0120AII Common Core State Standards

1 The expression $\sqrt[4]{81x^8y^6}$ is equivalent to

1) $3x^2y^{\frac{3}{2}}$

2) $3x^4y^2$

3) $9x^2y^{\frac{3}{2}}$

4) $9x^4y^2$

2 Chet has \$1200 invested in a bank account modeled by the function $P(n) = 1200(1.002)^n$, where $P(n)$ is the value of his account, in dollars, after n months. Chet's debt is modeled by the function $Q(n) = 100n$, where $Q(n)$ is the value of debt, in dollars, after n months. After n months, which function represents Chet's net worth, $R(n)$?

1) $R(n) = 1200(1.002)^n + 100n$

2) $R(n) = 1200(1.002)^{12n} + 100n$

3) $R(n) = 1200(1.002)^n - 100n$

4) $R(n) = 1200(1.002)^{12n} - 100n$

3 Emmeline is working on one side of a polynomial identity proof used to form Pythagorean triples. Her work is shown below:

$$(5x)^2 + (5x^2 - 5)^2$$

Step 1: $25x^2 + (5x^2 - 5)^2$

Step 2: $25x^2 + 25x^2 + 25$

Step 3: $50x^2 + 25$

Step 4: $75x^2$

What statement is true regarding Emmeline's work?

1) Emmeline's work is entirely correct.

2) There is a mistake in step 2, only.

3) There are mistakes in step 2 and step 4.

4) There is a mistake in step 4, only.

4 Susan won \$2,000 and invested it into an account with an annual interest rate of 3.2%. If her investment were compounded monthly, which expression best represents the value of her investment after t years?

1) $2000(1.003)^{12t}$

2) $2000(1.032)^{\frac{t}{12}}$

3) $2064^{\frac{t}{12}}$

4) $\dfrac{2000(1.032)^t}{12}$
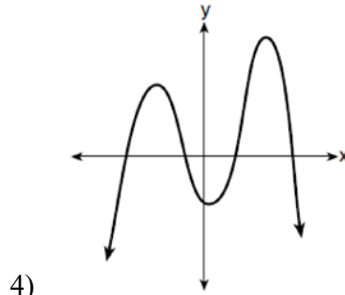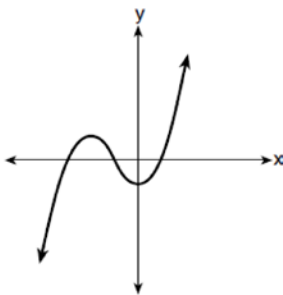
5 Consider the end behavior description below.
- as $x \to -\infty, f(x) \to \infty$
- as $x \to \infty, f(x) \to -\infty$

Which function satisfies the given conditions?

1) $f(x) = x^4 + 2x^2 + 1$

3) $f(x) = -x^3 + 2x - 6$

2)

4)

6 The expression $(x + a)^2 + 5(x + a) + 4$ is equivalent to
1) $(a + 1)(a + 4)$ 3) $(x + a + 1)(x + a + 4)$
2) $(x + 1)(x + 4)$ 4) $x^2 + a^2 + 5x + 5a + 4$

7 Given $x \neq -2$, the expression $\dfrac{2x^2 + 5x + 8}{x + 2}$ is equivalent to

1) $2x^2 + \dfrac{9}{x + 2}$ 3) $2x + 1 + \dfrac{6}{x + 2}$

2) $2x + \dfrac{7}{x + 2}$ 4) $2x + 9 - \dfrac{10}{x + 2}$

8 Which situation best describes conditional probability?
1) finding the probability of an event   3) finding the probability of two
    occurring two or more times   independent events occurring at the same
                                                       time
2) finding the probability of an event   4) finding the probability of an event
    occurring only once   occurring given another event had
                                                       already occurred

9 Which expression is *not* a solution to the equation $2^t = \sqrt{10}$?

1) $\dfrac{1}{2} \log_2 10$ 3) $\log_4 10$

2) $\log_2 \sqrt{10}$ 4) $\log_{10} 4$

10 What is the solution set of $x = \sqrt{3x + 40}$?
1) $\{-5, 8\}$ 3) $\{-4, 10\}$
2) $\{8\}$ 4) $\{\ \}$

11 Consider the data in the table below.

	Right Handed	Left Handed
Male	87	13
Female	89	11

What is the probability that a randomly selected person is male given the person is left handed?

