, \quad 325e^{-0.0735(5)} + 75 \approx 300, \quad 270 = (450 - 75)e^{-8r} + 75, \quad 325e^{-0.0735t} + 75 = 375e^{-0.0817t} + 75$$

$$r \approx 0.0817 \qquad t \approx 17$$

PTS: 6

REF: 012337aai

NAT: A.CED.A.1

TOP: Exponential Decay