

ALGEBRA
II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, June 21, 2023 — 9:15 a.m. to 12:15 p.m., only

Student Name _____

School Name _____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

- 1 The population of Austin, Texas from 1850 to 2010 is summarized in the table below.

Use this space for computations.

Year	1850	1870	1890	1910	1930	1950	1970	1990	2010
Population	629	4428	14,575	29,860	53,120	132,459	251,808	494,290	790,390

Over which period of time was the average rate of change in population the greatest?

- (1) 1850 to 1910 (3) 1950 to 1970
(2) 1990 to 2010 (4) 1890 to 1970
- 2 Which expression is *not* equivalent to $36x^6 - 25y^4$?
- (1) $6^2(x^3)^2 - 5^2(y^2)^2$ (3) $(6x^6 - 5y^4)(6x^6 + 5y^4)$
(2) $(6x^3 - 5y^2)(6x^3 + 5y^2)$ (4) $(3 \cdot 2x^3 - 5y^2)(3 \cdot 2x^3 + 5y^2)$
- 3 What are the zeros of $s(x) = x^4 - 9x^2 + 3x^3 - 27x - 10x^2 + 90$?
- (1) $\{-3, -2, 5\}$ (3) $\{-3, -2, 3, 5\}$
(2) $\{-2, 3, 5\}$ (4) $\{-5, -3, 2, 3\}$

**Use this space for
computations.**

4 If θ is an angle in standard position whose terminal side passes through the point $(-2, -3)$, what is the numerical value of $\tan \theta$?

(1) $\frac{2}{3}$

(3) $-\frac{2}{\sqrt{13}}$

(2) $\frac{3}{2}$

(4) $-\frac{3}{\sqrt{13}}$

5 The average monthly temperature, $T(m)$, in degrees Fahrenheit, over a 12 month period, can be modeled by $T(m) = -23 \cos\left(\frac{\pi}{6}m\right) + 56$, where m is in months. What is the range of temperatures, in degrees Fahrenheit, of this function?

(1) $[-23, 23]$

(3) $[-23, 56]$

(2) $[33, 79]$

(4) $[-79, 33]$

6 Which expression is an equivalent form of $a\sqrt[5]{a^4}$?

(1) a

(3) $a^{\frac{9}{4}}$

(2) $a^{\frac{9}{5}}$

(4) $a^{\frac{1}{5}}$

7 The expression $3i(ai - 6i^2)$ is equivalent to

(1) $3a + 18i$

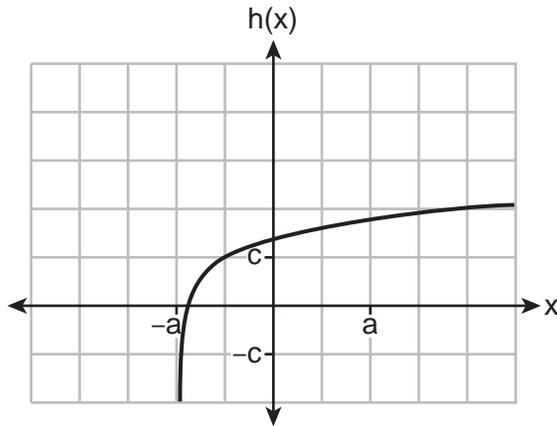
(3) $-3a + 18i$

(2) $3a - 18i$

(4) $-3a - 18i$

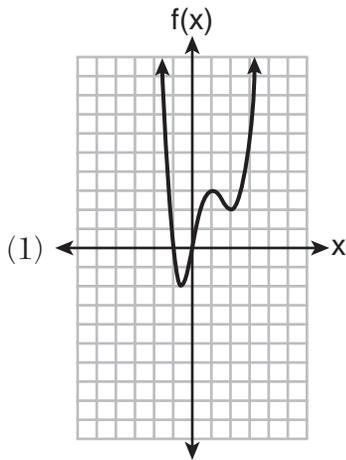
Use this space for computations.

8 Which equation best represents the graph below?

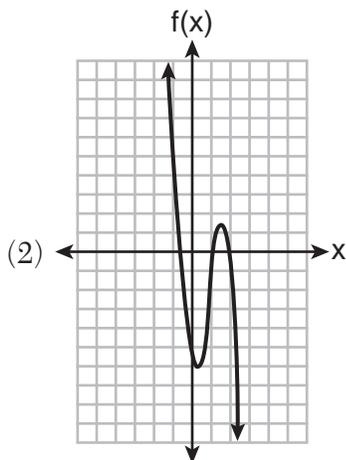


- (1) $h(x) = \log(x + a) + c$ (3) $h(x) = \log(x + a) - c$
 (2) $h(x) = \log(x - a) + c$ (4) $h(x) = \log(x - a) - c$

9 Which function has the characteristic as $x \rightarrow -\infty, f(x) \rightarrow -\infty$?



(3) $f(x) = 5(4)^{-x}$



(4) $f(x) = -\log_5(-x)$

**Use this space for
computations.**

10 The expression $(x^2 + 3)^2 - 2(x^2 + 3) - 24$ is equivalent to

- (1) $(x^2 + 9)(x^2 - 1)$ (3) $x^4 - 2x^2 - 21$
(2) $(x^2 - 3)(x^2 + 7)$ (4) $x^4 + 4x^2 - 9$

11 What is the solution for the system of equations below?

$$\begin{aligned}x + y + z &= 2 \\x - 2y - z &= -4 \\x - 9y + z &= -18\end{aligned}$$

- (1) $(-2, 2, 2)$ (3) $(0, 2, 0)$
(2) $(-2, -2, 6)$ (4) $(0, 2, 4)$

12 The roots of the equation $x^2 - 4x = -13$ are

- (1) $2 \pm 3i$ (3) $2 \pm \sqrt{17}$
(2) $2 \pm 6i$ (4) $2 \pm i\sqrt{13}$

13 Which expression is equivalent to $\frac{2x^3 + 2x - 7}{2x + 4}$?

- (1) $x^2 - 2x + 5 - \frac{27}{2x + 4}$ (3) $x^2 + 2x + 5 + \frac{13}{2x + 4}$
(2) $x^2 - 1 - \frac{3}{2x + 4}$ (4) $x^2 + 2x - 3 + \frac{5}{2x + 4}$

Use this space for
computations.

19 The solution set of $\frac{x+3}{x-5} + \frac{6}{x+2} = \frac{6+10x}{(x-5)(x+2)}$ is

(1) $\{-6\}$

(3) $\{-6,5\}$

(2) $\{5\}$

(4) $\{-5,6\}$

20 Given x and y are positive, which expressions are equivalent to $\frac{x^3}{y}$?

I. $\left(\frac{y}{x^3}\right)^{-1}$

II. $\sqrt[3]{x^9}(y^{-1})$

III. $\frac{x^6\sqrt[4]{y^8}}{x^3y^3}$

(1) I and II, only

(3) II and III, only

(2) I and III, only

(4) I, II, and III

21 Given the inverse function $f^{-1}(x) = \frac{2}{3}x + \frac{1}{6}$, which function represents $f(x)$?

(1) $f(x) = -\frac{2}{3}x + \frac{1}{6}$

(3) $f(x) = \frac{3}{2}x - \frac{1}{4}$

(2) $f(x) = -\frac{3}{2}x + \frac{1}{4}$

(4) $f(x) = \frac{3}{2}x - \frac{1}{6}$

Use this space for
computations.

22 How many equations below are identities?

- $x^2 + y^2 = (x^2 - y^2) + (2xy)^2$
- $x^3 + y^3 = (x - y) + (x^2 - xy + y^2)$
- $x^4 + y^4 = (x - y)(x - y)(x^2 + y^2)$

(1) 1

(3) 3

(2) 2

(4) 0

23 If the focus of a parabola is $(0, 6)$ and the directrix is $y = 4$, what is an equation for the parabola?

(1) $y^2 = 4(x - 5)$

(3) $y^2 = 8(x - 5)$

(2) $x^2 = 4(y - 5)$

(4) $x^2 = 8(y - 6)$

24 John and Margaret deposit \$500 into a savings account for their son on his first birthday. They continue to make a deposit of \$500 on the child's birthday, with the last deposit being made on the child's 21st birthday. If the account pays 4% annual interest, which equation represents the amount of money in the account after the last deposit is made?

(1) $S_{21} = 500(1.04)^{21}$

(3) $S_{21} = 500(1.04)^{20} + 500$

(2) $S_{21} = \frac{500(1 - 1.04^{21})}{1 - 1.04}$

(4) $S_{21} = \frac{500(1 - 0.04^{21})}{1 - 1.04}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

- 25** The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

26 Determine the solution of $\sqrt{3x + 7} = x - 1$ algebraically.

27 The population of bacteria, $P(t)$, in hundreds, after t hours can be modeled by the function $P(t) = 37e^{0.0532t}$. Determine whether the population is increasing or decreasing over time. Explain your reasoning.

28 The polynomial function $g(x) = x^3 + ax^2 - 5x + 6$ has a factor of $(x - 3)$. Determine the value of a .

29 Write a recursive formula for the sequence 189, 63, 21, 7,

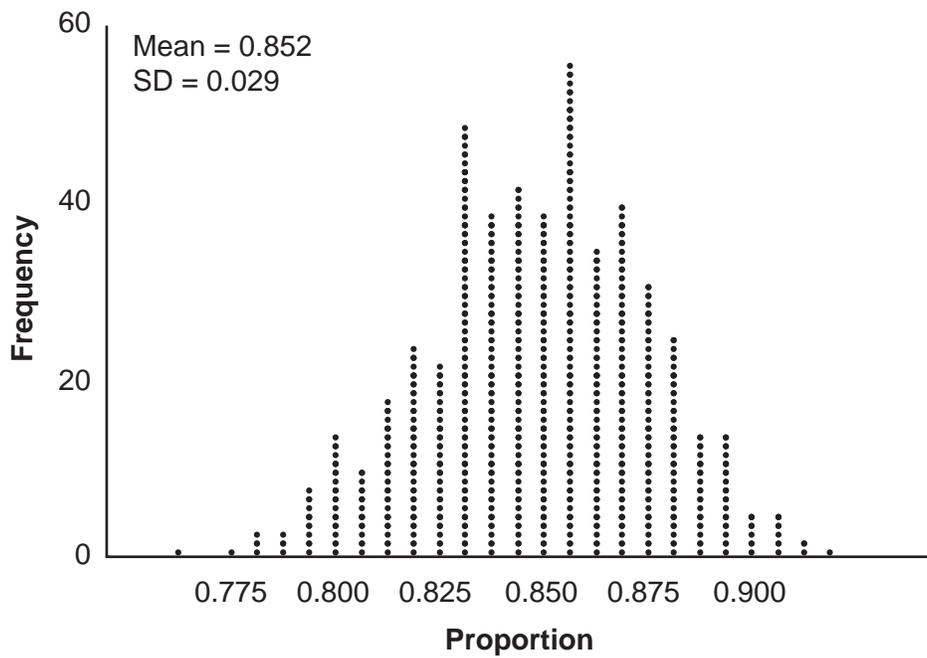
30 Solve algebraically for x to the *nearest thousandth*:

$$2e^{0.49x} = 15$$

31 For all values of x for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



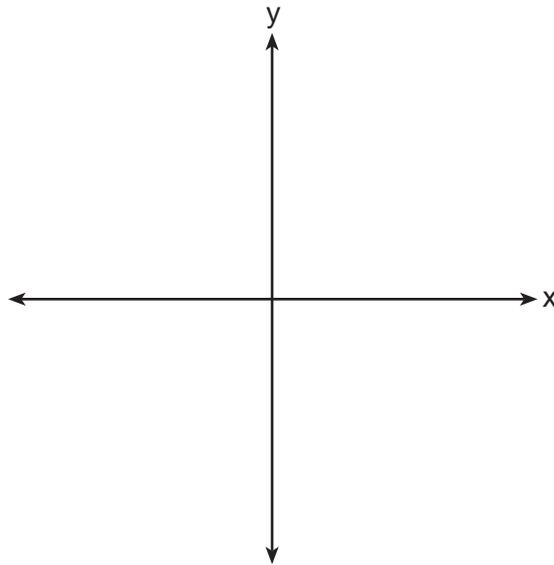
Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

- 33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

Sketch $y = p(x)$ on the set of axes below.