1) $\dfrac{13}{200}$ 3) $\dfrac{13}{50}$

2) $\dfrac{13}{100}$ 4) $\dfrac{13}{24}$

12 The function $N(x) = 90(0.86)^x + 69$ can be used to predict the temperature of a cup of hot chocolate in degrees Fahrenheit after x minutes. What is the approximate average rate of change of the temperature of the hot chocolate, in degrees per minute, over the interval $[0,6]$?
1) -8.93 3) 0.11
2) -0.11 4) 8.93

13 A recursive formula for the sequence $40, 30, 22.5, \ldots$ is

1) $g_n = 40\left(\dfrac{3}{4}\right)^n$
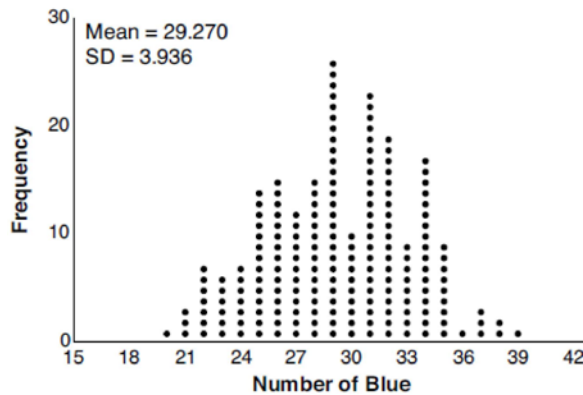
2) $g_1 = 40$
$g_n = g_{n-1} - 10$

3) $g_n = 40\left(\dfrac{3}{4}\right)^{n-1}$

4) $g_1 = 40$
$g_n = \dfrac{3}{4} g_{n-1}$

14 The J& B candy company claims that 45% of the candies it produces are blue, 30% are brown, and 25% are yellow. Each bag holds 65 candies. A simulation was run 200 times, each of sample size 65, based on the premise that 45% of the candies are blue. The results of the simulation are shown below.

Bonnie purchased a bag of J& B's candy and counted 24 blue candies. What inference can be made regarding a bag of J& B's with only 24 blue candies?

1) The company is not meeting their production standard.

2) Bonnie's bag was a rarity and the company should not be concerned.

3) The company should change their claim to 37% blue candies are produced.

4) Bonnie's bag is within the middle 95% of the simulated data supporting the company's claim.

15 Which investigation technique is most often used to determine if a single variable has an impact on a given population?

1) observational study

2) random survey

3) controlled experiment

4) formal interview

16 As θ increases from $-\dfrac{\pi}{2}$ to 0 radians, the value of $\cos \theta$ will

1) decrease from 1 to 0

2) decrease from 0 to −1

3) increase from −1 to 0

4) increase from 0 to 1

17 Consider the following patterns:

     I. $16, -12, 9, -6.75, \ldots$

     II. $1, 4, 9, 16, \ldots$

     III. $6, 18, 30, 42, \ldots$

     IV. $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{5}, \ldots$

Which pattern is geometric?

1)	I	3)	III
2)	II	4)	IV

18 Consider the system below.

$$x + y + z = 9$$
$$x - y - z = -1$$
$$x - y + z = 21$$

Which value is *not* in the solution, (x, y, z), of the system?

1)	-8	3)	11
2)	-6	4)	4

19 Which statement regarding polynomials and their zeros is true?

1) $f(x) = (x^2 - 1)(x + a)$ has zeros of 1 and $-a$, only.

3) $f(x) = (x^2 + 25)(x + a)$ has zeros of ± 5 and $-a$.

2) $f(x) = x^3 - ax^2 + 16x - 16a$ has zeros of 4 and a, only.

4) $f(x) = x^3 - ax^2 - 9x + 9a$ has zeros of ± 3 and a.

20 If a solution of $2(2x - 1) = 5x^2$ is expressed in simplest $a + bi$ form, the value of b is

1)	$\dfrac{\sqrt{6}}{5}i$	3)	$\dfrac{1}{5}i$
2)	$\dfrac{\sqrt{6}}{5}$	4)	$\dfrac{1}{5}$

21 Which value, to the *nearest tenth*, is the *smallest* solution of $f(x) = g(x)$ if $f(x) = 3\sin\left(\dfrac{1}{2}x\right) - 1$ and

$g(x) = x^3 - 2x + 1$?

1)	-3.6	3)	-1.8
2)	-2.1	4)	1.4

22 Expressed in simplest $a + bi$ form, $(7 - 3i) + (x - 2i)^2 - (4i + 2x^2)$ is

1)	$(3 - x^2) - (4x + 7)i$	3)	$(3 - x^2) - 7i$
2)	$(3 + 3x^2) - (4x + 7)i$	4)	$(3 + 3x^2) - 7i$

23 Written in simplest form, the fraction $\dfrac{x^3 - 9x}{9 - x^2}$, where $x \neq \pm 3$, is equivalent to

1) $-x$

2) x

3) $\dfrac{-x(x+3)}{(3+x)}$

4) $\dfrac{x(x-3)}{(3-x)}$

24 According to a study, 45% of Americans have type O blood. If a random number generator produces three-digit values from 000 to 999, which values would represent those having type O blood?
   1) between 000 and 045, inclusive
   2) between 000 and 444, inclusive
   3) between 000 and 449, inclusive
   4) between 000 and 450, inclusive

25 For n and $p > 0$, is the expression $\left(p^2 n^{\frac{1}{2}}\right)^8 \sqrt{p^5 n^4}$ equivalent to $p^{18} n^6 \sqrt{p}$? Justify your answer.

26 Show why $x - 3$ is a factor of $m(x) = x^3 - x^2 - 5x - 3$. Justify your answer.

27 Describe the transformation applied to the graph of $p(x) = 2^x$ that forms the new function $q(x) = 2^{x-3} + 4$.

28 The parabola $y = -\dfrac{1}{20}(x - 3)^2 + 6$ has its focus at $(3, 1)$. Determine and state the equation of the directrix. (The use of the grid below is optional.)
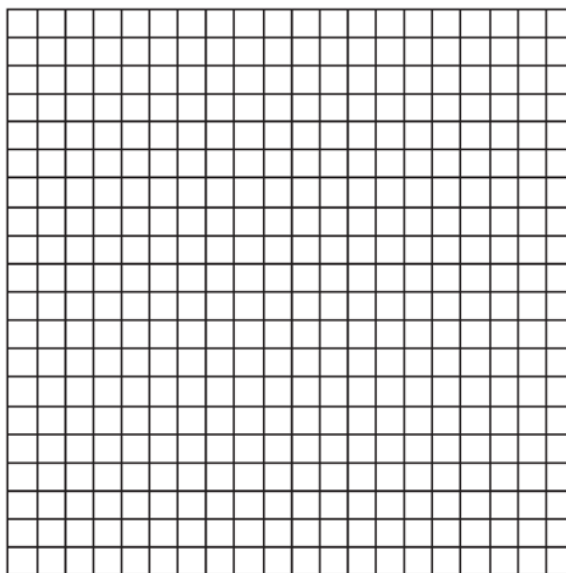
29 Given the geometric series $300 + 360 + 432 + 518.4 + \ldots$, write a geometric series formula, S_n, for the sum of the first n terms. Use the formula to find the sum of the first 10 terms, to the *nearest tenth*.