34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	Patron
Method of Donation	Phone calls	400	672
	Online	1200	2016

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

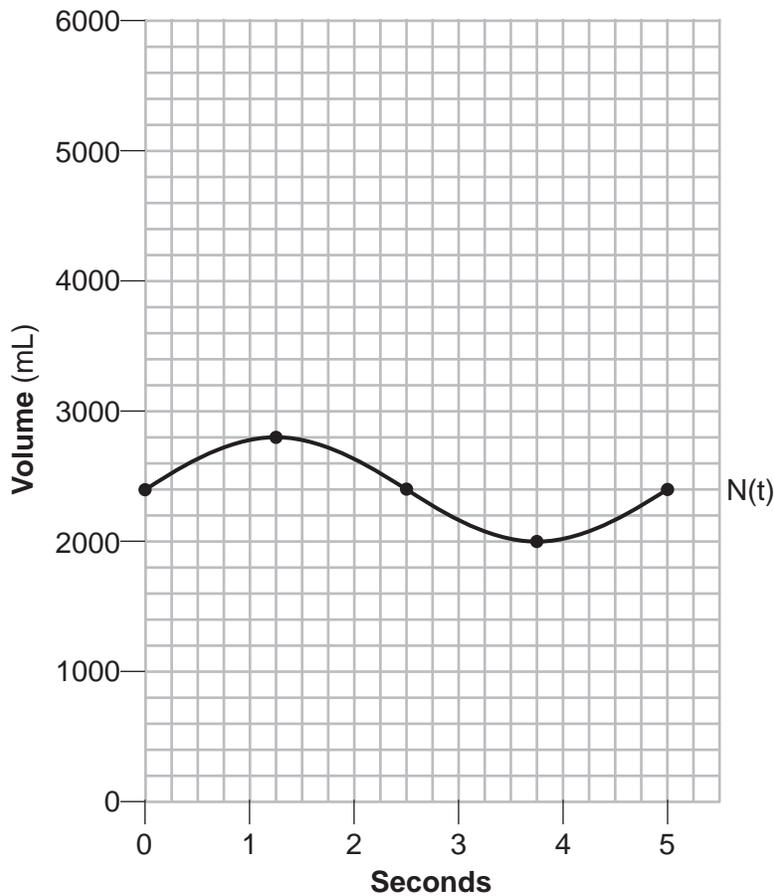
Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now.

State the solution to the equation $P(x) = F(x)$, rounded to the *nearest year*. Interpret the meaning of this value within the given context.

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

Question 37 is continued on the next page.

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

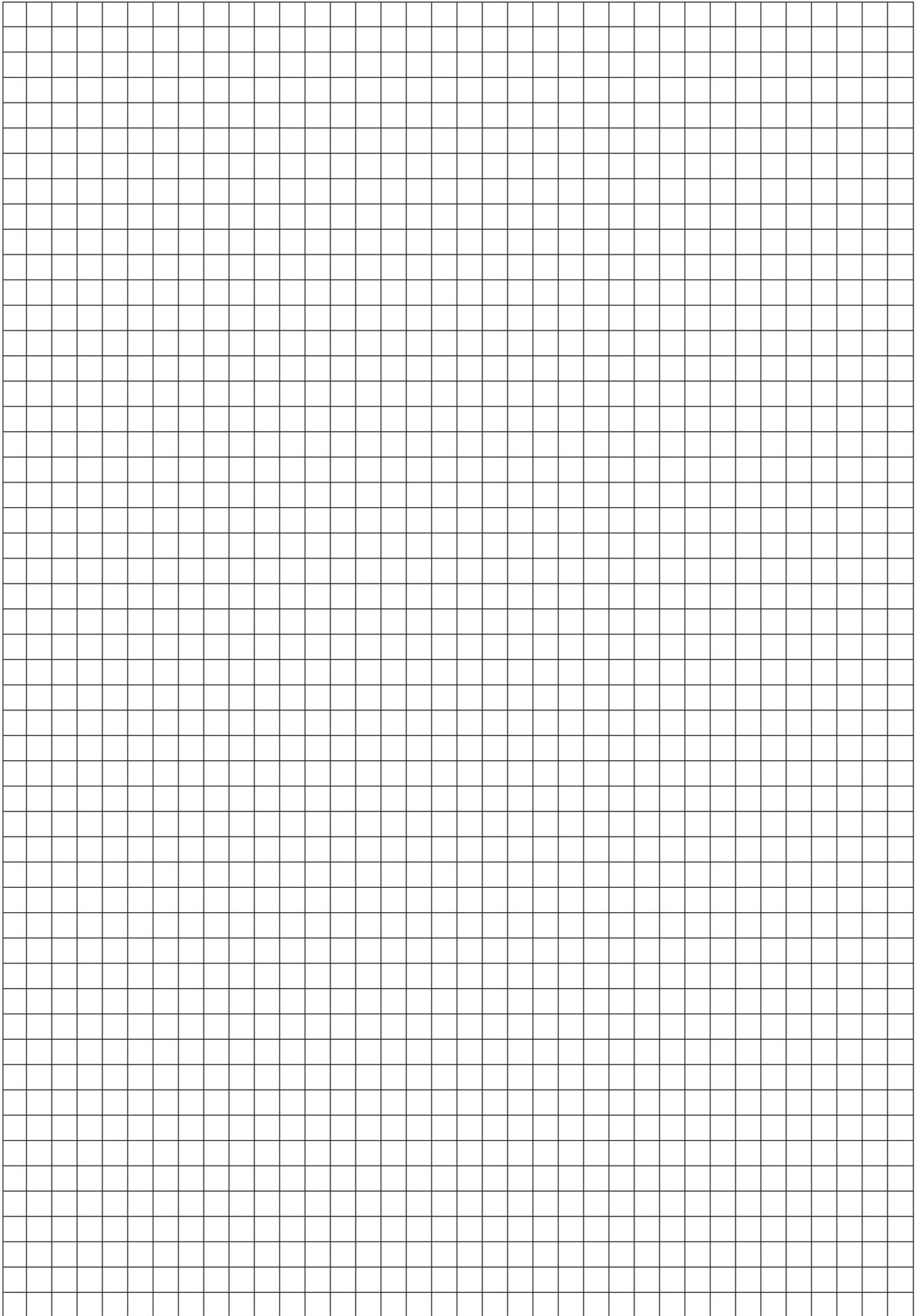
Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

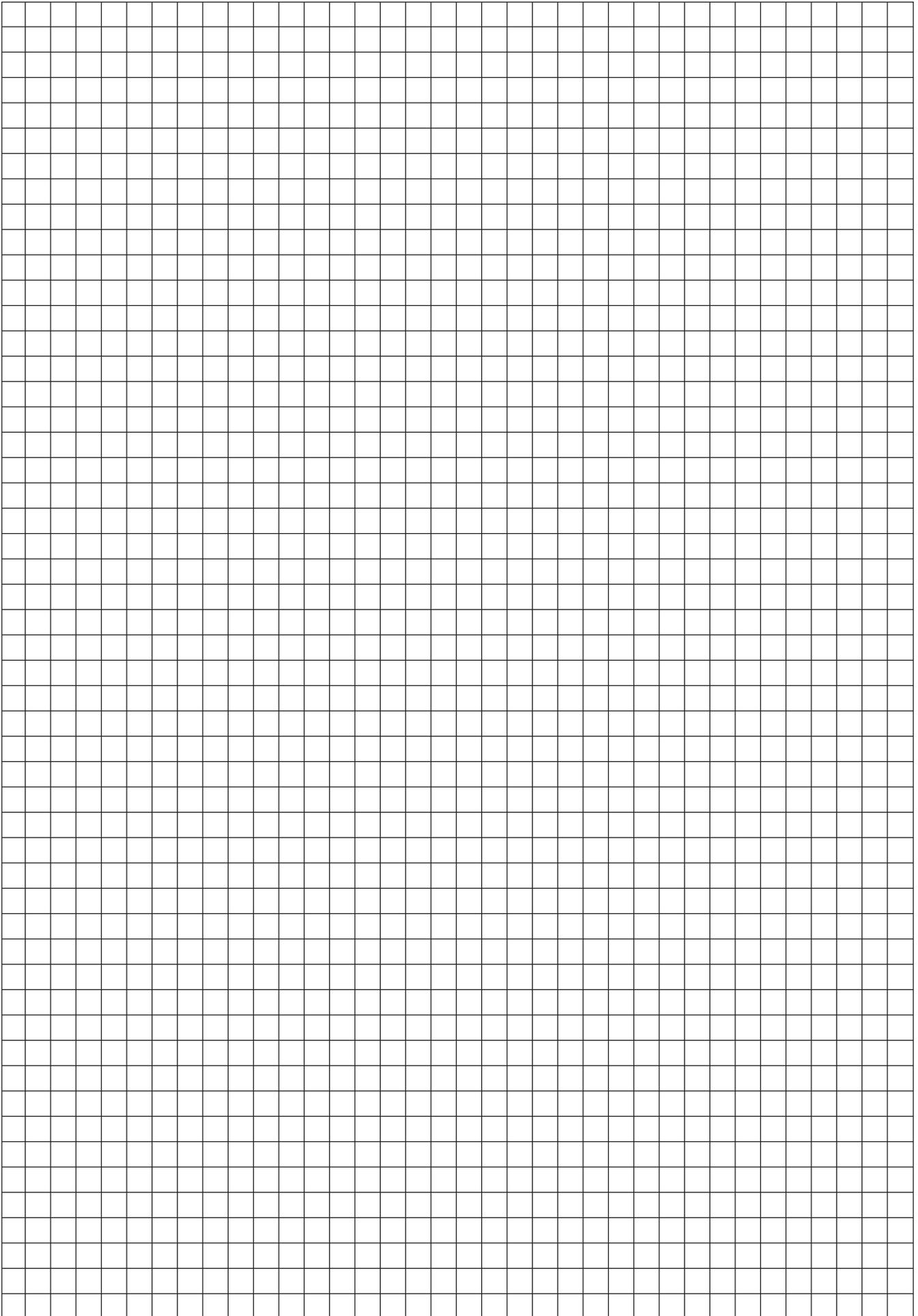
Scrap Graph Paper — this sheet will *not* be scored.

Tear Here

Tear Here



Scrap Graph Paper — this sheet will *not* be scored.