30 Visible light can be represented by sinusoidal waves. Three visible light waves are shown in the graph below. The midline of each wave is labeled ℓ.

Nanometers
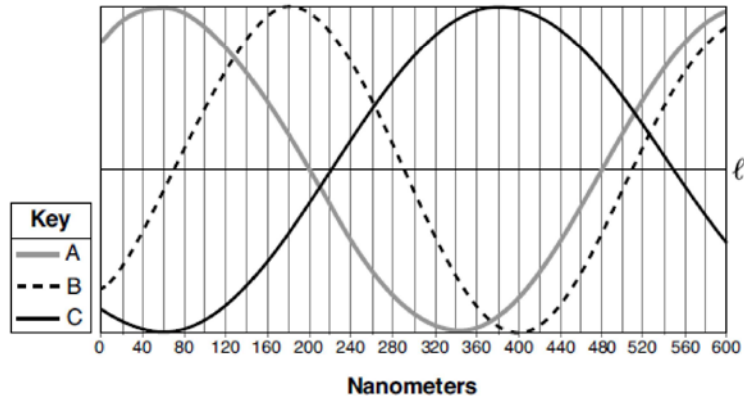
Based on the graph, which light wave has the longest period? Justify your answer.

31 Biologists are studying a new bacterium. They create a culture with 100 of the bacteria and anticipate that the number of bacteria will double every 30 hours. Write an equation for the number of bacteria, B, in terms of the number of hours, t, since the experiment began.

32 Graph $y = x^3 - 4x^2 + 2x + 7$ on the set of axes below.
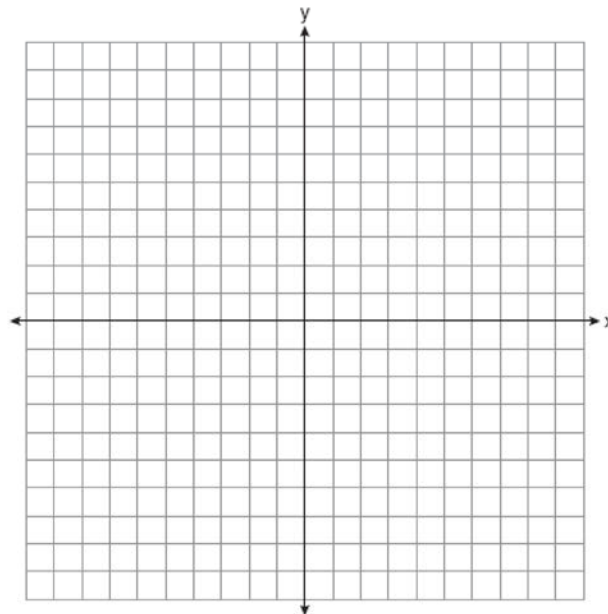
33 Sonja is cutting wire to construct a mobile. She cuts 100 inches for the first piece, 80 inches for the second piece, and 64 inches for the third piece. Assuming this pattern continues, write an explicit equation for a_n, the length in inches of the nth piece. Sonja only has 40 feet of wire to use for the project and wants to cut 20 pieces total for the mobile using her pattern. Will she have enough wire? Justify your answer.

34   Graph the following function on the axes below.
$$f(x) = \log_3(2-x)$$
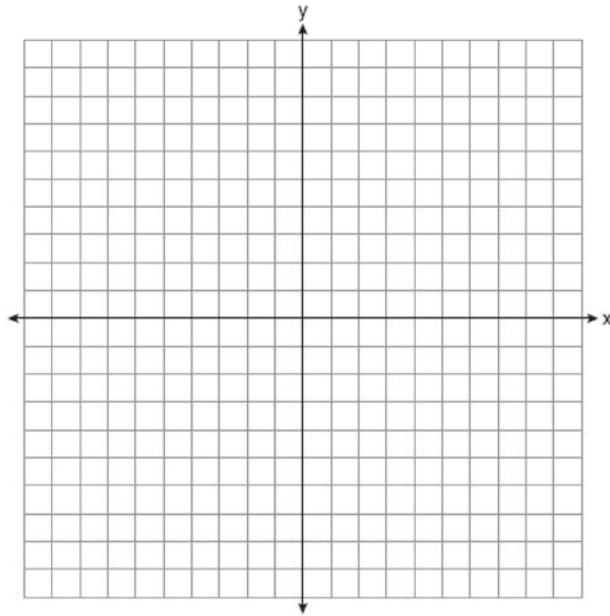
State the domain of f. State the equation of the asymptote.