Tear Here

Tear Here

High School Math Reference Sheet

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n - 1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian = $\frac{180}{\pi}$ degrees
Degrees	1 degree = $\frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

Tear Here

Tear Here

Tear Here

Tear Here

Regents Examination in Algebra II – June 2023**Scoring Key: Part I (Multiple-Choice Questions)**

Examination	Date	Question Number	Scoring Key	Question Type	Credit	Weight
Algebra II	June '23	1	2	MC	2	1
Algebra II	June '23	2	3	MC	2	1
Algebra II	June '23	3	4	MC	2	1
Algebra II	June '23	4	2	MC	2	1
Algebra II	June '23	5	2	MC	2	1
Algebra II	June '23	6	2	MC	2	1
Algebra II	June '23	7	3	MC	2	1
Algebra II	June '23	8	1	MC	2	1
Algebra II	June '23	9	4	MC	2	1
Algebra II	June '23	10	2	MC	2	1
Algebra II	June '23	11	3	MC	2	1
Algebra II	June '23	12	1	MC	2	1
Algebra II	June '23	13	1	MC	2	1
Algebra II	June '23	14	3	MC	2	1
Algebra II	June '23	15	3	MC	2	1
Algebra II	June '23	16	4	MC	2	1
Algebra II	June '23	17	2	MC	2	1
Algebra II	June '23	18	1	MC	2	1
Algebra II	June '23	19	1	MC	2	1
Algebra II	June '23	20	4	MC	2	1
Algebra II	June '23	21	3	MC	2	1
Algebra II	June '23	22	4	MC	2	1
Algebra II	June '23	23	2	MC	2	1
Algebra II	June '23	24	2	MC	2	1

Regents Examination in Algebra II – June 2023**Scoring Key: Parts II, III, and IV (Constructed-Response Questions)**

Examination	Date	Question Number	Scoring Key	Question Type	Credit	Weight
Algebra II	June '23	25	-	CR	2	1
Algebra II	June '23	26	-	CR	2	1
Algebra II	June '23	27	-	CR	2	1
Algebra II	June '23	28	-	CR	2	1
Algebra II	June '23	29	-	CR	2	1
Algebra II	June '23	30	-	CR	2	1
Algebra II	June '23	31	-	CR	2	1
Algebra II	June '23	32	-	CR	2	1
Algebra II	June '23	33	-	CR	4	1
Algebra II	June '23	34	-	CR	4	1
Algebra II	June '23	35	-	CR	4	1
Algebra II	June '23	36	-	CR	4	1
Algebra II	June '23	37	-	CR	6	1

Key

MC = Multiple-choice question

CR = Constructed-response question

The chart for determining students' final examination scores for the **June 2023 Regents Examination in Algebra II** will be posted on the Department's web site at: <https://www.nysedregents.org/algebratwo/> on the day of the examination. Conversion charts provided for the previous administrations of the Regents Examination in Algebra II must NOT be used to determine students' final scores for this administration.

FOR TEACHERS ONLY

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, June 21, 2023 — 9:15 a.m. to 12:15 p.m., only

RATING GUIDE

Updated information regarding the rating of this examination may be posted on the New York State Education Department's web site during the rating period. Check this web site at: <https://www.nysed.gov/state-assessment/high-school-regents-examinations> and select the link "Scoring Information" for any recently posted information regarding this examination. This site should be checked before the rating process for this examination begins and several times throughout the Regents Examination period.

The Department is providing supplemental scoring guidance, the "Model Response Set," for the Regents Examination in Algebra II. This guidance is intended to be part of the scorer training. Schools are encouraged to incorporate the Model Response Sets into the scorer training or to use them as additional information during scoring. While not reflective of all scenarios, the model responses selected for the Model Response Set illustrate how less common student responses to constructed-response questions may be scored. The Model Response Set will be available on the Department's web site at <https://www.nysedregents.org/algebratwo/>.

Note: The rubric definition for a 0-credit response has been updated based on feedback from New York State mathematics educators.

Mechanics of Rating

The following procedures are to be followed for scoring student answer papers for the Regents Examination in Algebra II. More detailed information about scoring is provided in the publication *Information Booklet for Scoring the Regents Examination in Algebra II*.

Do *not* attempt to correct the student's work by making insertions or changes of any kind. In scoring the constructed-response questions, use check marks to indicate student errors. Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Each student's answer paper is to be scored by a minimum of three mathematics teachers. No one teacher is to score more than approximately one-third of the constructed-response questions on a student's paper. Teachers may not score their own students' answer papers. On the student's separate answer sheet, for each question, record the number of credits earned and the teacher's assigned rater/scorer letter.

Schools are not permitted to rescore any of the constructed-response questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Raters should record the student's scores for all questions and the total raw score on the student's separate answer sheet. Then the student's total raw score should be converted to a scale score by using the conversion chart that will be posted on the Department's web site at: <https://www.nysed.gov/state-assessment/> by Wednesday, June 21, 2023. Because scale scores corresponding to raw scores in the conversion chart may change from one administration to another, it is crucial that, for each administration, the conversion chart provided for that administration be used to determine the student's final score. The student's scale score should be entered in the box provided on the student's separate answer sheet. The scale score is the student's final examination score.

General Rules for Applying Mathematics Rubrics

I. General Principles for Rating

The rubrics for the constructed-response questions on the Regents Examination in Algebra II are designed to provide a systematic, consistent method for awarding credit. The rubrics are not to be considered all-inclusive; it is impossible to anticipate all the different methods that students might use to solve a given problem. Each response must be rated carefully using the teacher's professional judgment and knowledge of mathematics; all calculations must be checked. The specific rubrics for each question must be applied consistently to all responses. In cases that are not specifically addressed in the rubrics, raters must follow the general rating guidelines in the publication *Information Booklet for Scoring the Regents Examination in Algebra II*, use their own professional judgment, confer with other mathematics teachers, and/or contact the State Education Department for guidance. During each Regents Examination administration period, rating questions may be referred directly to the Education Department. The contact numbers are sent to all schools before each administration period.

II. Full-Credit Responses

A full-credit response provides a complete and correct answer to all parts of the question. Sufficient work is shown to enable the rater to determine how the student arrived at the correct answer.

When the rubric for the full-credit response includes one or more examples of an acceptable method for solving the question (usually introduced by the phrase “such as”), it does not mean that there are no additional acceptable methods of arriving at the correct answer. Unless otherwise specified, mathematically correct alternative solutions should be awarded credit. The only exceptions are those questions that specify the type of solution that must be used; e.g., an algebraic solution or a graphic solution. A correct solution using a method other than the one specified is awarded half the credit of a correct solution using the specified method.

III. Appropriate Work

Full-Credit Responses: The directions in the examination booklet for all the constructed-response questions state: “Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.” The student has the responsibility of providing the correct answer **and** showing how that answer was obtained. The student must “construct” the response; the teacher should not have to search through a group of seemingly random calculations scribbled on the student paper to ascertain what method the student may have used.

Responses With Errors: Rubrics that state “Appropriate work is shown, but...” are intended to be used with solutions that show an essentially complete response to the question but contain certain types of errors, whether computational, rounding, graphing, or conceptual. If the response is incomplete; i.e., an equation is written but not solved or an equation is solved but not all of the parts of the question are answered, appropriate work has **not** been shown. Other rubrics address incomplete responses.

IV. Multiple Errors

Computational Errors, Graphing Errors, and Rounding Errors: Each of these types of errors results in a 1-credit deduction. Any combination of two of these types of errors results in a 2-credit deduction. No more than 2 credits should be deducted for such mechanical errors in a 4-credit question and no more than 3 credits should be deducted in a 6-credit question. The teacher must carefully review the student's work to determine what errors were made and what type of errors they were.

Conceptual Errors: A conceptual error involves a more serious lack of knowledge or procedure. Examples of conceptual errors include using the incorrect formula for the area of a figure, choosing the incorrect trigonometric function, or multiplying the exponents instead of adding them when multiplying terms with exponents.

If a response shows repeated occurrences of the same conceptual error, the student should not be penalized twice. If the same conceptual error is repeated in responses to other questions, credit should be deducted in each response.

For 4- and 6-credit questions, if a response shows one conceptual error and one computational, graphing, or rounding error, the teacher must award credit that takes into account both errors. Refer to the rubric for specific scoring guidelines.

Part II

For each question, use the specific criteria to award a maximum of 2 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(25) [2] A correct explanation is written that implies randomization.

[1] One conceptual error is made.

or

[1] A model is chosen that will gather data but does not contain a randomized collection process.

[0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.

(26) [2] 6 and correct algebraic work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] Appropriate work is shown, but -1 is not rejected.

or

[1] 6, but a method other than algebraic is shown.

or

[1] 6, but no work is shown.

[0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.

- (27) [2] Increasing, and a correct explanation is written.
- [1] Appropriate work is shown, but one computational error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- or*
- [1] Increasing, but the explanation is incomplete.
- [0] Increasing, but no explanation is written.
- or*
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.
- (28) [2] $a = -2$, and correct work is shown.
- [1] Appropriate work is shown, but one computational error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- or*
- [1] $a = -2$, but no work is shown.
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.
- (29) [2] $a_1 = 189$, $a_n = \frac{1}{3}a_{n-1}$, or an equivalent recursive formula.
- [1] Appropriate work is shown, but one computational error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- [0] $a_1 = 189$, but no further correct work is shown.
- or*
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.

- (30) [2] 4.112, and correct algebraic work is shown.
- [1] Appropriate work is shown, but one computational or rounding error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- or*
- [1] 4.112, but a method other than algebraic is used.
- or*
- [1] 4.112, but no work is shown.
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.
-
- (31) [2] $-\frac{(x + 3)(2x + 1)}{x}$ or an equivalent answer in simplest form, and correct work is shown.
- [1] Appropriate work is shown, but one computational, factoring, or simplification error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- or*
- [1] $-\frac{(x + 3)(2x + 1)}{x}$, but no work is shown.
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.

- (32) [2] A negative response is indicated, and a correct explanation is written.
- [1] Appropriate work is shown, but one computational error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- or*
- [1] A correct explanation is written, but a negative response is not clearly indicated.
- or*
- [1] No, but an incomplete explanation is written.
- [0] No, but the explanation is missing or incorrect.
- or*
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.
-

Part III

For each question, use the specific criteria to award a maximum of 4 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

- (33) [4] $p(x) = (x - 2)(x - 3)(x + 6)$ or equivalent, and a correct graph is sketched.
- [3] One computational, factoring, graphing, or notation error is made.
- [2] Two or more computational, factoring, graphing, or notation errors are made.
- or*
- [2] One conceptual error is made.
- or*
- [2] $p(x) = (x - 2)(x - 3)(x + 6)$, but no further correct work is shown.
- or*
- [2] A correct graph is sketched, but no further correct work is shown.
- [1] One conceptual error and one computational, factoring, graphing, or notation error are made.
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.

- (34) [4] 0.373 and correct work is shown, and yes and a correct justification is given.
- [3] Appropriate work is shown, but one computational, simplification, or rounding error is made.
- or***
- [3] Appropriate work is shown to find 0.373, but an incomplete justification is given.
- [2] Appropriate work is shown, but two or more computational, simplification, or rounding errors are made.
- or***
- [2] Appropriate work is shown, but one conceptual error is made.
- or***
- [2] Appropriate work is shown to find 0.373, but no further correct work is shown.
- or***
- [2] Yes and a correct justification is given, but no further correct work is shown.
- [1] Appropriate work is shown, but one conceptual error and one computational, simplification, or rounding error are made.
- or***
- [1] 0.373, but no work is shown.
- [0] Yes, but no justification is given.
- or***
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.

- (35) [4] $(0, 7)$ and $(4, -1)$ and correct algebraic work is shown.
- [3] Appropriate work is shown, but one computational or factoring error is made.
- or***
- [3] Appropriate work is shown, but only the x -values or y -values are stated.
- [2] Appropriate work is shown, but two or more computational or factoring errors are made.
- or***
- [2] Appropriate work is shown, but one conceptual error is made.
- or***
- [2] A correct quadratic equation in one variable, in standard form, is written.
- or***
- [2] $(0, 7)$ and $(4, -1)$, but a method other than algebraic is used.
- [1] Appropriate work is shown, but one conceptual error and one computational or factoring error are made.
- or***
- [1] $(0, 7)$ and $(4, -1)$, but no work is shown.
- [0] $(0, 7)$ or $(4, -1)$, but no work is shown.
- or***
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.