35   Algebraically solve the following system of equations.
$$(x-2)^2 + (y-3)^2 = 16$$
$$x + y - 1 = 0$$

36   The table below gives air pressures in kPa at selected altitudes above sea level measured in kilometers.

x	Altitude (km)	0	1	2	3	4	5
y	Air Pressure (kPa)	101	90	79	70	62	54

Write an exponential regression equation that models these data rounding all values to the *nearest thousandth*. Use this equation to algebraically determine the altitude, to the *nearest hundredth* of a kilometer, when the air pressure is 29 kPa.

37   Sarah is fighting a sinus infection. Her doctor prescribed a nasal spray and an antibiotic to fight the infection. The active ingredients, in milligrams, remaining in the bloodstream from the nasal spray, $n(t)$, and the antibiotic, $a(t)$, are modeled in the functions below, where t is the time in hours since the medications were taken.
$$n(t) = \frac{t+1}{t+5} + \frac{18}{t^2 + 8t + 15}$$

$$a(t) = \frac{9}{t+3}$$

Determine which drug is made with a greater initial amount of active ingredient. Justify your answer. Sarah's doctor told her to take both drugs at the same time. Determine algebraically the number of hours after taking the medications when both medications will have the same amount of active ingredient remaining in her bloodstream.

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Answer Section

1 ANS: 1

$$\sqrt[4]{81x^8y^6} = 81^{\frac{1}{4}} x^{\frac{8}{4}} y^{\frac{6}{4}} = 3x^2 y^{\frac{3}{2}}$$

   PTS: 2   REF: 012001aii   NAT: N.RN.A.2   TOP: Radicals and Rational Exponents
   KEY: variables

2 ANS: 3   PTS: 2   REF: 012002aii   NAT: F.BF.A.1
   TOP: Operations with Functions

3 ANS: 3   PTS: 2   REF: 012003aii   NAT: A.APR.C.4
   TOP: Polynomial Identities

4 ANS: 1

$$2000\left(1 + \frac{.032}{12}\right)^{12t} \approx 2000(1.003)^{12t}$$

   PTS: 2   REF: 012004aii   NAT: F.BF.A.1   TOP: Modeling Exponential Functions

5 ANS: 3   PTS: 2   REF: 012005aii   NAT: F.IF.B.4
   TOP: Graphing Polynomial Functions

6 ANS: 3

$(x + a)^2 + 5(x + a) + 4 \;$ let $u = x + a$

$$u^2 + 5u + 4$$
$$(u + 4)(u + 1)$$
$$(x + a + 4)(x + a + 1)$$

   PTS: 2   REF: 012006aii   NAT: A.SSE.A.2   TOP: Factoring Polynomials
   KEY: multivariable

7 ANS: 3

$$\require{enclose}\begin{array}{r}2x + 1 \\ x + 2 \enclose{longdiv}{2x^2 + 5x + 8} \\ \underline{2x^2 + 4x} \\ x + 8 \\ \underline{x + 2} \\ 6\end{array}$$

   PTS: 2   REF: 012007aii   NAT: A.APR.D.6   TOP: Rational Expressions
   KEY: division

8 ANS: 4   PTS: 2   REF: 012008aii   NAT: S.CP.A.3
   TOP: Conditional Probability

9 ANS: 4

$\log 2^t = \log \sqrt{10}$ 2) $\dfrac{\log \sqrt{10}}{\log 2} = \log_2 \sqrt{10}$, 1) $\log_2 \sqrt{10} = \log_2 10^{\frac{1}{2}} = \dfrac{1}{2}\log_2 10$, 3) $\log_4 10 = \dfrac{\log_2 10}{\log_2 4} = \dfrac{1}{2}\log_2 10$

$t \log 2 = \log \sqrt{10}$

$t = \dfrac{\log \sqrt{10}}{\log 2}$

PTS: 2   REF: 012009aii   NAT: F.LE.A.4   TOP: Exponential Equations
KEY: without common base

10 ANS: 2

        $x^2 = 3x + 40$. $x = -5$ is an extraneous solution.

$x^2 - 3x - 40 = 0$

$(x - 8)(x + 5) = 0$

       $x = 8, -5$

PTS: 2   REF: 012010aii   NAT: A.REI.A.2   TOP: Solving Radicals
KEY: extraneous solutions

11 ANS: 4

$\dfrac{13}{13 + 11} = \dfrac{13}{24}$

PTS: 2   REF: 012011aii   NAT: S.CP.A.4   TOP: Conditional Probability

12 ANS: 1

$\dfrac{N(6) - N(0)}{6 - 0} \approx -8.93$

PTS: 2   REF: 012012aii   NAT: F.IF.B.6   TOP: Rate of Change

13 ANS: 4
(1) and (3) are not recursive

PTS: 2   REF: 012013aii   NAT: F.LE.A.2   TOP: Sequences
KEY: recursive

14 ANS: 4   PTS: 2   REF: 012014aii   NAT: S.IC.B.5
TOP: Analysis of Data

15 ANS: 3   PTS: 2   REF: 012015aii   NAT: S.IC.B.3
TOP: Analysis of Data   KEY: type

16 ANS: 4   PTS: 2   REF: 012016aii   NAT: F.IF.B.4
TOP: Graphing Trigonometric Functions   KEY: increasing/decreasing

17 ANS: 1

$\dfrac{-12}{16} = \dfrac{9}{-12} = \dfrac{-6.75}{9}$

PTS: 2   REF: 012017aii   NAT: F.IF.A.3   TOP: Sequences
KEY: difference or ratio

18 ANS: 1

$x + y + z = 9$ $4 - y - z = -1$ $4 - 6 + z = 9$

$\underline{x - y - z = -1}$ $4 - y + z = 21$ $z = 11$

$\quad 2x = 8$ $-y - z = -5$

$\quad\; x = 4$ $\underline{-y + z = 17}$

$\qquad\qquad\quad -2y = 12$

$\qquad\qquad\qquad\; y = -6$

PTS: 2   REF: 012018aii   NAT: A.REI.C.6   TOP: Solving Linear Systems
KEY: three variables

19 ANS: 4

1) -1 is also a zero. 2) $x^2(x - a) + 16(x - a) = (x^2 + 16)(x - a)$ a is the only zero. 3) $-a$ is the only zero. 4) $x^2(x - a) - 9(x - a) = (x^2 - 9)(x - a)$.

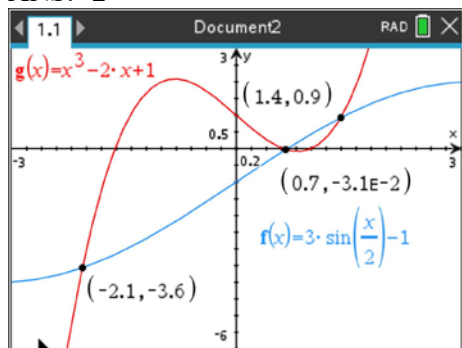PTS: 2   REF: 012019aii   NAT: A.APR.B.3   TOP: Solving Polynomial Equations