- (36) [4] $P(x) = 500(0.97)^x$, $F(x) = 200e^{0.02x}$ or equivalent, 18, and a correct interpretation is written.
- [3] Appropriate work is shown, but one computational, rounding, or notation error is made.
- [2] Appropriate work is shown, but two computational, rounding, or notation errors are made.
- or**
- [2] Appropriate work is shown, but one conceptual error is made.
- or**
- [2] $P(x) = 500(0.97)^x$ and $F(x) = 200e^{0.02x}$ are written, but no further correct work is shown.
- [1] 18, but no further correct work is shown.
- or**
- [1] A correct interpretation is written, but no further correct work is shown.
- or**
- [1] $P(x) = 500(0.97)^x$ or $F(x) = 200e^{0.02x}$ is written, but no further correct work is shown.
- or**
- [1] Appropriate work is shown, but one conceptual error and one computational, rounding or notation error is made.
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.
-

Part IV

For each question, use the specific criteria to award a maximum of 6 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

- (37) [6] $N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + 2400$, a correct graph is drawn, and 4 is stated.
- [5] Appropriate work is shown, but one graphing or notation error is made.
- [4] Appropriate work is shown, but two graphing or notation errors are made.
- or*
- [4] Appropriate work is shown, but one conceptual error is made.
- [3] Appropriate work is shown, but three or more graphing or notation errors are made.
- or*
- [3] Appropriate work is shown, but one conceptual and one graphing or notation error are made.
- or*
- [3] A correct graph is drawn, but no further correct work is shown.
- [2] Appropriate work is shown, but two conceptual errors are made.
- or*
- [2] Appropriate work is shown to find, $N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + 2400$, but no further correct work is shown.
- [1] Appropriate work is shown, but two conceptual errors and one graphing or notation error are made.
- or*
- [1] 4 is stated, but no further correct work is shown.
- [0] A zero response does not contain enough relevant course-level work to receive any credit, does not satisfy the criteria for one or more credits, or is a correct response that was obtained by an obviously incorrect procedure.
-

**Map to the Learning Standards
Algebra II
June 2023**

Question	Type	Credits	Cluster
1	Multiple Choice	2	F-IF.B
2	Multiple Choice	2	A-SSE.A
3	Multiple Choice	2	A-APR.B
4	Multiple Choice	2	F-TF.A
5	Multiple Choice	2	F-IF.B
6	Multiple Choice	2	N-RN.A
7	Multiple Choice	2	N-CN.A
8	Multiple Choice	2	F-IF.C
9	Multiple Choice	2	F-IF.C
10	Multiple Choice	2	A-SSE.A
11	Multiple Choice	2	A-REI.C
12	Multiple Choice	2	A-REI.B
13	Multiple Choice	2	A-APR.D
14	Multiple Choice	2	S-ID.B
15	Multiple Choice	2	A-SSE.B
16	Multiple Choice	2	S-ID.A
17	Multiple Choice	2	S-IC.B
18	Multiple Choice	2	F-BF.B
19	Multiple Choice	2	A-REI.A
20	Multiple Choice	2	N-RN.A

21	Multiple Choice	2	F-BF.B
22	Multiple Choice	2	A-APR.C
23	Multiple Choice	2	G-GPE.A
24	Multiple Choice	2	A-SSE.B
25	Constructed Response	2	S-IC.B
26	Constructed Response	2	A-REI.A
27	Constructed Response	2	F-IF.C
28	Constructed Response	2	A-APR.B
29	Constructed Response	2	F-BF.A
30	Constructed Response	2	F-LE.A
31	Constructed Response	2	A-SSE.A
32	Constructed Response	2	S-IC.A
33	Constructed Response	4	A-APR.B
34	Constructed Response	4	S-CP.A
35	Constructed Response	4	A-REI.C
36	Constructed Response	4	F-BF.A
37	Constructed Response	6	F-IF.B

Regents Examination in Algebra II
June 2023
Chart for Converting Total Test Raw Scores to
Final Examination Scores (Scale Scores)

The *Chart for Determining the Final Examination Score for the June 2023 Regents Examination in Algebra II* will be posted on the Department’s web site at: <https://www.nysed.gov/state-assessment/> by Wednesday, June 21, 2023. Conversion charts provided for previous administrations of the Regents Examination in Algebra II must NOT be used to determine students’ final scores for this administration.

Online Submission of Teacher Evaluations of the Test to the Department

Suggestions and feedback from teachers provide an important contribution to the test development process. The Department provides an online evaluation form for State assessments. It contains spaces for teachers to respond to several specific questions and to make suggestions. Instructions for completing the evaluation form are as follows:

1. Go to <https://www.surveymonkey.com/r/8LNLLDW>.
2. Select the test title.
3. Complete the required demographic fields.
4. Complete each evaluation question and provide comments in the space provided.
5. Click the SUBMIT button at the bottom of the page to submit the completed form.

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, June 21, 2023 — 9:15 a.m. to 12:15 p.m.,

MODEL RESPONSE SET

Table of Contents

Question 25	2
Question 26	8
Question 27	15
Question 28	20
Question 29	25
Question 30	30
Question 31	36
Question 32	42
Question 33	48
Question 34	57
Question 35	67
Question 36	76
Question 37	83

Question 25

25 The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

Pick random names from a list of students
and ask them how they will pay.

Score 2: The student gave a complete and correct response.

Question 25

25 The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

The college should take a random
Sample Survey of students walking onto
Campus.

Score 2: The student gave a complete and correct response.

Question 25

25 The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

The office can make a survey at the mall or a public place and this will get them an unbiased information because they are randomly choosing people.

Score 1: The student did not survey an appropriate sample.

Question 25

25 The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

ASK a certain amount of students in each grade
how they make their payments when buying books.

Score 1: The student did not describe a random selection process.

Question 25

25 The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

Take a survey

Score 0: The student did not show enough correct work to receive any credit.

Question 25

25 The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

They can test it out
by samples

Score 0: The student did not show enough correct work to receive any credit.

Question 26

26 Determine the solution of $\sqrt{3x + 7} = x - 1$ algebraically.

$$3x + 7 = x^2 - 2x + 1$$

$$0 = x^2 - 5x - 6$$

$$0 = (x - 6)(x + 1)$$

$$x = 6$$

Score 2: The student gave a complete and correct response.

Question 26

26 Determine the solution of $\sqrt{3x+7} = x-1$ algebraically.

$$(\sqrt{3x+7})^2 = (x-1)^2$$

$$3x+7 = x^2 - 2x + 1$$

$$x^2 - 5x - 6 = 0$$

$$(x-1)(x-1)$$

$$x^2 - x - x + 1$$

$$x^2 - 2x + 1$$

$$x = \frac{5 \pm \sqrt{(5)^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - (-24)}}{2}$$

$$x = \frac{5 \pm 7}{2}$$

$$x = 6$$

$$x = -1$$

extraneous
root

$$\sqrt{3(6)+7} = 6-1$$

$$5 = 5$$

$$\sqrt{3(-1)+7} = -1-1$$

$$2 \neq -2$$

Score 2: The student gave a complete and correct response.

Question 26

26 Determine the solution of $\sqrt{3x+7} = x-1$ algebraically.

$$(\sqrt{3x+7})^2 = (x-1)^2 \quad (x-1)(x-1)$$

$$\begin{array}{r} 3x+7 = x^2 - 2x+1 \\ -3x-7 \quad -3x-7 \\ \hline \end{array}$$

$$0 = x^2 - 5x - 6$$

$$(x-6)(x+1)$$

$$\boxed{x=6} \quad \boxed{x=-1}$$

Score 1: The student did not reject -1 .

Question 26

26 Determine the solution of $\sqrt{3x+7} = x-1$ algebraically.

$$(\sqrt{3x+7})^2 = (x-1)^2 \quad (x-1)(x-1)$$

$$x^2 - 2x + 1$$

$$3x+7 = x^2 - 2x + 1$$

$$-x^2 + 5x + 6 = 0$$

$$25 - 4(-1)(6)$$

$$\frac{-5 \pm 7}{-2} = \{-1, 6\}$$

$$\sqrt{3(-1)+7} = -1-1$$

$$\sqrt{-4} = -2$$

$$\pm 2 = -2 \quad \checkmark$$

$$\sqrt{3(6)+7} = 6-1$$

$$\pm 5 = 5 \quad \checkmark$$

$$\{-1, 6\}$$

Score 1: The student incorrectly found the square root of 4.

Question 26

26 Determine the solution of $\sqrt{3x+7} = x-1$ algebraically.

x	1	
x^2	$1x$	x
$-6x$	-6	-6

\downarrow
 $-5x$

$$\begin{array}{r} 3x+7 = x^2-2x+1 \\ -3x-7 \quad -3x-7 \\ \hline 0 = x^2-5x-6 \end{array}$$

$(x+1)(x-6)$	
$x \neq -1$	$x \neq -6$

unconstitutional

x	-1	
x^2	$-1x$	x
$-1x$	1	-1

Score 1: The student made an error finding x , but then rejected correctly.

Question 26

26 Determine the solution of $\sqrt{3x+7} = (x-1)$ algebraically.

$$3x+7 = (x-1)(x-1)$$

$$3x+7 = x^2 - x - x + 1$$

$$3x+7 = x^2 + 2x + 1$$

$$-3x - 7$$

$$x^2 - x - 6$$

$$(x-3)(x+2)$$

$$x-3=0 \quad | \quad x+2=0$$

$x=3$	$x=-2$
-------	--------

Score 0: The student made a computational error and did not reject correctly.

Question 26

26 Determine the solution of $\sqrt{3x+7} = x-1$ algebraically.

$$\sqrt{3x+7} = x-1$$
$$3x+7 = (x-1)(x-1)$$

$$\sqrt{3x+7} = x-1$$
$$+1 \quad +1$$

$$(\sqrt{3x+7} + 1)^2 = (x)^2$$

$$3x+7+1 = x^2$$

$$3x+8 = x^2$$
$$-3x \quad -3x$$

$$8 = x^2 - 3x$$
$$-8 \quad -8$$

$$x^2 - 3x - 8$$

$$+ 2$$
$$6 \quad 1$$

Score 0: The student did not do enough correct work to receive any credit.

Question 27

27 The population of bacteria, $P(t)$, in hundreds, after t hours can be modeled by the function $P(t) = 37e^{0.0532t}$. Determine whether the population is increasing or decreasing over time. Explain your reasoning.

The population is increasing over time. If you graph the equation, as the x values increase, the y values increase in greater intervals.

Score 2: The student gave a complete and correct response.

Question 27

27 The population of bacteria, $P(t)$, in hundreds, after t hours can be modeled by the function $P(t) = 37e^{0.0532t}$. Determine whether the population is increasing or decreasing over time. Explain your reasoning.

$$e^{.0532} \approx 1.0546 \text{ is greater than } 1 \text{ so}$$

Increasing

Score 2: The student gave a complete and correct response.

Question 27

27 The population of bacteria, $P(t)$, in hundreds, after t hours can be modeled by the function $P(t) = 37e^{0.0532t}$. Determine whether the population is increasing or decreasing over time. Explain your reasoning.

Increasing - the values steadily increase

Score 1: The student gave an incomplete explanation.

Question 27

27 The population of bacteria, $P(t)$, in hundreds, after t hours can be modeled by the function $P(t) = 37e^{0.0532t}$. Determine whether the population is increasing or decreasing over time. Explain your reasoning.

It increase because its getting multiplied

Score 0: The student did not give enough of an explanation to receive any credit.

Question 27

27 The population of bacteria, $P(t)$, in hundreds, after t hours can be modeled by the function $P(t) = 37e^{0.0532t}$. Determine whether the population is increasing or decreasing over time. Explain your reasoning.

The population of the bacteria
will decrease because of
the exponent.

Score 0: The student stated decrease and wrote an incomplete explanation.

Question 28

28 The polynomial function $g(x) = x^3 + ax^2 - 5x + 6$ has a factor of $(x - 3)$. Determine the value of a .

$$0 = 27 + 9a - 15 + 6$$

$$0 = 18 + 9a$$

$$a = -2$$

Score 2: The student gave a complete and correct response.

Question 28

28 The polynomial function $g(x) = x^3 + ax^2 - 5x + 6$ has a factor of $(x - 3)$. Determine the value of a .

$$g(3) = (-3)^3 + a(-3)^2 - 5(-3) + 6$$

$$0 = -27 + 9a + 15 + 6$$

$$0 = 9a - 6$$

$$\frac{6}{9} = \frac{9a}{9}$$

$$\frac{2}{3} = a$$

Score 1: The student used -3 for x .

Question 28

28 The polynomial function $g(x) = x^3 + ax^2 - 5x + 6$ has a factor of $(x - 3)$. Determine the value of a .

$$g(x) = x^3 + ax^2 - 5x + 6$$

$$~~x^2(x+a) \parallel (-5x+6)~~$$

$$~~x+a = -5x+6~~$$

$$a = -2$$

Score 1: The student found the correct answer with no correct work.

Question 28

28 The polynomial function $g(x) = x^3 + ax^2 - 5x + 6$ has a factor of $(x - 3)$. Determine the value of a .

$$\begin{aligned} 3 &= 3^3 + a3^2 - 5(3) + 6 & 3 &= 3^3 + a3^2 - 5(3) + 6 \\ 3 &= 27 + a9 - 15 + 6 & 3 &= 27 + a9 - 15 + 6 \\ \frac{3}{-27} & & 3 &= 27 + a9 - 9 \\ -24 &= a9 - 15 + 6 \\ + 15 & & & \\ -9 &= a9 + 6 \end{aligned}$$

Score 0: The student did not show enough correct work to receive any credit.

Question 29

29 Write a recursive formula for the sequence 189, 63, 21, 7,

$$A_n = A_{n-1} \div 3.$$

$$A_1 = 189$$

Score 2: The student gave a complete and correct response.

Question 29

29 Write a recursive formula for the sequence 189, 63, 21, 7, ...

$$\frac{189}{3} \rightarrow \frac{63}{3} \rightarrow \frac{21}{3}$$

$$r = \frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}$$

$$a_n = 189 \left(\frac{1}{3}\right)^{n-1}$$

Score 1: The student wrote an explicit formula.

Question 29

29 Write a recursive formula for the sequence 189, 63, 21, 7,

$$a_n = \frac{a_{n-1}}{3}$$

$$\frac{189}{?} = 63$$

$$\frac{189}{3} = 63$$

$$\frac{63}{3} = 21$$

$$\frac{21}{3} = 7$$

Score 1: The student did not state a_1 .

Question 29

29 Write a recursive formula for the sequence 189, 63, 21, 7, ...

$\underbrace{\quad\quad\quad}$
 -3

$$f(1) = 189$$

$$f(2) = 63$$

$$f(x-1) = x \div 3$$

Score 0: The student did not show enough correct work to receive any credit.

Question 29

29 Write a recursive formula for the sequence 189, 63, 21, 7,

$$a_n = a_1 (3)^{n-1}$$

$$a_n = 189 (3)^{n-1}$$

Score 0: The student wrote an incorrect explicit formula.

Question 30

30 Solve algebraically for x to the nearest thousandth:

$$2e^{0.49x} = 15$$

$$\frac{2e^{0.49x}}{2} = \frac{15}{2}$$

$$e^{0.49x} = 7.5$$

$$\frac{0.49x \ln e}{\ln e} = \frac{\ln 7.5}{\ln e}$$

$$= 1$$

$$\frac{0.49x}{0.49} = \frac{2.014903021}{0.49}$$

$$x = 4.112046981$$

$$x \approx 4.112$$

Score 2: The student gave a complete and correct response.

Question 30

30 Solve algebraically for x to the *nearest thousandth*:

$$\frac{2e^{0.49x}}{2} = \frac{15}{2}$$

$$e^{0.49x} = 7.5$$

$$.49x = \log_e 7.5$$

$$x = \frac{\log_e 7.5}{.49}$$

$$x = 4.112$$

Score 2: The student gave a complete and correct response.

Question 30

30 Solve algebraically for x to the nearest thousandth:

$$\frac{2e^{0.49x}}{2} = \frac{15}{2}$$

$$\ln \cancel{2} \cdot 0.49x = \ln 7.5$$

$$\frac{.049x}{.049} = \frac{\ln 7.5}{.049}$$

$$x = 41.120$$

Score 1: The student made a transcription error.

Question 30

30 Solve algebraically for x to the nearest thousandth:

$$2e^{0.49x} = 15$$

$$\ln(15) = .49x$$

$$\frac{5.416100402}{.49} = \frac{.49x}{.49}$$

$$11.0532653 = x$$

$$x = 11.053$$

Score 1: The student multiplied by 2 instead of dividing by 2.

Question 30

30 Solve algebraically for x to the nearest thousandth:

$$\frac{2e^{0.49x}}{2} = \frac{15}{2}$$

$$\frac{e^{0.49x}}{e} = \frac{7.5}{e}$$

$$\frac{0.49x}{0.49} = \frac{2.759095}{0.49}$$

$$X = 5.63080$$

$$X \approx 5.63$$

Score 0: The student did not show enough correct work to receive any credit.

Question 30

30 Solve algebraically for x to the *nearest thousandth*:

$$\ln(2e^{0.49x}) = \ln(15)$$

$$x = 4,113$$

Score 0: The student did not show enough correct work to receive any credit.

Question 31

31 For all values of x for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

$$\frac{x^2(2x+1) - 9(2x+1)}{x(3-x)}$$
$$= \frac{(x+3)\cancel{(x-3)}(2x+1)}{x\cancel{(3-x)}}$$
$$= \frac{(x+3)(2x+1)}{x}$$

Score 2: The student gave a complete and correct response.

Question 31

31 For all values of x for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

$$\frac{x^2(2x+1) - 9(x+1)}{-x(3+x)}$$

$$\frac{(x^2 - 9)(2x+1)}{-x(3+x)}$$

$$\frac{(x+3)(x-3)(2x+1)}{-x(3+x)}$$

$$\frac{(2x+1)(x-3)}{-x}$$

Score 1: The student made a factoring error.

Question 31

31 For all values of x for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

$$\frac{2x^3 + x^2 - 18x - 9}{x(3-x)}$$

$$\begin{array}{r} -2x+5 \\ \hline -x^2+3x \overline{) 2x^3+x^2-18x-9} \\ \underline{-(-2x^3+6x^2)} \quad \downarrow \quad \downarrow \\ 5x^2-18x \\ \underline{-(-5x^2+15x)} \\ -3x-9 \end{array}$$

$$2x^3 + x^2 - 18x - 9$$

$$(2x+1)(x^2-9)$$

$$2x^3 - 18x + x^2 - 9$$

$$\frac{(2x+1)(x+3)(x-3)}{x(3-x)}$$

Score 1: The student did not leave the answer in simplest form.

Question 31

31 For all values of x for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

$$x^2(2x+1) \mid -9(2x+1)$$

$$(x^2 - 9)(2x+1)(2x+1)$$

$$\frac{(x+3)(x-3)(2x+1)}{3x-x^2}$$

Score 1: The student only factored the numerator correctly.

Question 31

31 For all values of x for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

$$\frac{x^2(2x) - 9(2x)}{3x - x^2}$$

$$\frac{(x^2 - 9)}{-x^2 + 3x}$$

$$\frac{(x+3)(x-3)}{-x^2 + 3x}$$

Score 0: The student did not show enough correct work to receive any credit.

Question 31

31 For all values of x for which the expression is defined, write the expression below in simplest form.

$$\frac{(2x^3 + x^2 - 18x - 9)}{3x - x^2}$$

$$\frac{x^2(2x) - 9(2x)}{x(3-x)}$$

$$x(3-x)$$

$$\frac{(x^2-9)(2x)}{x(3-x)}$$

$$x(3-x)$$

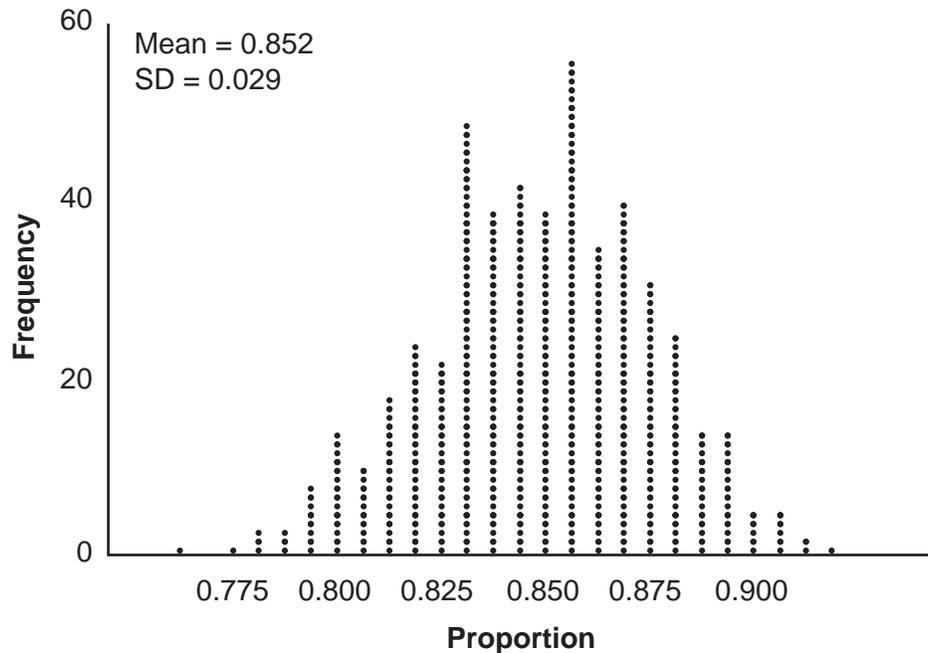
$$\frac{(x+3)(x-3)(2x)}{x(3-x)}$$

$$\frac{(-x+3)(2x)}{x}$$

Score 0: The student made multiple errors.

Question 32

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



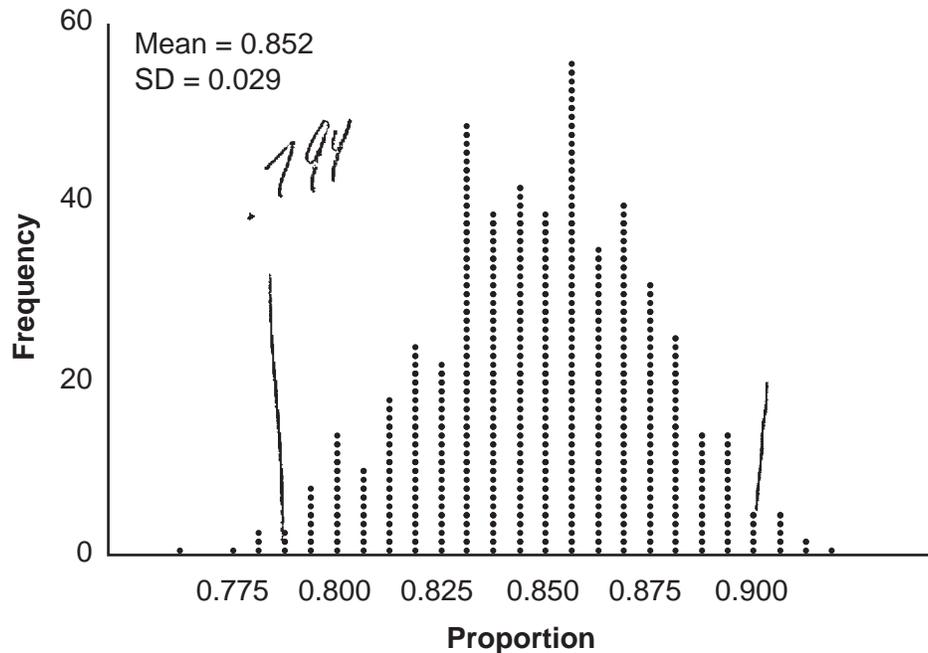
Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is incorrect? Explain.