20 ANS: 2

$5x^2 - 4x + 2 = 0$ $\dfrac{4 \pm \sqrt{(-4)^2 - 4(5)(2)}}{2(5)} = \dfrac{4 \pm \sqrt{-24}}{10} = \dfrac{4 \pm 2i\sqrt{6}}{10} = \dfrac{2 \pm i\sqrt{6}}{5}$

PTS: 2   REF: 012020aii   NAT: A.REI.B.4   TOP: Solving Quadratics
KEY: complex solutions | quadratic formula

21 ANS: 2

PTS: 2   REF: 012021aii   NAT: A.REI.D.11   TOP: Other Systems

22 ANS: 1

$7 - 3i + x^2 - 4xi + 4i^2 - 4i - 2x^2 = 7 - 7i - x^2 - 4xi - 4 = 3 - x^2 - 4xi - 7i = (3 - x^2) - (4x + 7)i$

PTS: 2   REF: 012022aii   NAT: N.CN.A.2   TOP: Operations with Complex Numbers

23 ANS: 1

$$\frac{x(x^2-9)}{-(x^2-9)} = -x$$

PTS: 2   REF: 012023aii   NAT: A.APR.D.6   TOP: Rational Expressions
KEY: factoring

24 ANS: 3

between 000 and 449, inclusive$\rightarrow \dfrac{450}{1000} = 45\%$

PTS: 2   REF: 012024aii   NAT: S.IC.B.3   TOP: Analysis of Data
KEY: type

25 ANS:

$$\left(p^2 n^{\frac{1}{2}}\right)^8 \sqrt{p^5 n^4} = \left(p^{16} n^4\right) p^2 n^2 \sqrt{p} = p^{18} n^6 \sqrt{p}$$

PTS: 2   REF: 012025aii   NAT: N.RN.A.2   TOP: Radicals and Rational Exponents

26 ANS:

$m(3) = 3^3 - 3^2 - 5(3) - 3 = 27 - 9 - 15 - 3 = 0$ Since $m(3) = 0$, there is no remainder when $m(x)$ is divided by $x - 3$, and so $x - 3$ is a factor.
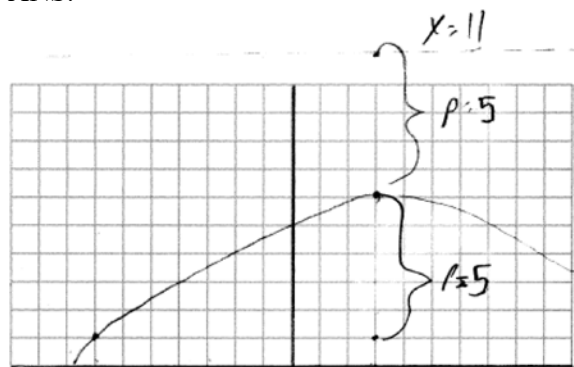
PTS: 2   REF: 012026aii   NAT: A.APR.B.2   TOP: Remainder Theorem

27 ANS:

Translation 3 units right and 4 units up

PTS: 2   REF: 012027aii   NAT: F.IF.C.7   TOP: Graphing Exponential Functions

28 ANS:

vertex $(3,6)$, focus $(3,1)$, $p = 5$, directrix $y = 6 + 5 = 11$

PTS: 2   REF: 012028aii   NAT: G.GPE.A.2   TOP: Graphing Quadratic Functions

29 ANS:

$$r = \frac{360}{300} = 1.2 \quad S_n = \frac{300 - 300(1.2)^n}{1 - 1.2} \quad S_{10} = \frac{300 - 300(1.2)^{10}}{1 - 1.2} \approx 7787.6$$

PTS: 2   REF: 012029aii   NAT: A.SSE.B.4   TOP: Series

4

30 ANS:
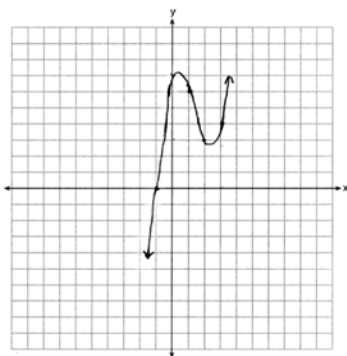Light wave C. The periods for A, B, and C are 280, 220 and 320.

PTS: 2   REF: 012030aii   NAT: F.IF.C.7   TOP: Graphing Trigonometric Functions
KEY: period

31 ANS:

$$B(t) = 100(2)^{\frac{t}{30}}$$

PTS: 2   REF: 012031aii   NAT: F.LE.A.2   TOP: Modeling Exponential Functions

32 ANS:

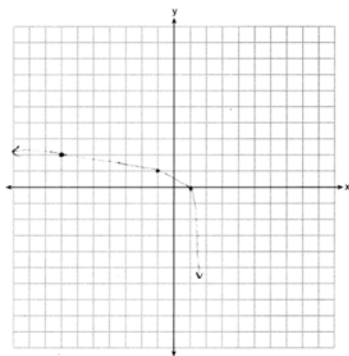PTS: 2   REF: 012032aii   NAT: F.IF.C.7   TOP: Graphing Polynomial Functions

33 ANS:

$$a_n = 100(.8)^{n-1} \quad S_{20} = \frac{100 - 100(.8)^{20}}{1 - .8} \approx 494 \quad \text{No, because } 494 > 40 \times 12.$$

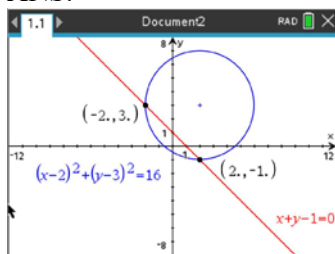PTS: 4   REF: 012033aii   NAT: A.SSE.B.4   TOP: Series

34 ANS:

Domain: $x < 2$, Asymptote $x = 2$

PTS: 4   REF: 012034aii   NAT: F.IF.C.7   TOP: Graphing Logarithmic Functions

35 ANS:

$$y = -x + 1 \quad y = -2 + 1 = -1 \quad (2, -1)$$

$$(x - 2)^2 + (-x + 1 - 3)^2 = 16 \qquad y = 2 + 1 = 3 \quad (-2, 3)$$

$$x^2 - 4x + 4 + x^2 + 4x + 4 = 16$$

$$2x^2 = 8$$

$$x = -2, 2$$

PTS: 4   REF: 012035aii   NAT: A.REI.C.7   TOP: Quadratic-Linear Systems

36 ANS:

$$y = 101.523(.883)^x \quad 29 = 101.523(.883)^x$$

$$\frac{29}{101.523} = (.883)^x$$

$$\log \frac{29}{101.523} = x \log(.883)$$

$$\frac{\log \dfrac{29}{101.523}}{\log(.883)} = x$$

$$x \approx 10.07$$

PTS: 4   REF: 012036aii   NAT: S.ID.B.6   TOP: Regression
KEY: exponential

37 ANS:

antibiotic $n(0) = \dfrac{0+1}{0+5} + \dfrac{18}{0^2 + 8(0) + 15} = \dfrac{3}{15} + \dfrac{18}{15} = \dfrac{21}{15}$ $\dfrac{t+1}{t+5} + \dfrac{18}{t^2 + 8t + 15} = \dfrac{9}{t+3}$

$a(0) = \dfrac{9}{0+3} = 3$ $\dfrac{(t+1)(t+3)}{(t+5)(t+3)} + \dfrac{18}{(t+3)(t+5)} = \dfrac{9(t+5)}{(t+3)(t+5)}$

$$t^2 + 4t + 3 + 18 = 9t + 45$$

$$t^2 - 5t - 24 = 0$$

$$(t - 8)(t + 3) = 0$$

$$t = 8$$

PTS: 6   REF: 012037aii   NAT: A.REI.A.2   TOP: Solving Rationals
KEY: rational solutions