No b/c the high school's sample falls within
the margin of error interval: 91% - 79.4%

Score 2: The student gave a complete and correct response.

Question 32

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

$$2(0.029) = .058$$

$$0.852 + .058 = .91$$

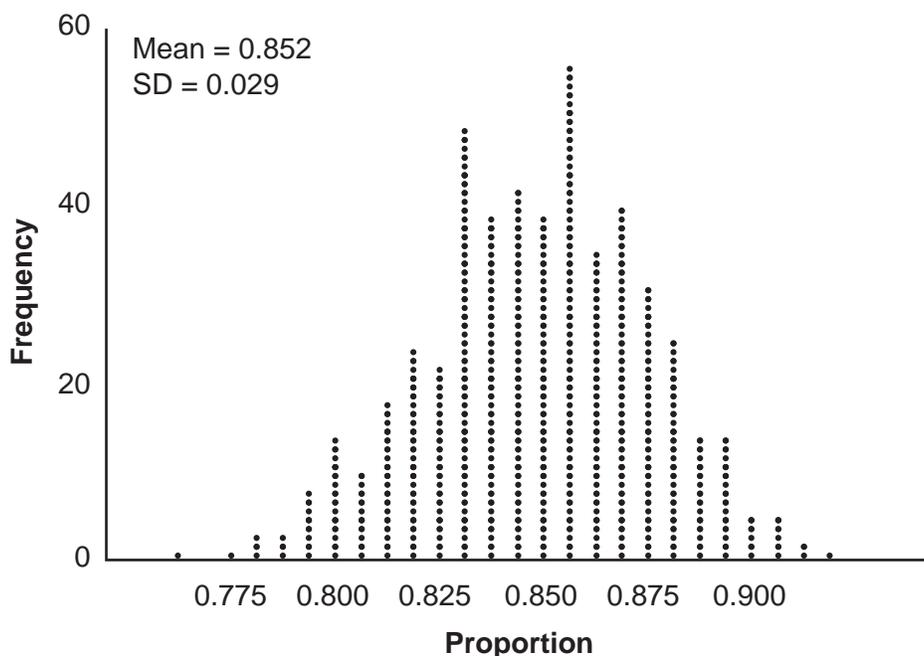
$$0.852 - .058 = .794$$

$[.794, .91]$ the results would not give the company reason to believe that their assumption is not correct because 88% falls in the confidence interval.

Score 2: The student gave a complete and correct response.

Question 32

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



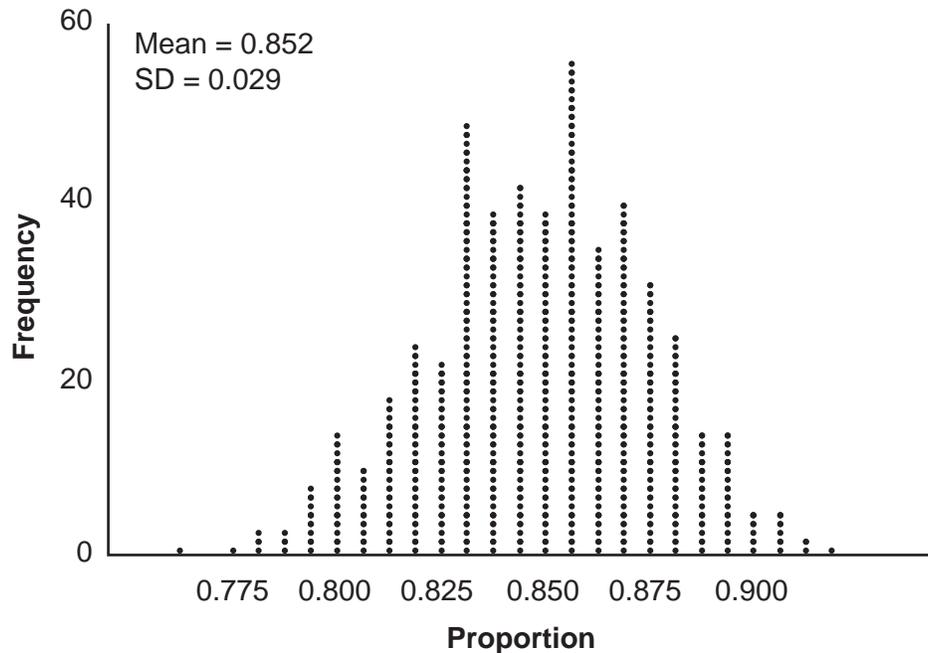
Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

$Mean \pm SD$
 $0.852 \pm 2(0.029)$
 $\leftarrow .794$ to $\rightarrow .91$
 NO
 it IS
 correct

Score 1: The student wrote an incomplete explanation.

Question 32

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



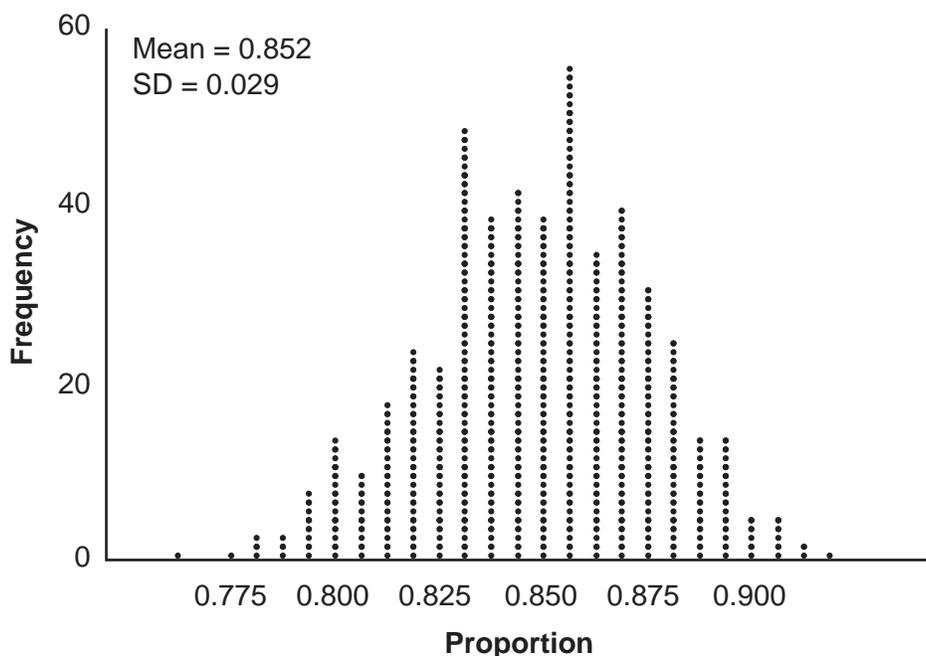
Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

It falls within the 95% confidence interval of the simulations so the app design company has no reason to believe their assumption is incorrect.

Score 1: The student gave an incomplete explanation.

Question 32

32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



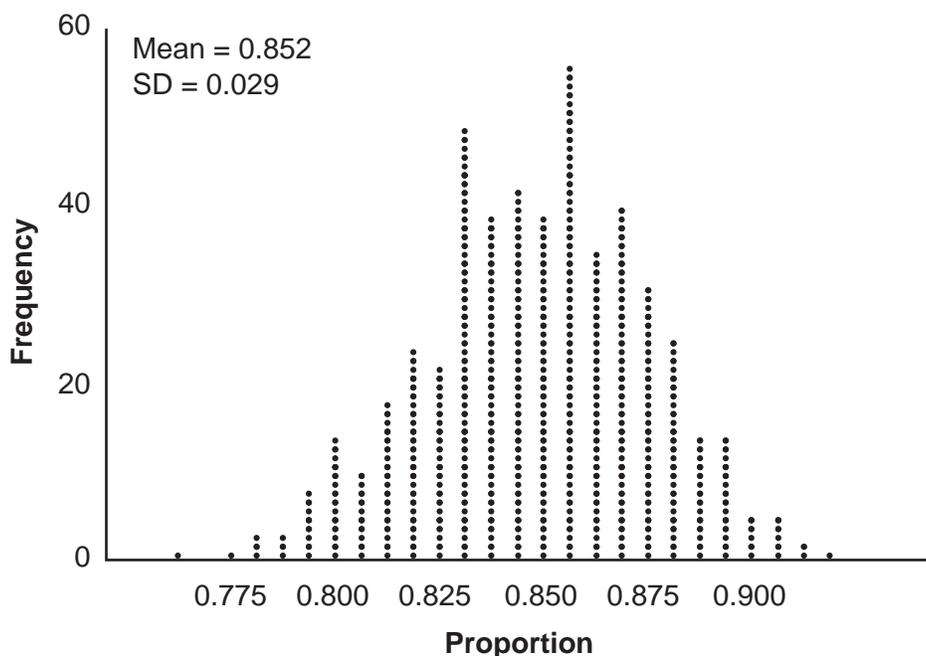
Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is incorrect? Explain.

The only reason to believe this assumption is not correct due to the simulation is the fact that the highest/most likely proportion shown is 86% and not 88%.

Score 0: The student did not show enough relevant course-level work to receive any credit.

Question 32

- 32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

$\frac{96}{150} = 0.64$ Since the p value is 64%
the data collected is not
significant and, due to chance
most likely
so the app design company is
incorrect.

Score 0: The student did not show enough relevant course-level work to receive any credit.

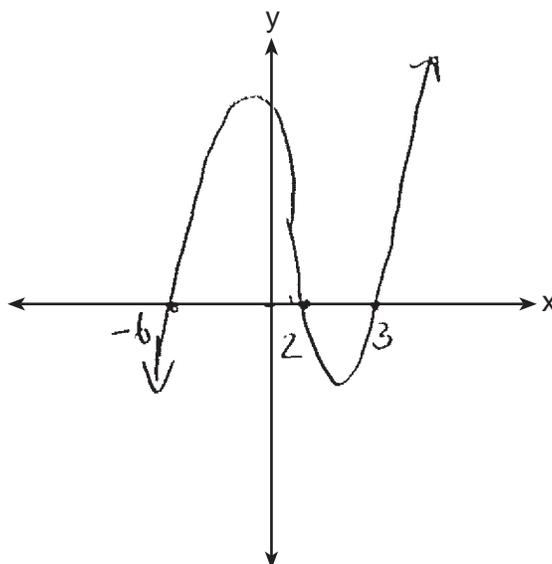
Question 33

33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

$$p(x) = (x-2)(x-3)(x+6)$$

$$\begin{array}{l} (x-2)(x-3) \\ x^2 - 3x - 2x + 6 \\ (x^2 - 5x + 6)(x+6) \\ x^3 - 5x^2 + 6x + 6x^2 + 36x + 36 \\ x^3 + x^2 + 41x + 36 \end{array}$$

Sketch $y = p(x)$ on the set of axes below.



Score 4: The student gave a complete and correct response.

Question 33

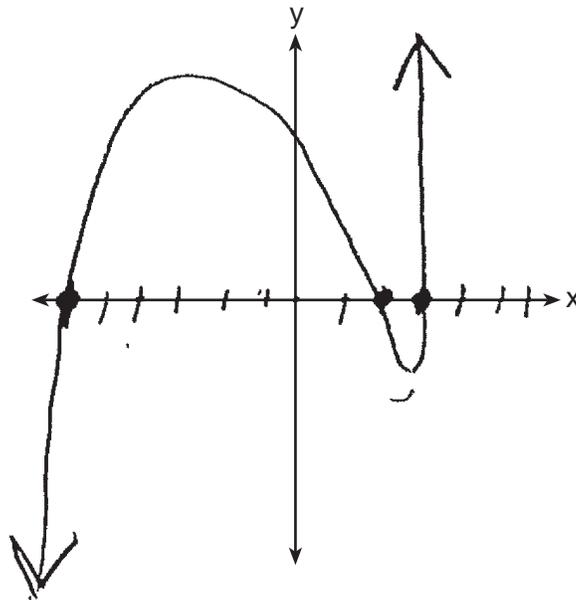
33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

$$p(x) = (x-2)(x-3)(x+6)$$
$$= x^2 - 5x + 6(x+6)$$

$$p(x) = x^3 + 6x^2 - 5x^2 - 30x + 6x + 36$$

$$p(x) = x^3 + x^2 - 24x + 36$$

Sketch $y = p(x)$ on the set of axes below.



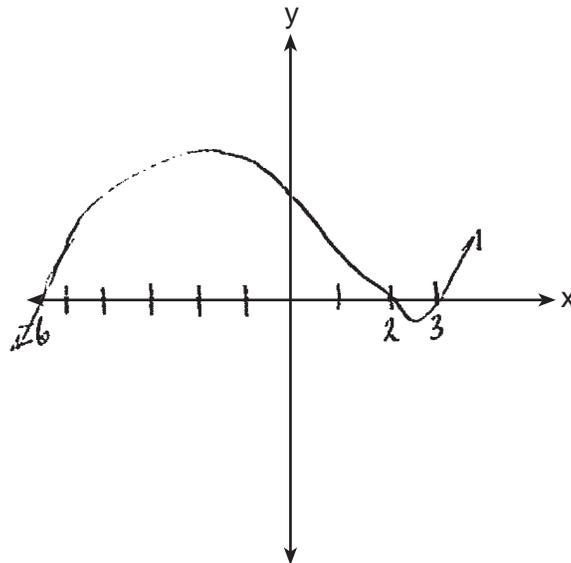
Score 4: The student gave a complete and correct response.

Question 33

33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

$$\begin{aligned} & (x-2)(x-3)(x-6) \\ & \quad x^2 - 3x - 2x + 6 \\ & (x-6)(x^2 - 5x + 6) \\ & \quad x^3 - 5x^2 + 6x - 6x^2 + 30x - 36 \\ & \boxed{p(x) = x^3 - 11x^2 + 36x - 36} \end{aligned}$$

Sketch $y = p(x)$ on the set of axes below.



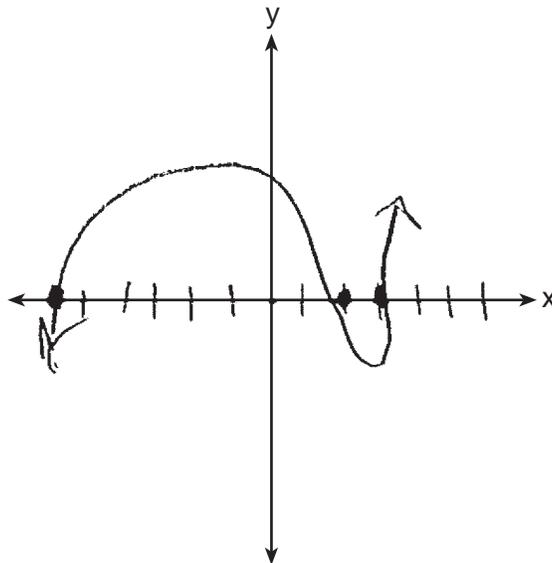
Score 3: The student incorrectly wrote one of the factors as $x - 6$.

Question 33

33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

$$(x-2)(x-3)(x+6)$$

Sketch $y = p(x)$ on the set of axes below.



Score 3: The student wrote an expression, not an equation for $p(x)$. The student drew an acceptable sketch through the zeros with appropriate end behavior.

Question 33

33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

$$(x-2)(x-3)(x+6) = P(x)$$

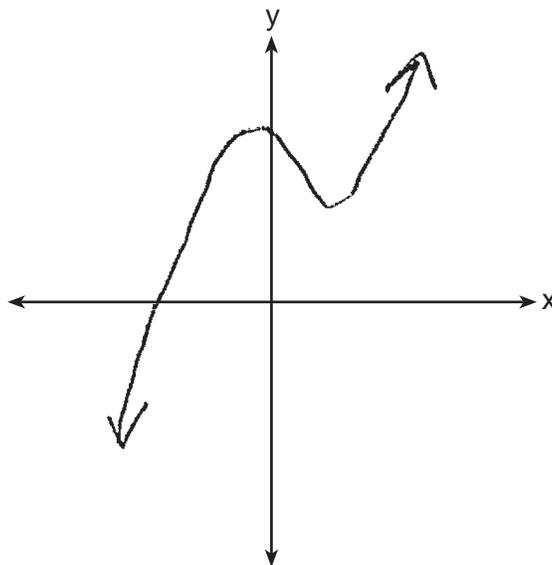
$$(x^2-3x-2x+6)(x+6) = P(x)$$

$$(x^2-5x+6)(x+6) = P(x)$$

$$x^3+6x^2-5x^2-30x+6x+36 = P(x)$$

$$x^3+x^2-24x+36 = P(x)$$

Sketch $y = p(x)$ on the set of axes below.



Score 2: The student only received credit for the equation.

Question 33

33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

$$x - (-6) \quad x - 2 \quad x - 3$$

$$(x - 2)(x - 3)(x + 6)$$

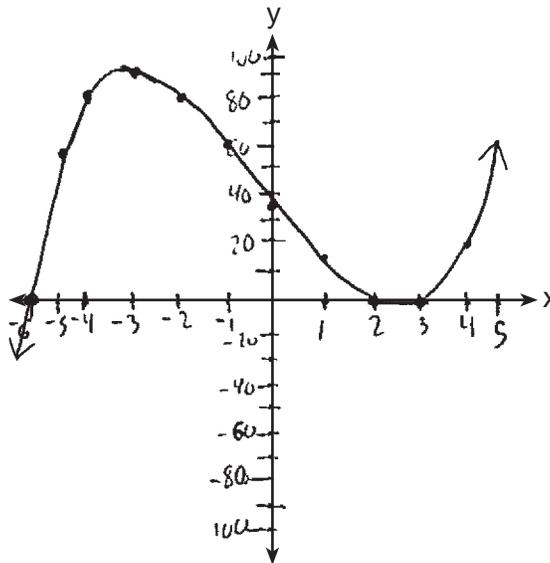
$$(x^2 - 3x - 2x + 6)(x + 6)$$

$$(x^2 - 5x + 6)(x + 6)$$

$$x^3 + 6x^2 - 5x^2 - 30x + 6x + 36$$

$$x^3 + x^2 - 24x + 36$$

Sketch $y = p(x)$ on the set of axes below.



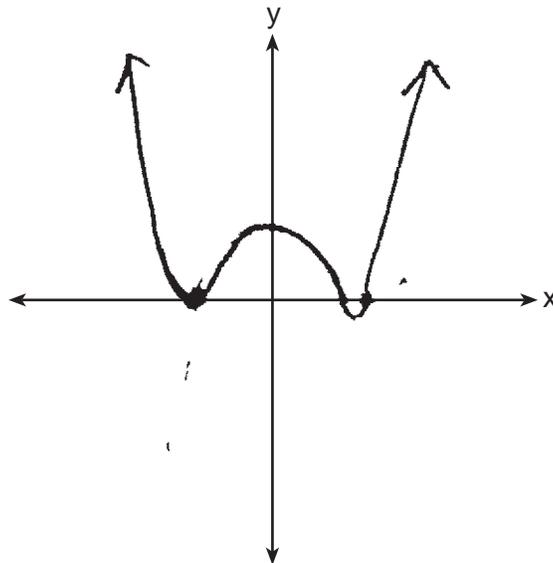
Score 2: The student did not write an equation for $p(x)$ and made one graphing error.

Question 33

33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

$$P(x) = x^4 + 7x^3 - 18x^2 + 252x + 216$$
$$x^4 + 12x^3 + 36x^2 - 5x^3 - 60x^2 - 180x + 6x^3 + 7x^2$$
$$(x^2 - 5x + 6)(x^2 + 12x + 36)$$
$$(x-2)(x-3)(x+6)(x+6)$$

Sketch $y = p(x)$ on the set of axes below.



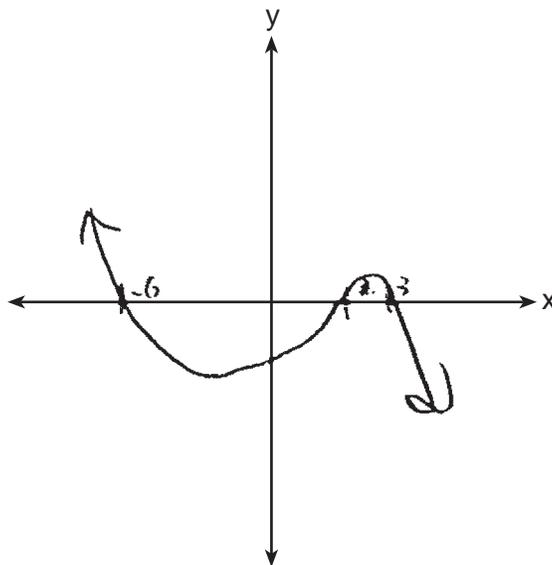
Score 1: The student received one credit for the sketch.

Question 33

33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

$$P(x) =$$

Sketch $y = p(x)$ on the set of axes below.



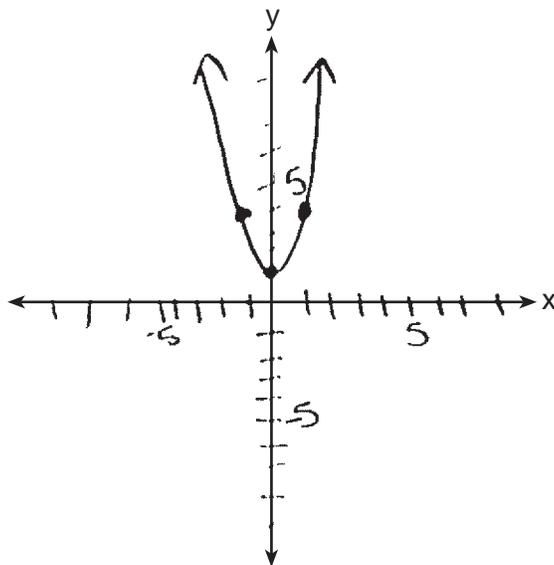
Score 1: The student received one credit for the sketch.

Question 33

33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

$$p(x) = 3x^2 + 1$$

Sketch $y = p(x)$ on the set of axes below.



Score 0: The student did not show enough relevant course-level work to receive any credit.

Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	Patron
Method of Donation	Phone calls	400	672
	Online	1200	2016

To the nearest thousandth find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\begin{array}{l}
 S = \text{supporter} \\
 O = \text{online}
 \end{array}
 \quad
 P(S|O) = \frac{1200}{3216} = \frac{25}{67} \approx \underline{.373} \text{ probability}$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(S) = \frac{1600}{4286} = \frac{25}{67} \approx .373 \quad 37.3\%$$

Yes, because the probability of being a supporter is equal to the probability of being a supporter of those donating online.

Score 4: The student gave a complete and correct response.

Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	Patron
Method of Donation	Phone calls	400	672
	Online	1200	2016
		1600	2688
		= 3216	
		1072	
		4288	

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{3216} = 0.373$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(O \cap S) \stackrel{?}{=} P(O) \cdot P(S)$$

$$\frac{1200}{4288} \stackrel{?}{=} \frac{3216}{4288} \cdot \frac{1600}{4288}$$

$$.2798507... \stackrel{?}{=} .2798507... \checkmark$$

Yes

Score 4: The student gave a complete and correct response.

Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	Patron
Method of Donation	Phone calls	400	672
	Online	1200	2016

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{3216}$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(A|B) = P(A)$$

$$\frac{1200}{3216} \approx \frac{1600}{4288} \dots \dots$$

$$.3731343284 = .3731343284$$

yes being a supporter is independent of donating online

Score 3: The student did not round the conditional probability.

Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category		T
		Supporter	Patron	
Method of Donation	Phone calls	400	672	1072
	Online	1200	2016	3216
T		1600	2688	4288

To the nearest thousandth, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$P(S|O) = 1200 / 3216 = \frac{25}{67} = .373$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(S \cap O) = \frac{1200}{4288} = \frac{75}{268}$$

$$P(S) \cdot P(O) = \frac{3216}{4288} \cdot \frac{1600}{4288} = \frac{75}{268}$$

Score 3: The student did not indicate a positive response to indicate independence.

Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	Patron
Method of Donation	Phone calls	400	672
	Online	1200	2016

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$P(S|O) = \frac{1200}{1600}$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(S) \cdot P(O) = P(S \cap O)$$

$$\frac{1600}{4288} \cdot \frac{3216}{4288} = \frac{1200}{4288}$$

$$.27985 = .27985$$

Yes b/c

$$P(S) \cdot P(O) = P(S \cap O)$$

Score 2: The student received no credit for the conditional probability.

Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	Patron
Method of Donation	Phone calls	400	672
	Online	1200	2016

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{3216} = .3731 = .373$$

$.373$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

No, as there are many more donors who are mostly donating online

Score 2: The student received no credit for determining independence.

Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category		Total
		Supporter	Patron	
Method of Donation	Phone calls	400	672	1072
	Online	1200	2016	3216
Total		1600	2688	4288

To the nearest thousandth, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

bottom

$$P(S|O) = \frac{1600}{3216} = .4975$$

1200

.498

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(S+O) = P(S) \cdot P(O)$$

$$\frac{1200}{1600} = \frac{1600}{4288} \cdot \frac{3216}{4288}$$

.75 ~~x~~ .2798507463

NOT independent

Score 2: The student made an error in the numerator of the conditional probability and an error calculating independence.

Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	Patron
Method of Donation	Phone calls	400	672
	Online	1200	2016
		1600	2688

3216
4241

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online. 510

$$\frac{1200}{3216}$$

$$37.31\%$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$P(A) \cdot P(B) = P(A) + P(B) - P(A \cup B)$$

$$(.7583)(.3772) = .28608$$

No they are not dependent because the possibility of $P(A) \cdot P(B) \neq P(A) + P(B) - P(A \cup B)$

Score 1: The student received one credit for the exact conditional probability, but showed no further correct work.

Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category		
		Supporter	Patron	
Method of Donation	Phone calls	400	672	1072
	Online	1200	2016	3216
		1600	2688	4288

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{1600} = \boxed{.750}$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$\frac{400}{1600} = \boxed{.25} \quad \frac{1200}{1600} = \boxed{.75}$$

INDEPENDENT yes because different numbers

Score 1: The student found the conditional probability of the reversed conditions but showed no further correct work.

Question 34

34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	Patron
Method of Donation	Phone calls	400	672
	Online	1200	2016

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{4288} \quad \text{0.280}$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

Independent because there's multiple ways to support

Score 0: The student did not find a conditional probability and did not show enough relevant course-level work to receive any credit.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)^2 + (-2x+7-3)^2 = 20$$

$$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$$

$$\frac{5x^2 - 20x + 20}{5} = \frac{20}{5}$$

$$x^2 - 4x + 4 = 4$$

$$\sqrt{(x-2)^2} = \sqrt{4} \quad y = -2x + 7$$

$$x - 2 = \pm 2$$

$$x = 2 \pm 2$$

$$x = 0, x = 4$$

$$y = -2(0) + 7$$

$$y = 0 + 7$$

$$y = 7$$

$$y = -2(4) + 7$$

$$y = -8 + 7$$

$$y = -1$$

$$\boxed{(0, 7) \quad (4, -1)}$$

Score 4: The student gave a complete and correct response.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)(x-2) + (y-3)(y-3) = 20$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 = \cancel{20}$$

$$\phantom{x^2 - 4x + 4 + y^2 - 6y + 9 = \cancel{20}} - 20 \phantom{x^2 - 4x + 4 + y^2 - 6y + 9 = \cancel{20}} - 20$$

$$x^2 - 4x + y^2 - 6y - 7 = 0$$

$$x^2 - 4x + (-2x+7)(-2x+7) - 6(-2x+7) - 7 = 0$$

~~$$x^2 - 4x + 4x^2 - 28x + 49 + 12x - 42 - 7 = 0$$~~

$$5x^2 - 20x = 0$$

$$5x(x - 4) = 0$$

$$5x = 0$$

$$\frac{\quad}{5}$$

$$x = 0$$

~~$$x - 4 = 0$$~~

~~$$+4 + 4$$~~

~~$$\hline$$~~

~~$$x = 4$$~~

$$y = -2(0) + 7$$

$$y = 7$$

$$y = -2(4) + 7$$

$$y = -1$$

Score 4: The student gave a complete and correct response.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)^2 + (-2x+7-3)^2 = 20 \rightarrow (x-2)^2 + (-2x+4)^2$$

$$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$$

$$5x^2 - 20x + 20 = 20$$

$$-20 \quad -20$$

$$5x^2 - 20x = 0$$

$$5x(x-4)$$

$$x = 4$$

$$y = -1$$

Score 3: The student found only one solution.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)^2 + (-2x+7-3)^2 = 20$$

$$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$$

$$5x^2 - 20x + 20 = 20$$

$$5x^2 - 20x = 0$$

$$5x(x-4) = 0$$

$x=0$	$x=4$
-------	-------

$$(0-2)^2 + (y-3)^2 = 20$$

$$4 + y^2 - 6y + 9 = 20$$

$$y^2 - 6y + 13 = 20$$

$$y^2 - 6y - 7 = 0$$

$$y^2 - y + 7y - 7$$

$$y(y-1) + 7(y-1)$$

$$(y-1)(y+7)$$

$y=1$

$y=-7$

Score 3: The student received credit for both x -values.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$\begin{array}{r} x-2 \\ x \begin{array}{|c|c|} \hline x^2 & -2x \\ \hline 2x & -4 \\ \hline \end{array} \\ -2 \end{array}$$

$$\begin{array}{r} x-3 \\ x \begin{array}{|c|c|} \hline x^2 & -3x \\ \hline -3x & 9 \\ \hline \end{array} \end{array}$$

$$x^2 - 4x + 4 + y^2 - 6x + 9 = 20$$

$$5x - 14 = 20$$

$$(-2x + 7 - 3)^2 = 20$$

$$(-2x + 4)^2 = 20$$

$$\begin{array}{r} -2x + 4 \\ -2x \begin{array}{|c|c|} \hline 4x^2 & -8x \\ \hline -8x & 16 \\ \hline \end{array} \\ +4 \end{array}$$

$$x^2 - 4x + 4 + 4x^2 - 16x + 16$$

$$5x^2 - 20x + 20 = 20$$

$$\begin{array}{r} 5x^2 - 20x + 20 \\ \hline -20 \quad -20 \\ \hline \end{array}$$

$$5x^2 - 20x = 0$$

$$5(x - 4) = 0$$

$$\begin{array}{r} x-4=0 \\ +4=4 \\ \hline x=4 \end{array}$$

$$x=4$$

Score 2: The student wrote a correct quadratic in standard form.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(y-3)^2 = 20 - (x-2)^2$$

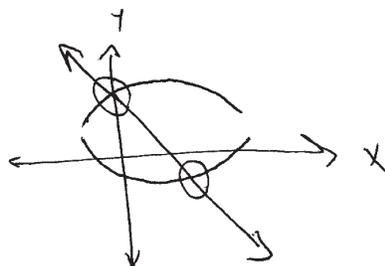
$$y-3 = \pm \sqrt{20 - (x-2)^2}$$

$$y = \pm \sqrt{20 - (x-2)^2} + 3$$

$$Y_1 = \sqrt{20 - (x-2)^2} + 3$$

$$Y_2 = -\sqrt{20 - (x-2)^2} + 3$$

$$Y_3 = -2x + 7$$



x	y ₁	y ₂	y ₃
0	7	-1	7
1			
2			
3			
4	7	-1	-1

(0, 7)
and
(4, -1)

Score 2: The student solved correctly but used a method other than algebraic.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)(x-2) + (-2x+7-3)(-2x+7-3) = 20$$

$$(x^2 - 4x + 4) + (4x^2 - 14x + 6x - 14x + 49 - 2)$$

$$+ 6x - 21 + 9$$
~~$$(x^2 - 4x + 4) + (4x^2 - 22x - 28 + 6x - 21 + 4)$$~~

$$(x^2 - 4x + 4) + (4x^2 - 16x - 40)$$

$$5x^2 - 20x - 36 = 20$$

$$5x^2 - 20x - 56 = 6$$

A B C

Score 1: The student made one computational error when attempting to put the equation in standard form, but showed no further correct work.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)(x-2) + (y-3)(y-3) = 20$$

$$(x-2)^2 + (-2x+7-3)^2 = 20$$

$$x^2 - 4 + -4x^2 + 14 - 6 = 20$$

$$\begin{array}{r} -4x^2 + 4 = 20 \\ -4 \quad -4 \end{array}$$

$$\begin{array}{r} -4x^2 = 16 \\ -4 \quad -4 \end{array}$$

$$\sqrt{x^2} = \sqrt{-4}$$

$$x = 2i$$

Score 0: The student did not show enough correct course-level work to receive any credit.

Question 35

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x - 2)^2 + (-2x + 7 - 3)^2 = 20$$

$$x - 4 + 2x + 7 - 6 = 20$$

$$\begin{array}{r} 3x - 3 = 20 \\ +3 \quad +3 \\ \hline \end{array}$$

$$\begin{array}{r} 3x = 23 \\ \hline 3 \quad 3 \end{array}$$

$$\boxed{x = 8}$$

$$(8, -9)$$

$$y = -2(8) + 7$$

$$y = -16 + 7$$

$$\boxed{y = -9}$$

Score 0: The student did not show enough course-level work to receive any credit.

Question 36

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now.

$$P(x) = 500 (.97)^x$$
$$F(x) = 200e^{.02x}$$

State the solution to the equation $P(x) = F(x)$, rounded to the *nearest year*. Interpret the meaning of this value within the given context.

$$x = 18$$

In 18 years the number of flamingos and palm trees will be equal.

Score 4: The student gave a complete and correct response.

Question 36

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now.

$$P(x) = 500 (1 - .03)^x$$

$$F(x) = 200 e^{.02(x)}$$

State the solution to the equation $P(x) = F(x)$, rounded to the *nearest year*. Interpret the meaning of this value within the given context.

When they have the exact same amount of flamingos and palm trees, it will be in 18 years.

Score 4: The student gave a complete and correct response.

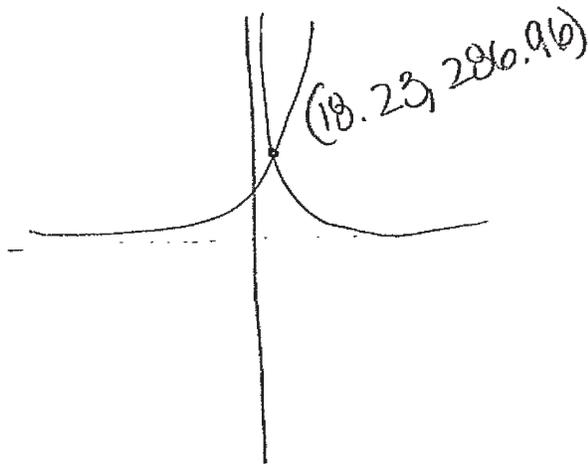
Question 36

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now.

$$P(x) = 500(1 - 0.03)^x$$
$$F(x) = 200(1 + 0.02)^x$$

State the solution to the equation $P(x) = F(x)$, rounded to the *nearest year*. Interpret the meaning of this value within the given context.



In 18 years the palm tree and flamingo population will be equal.

Score 3: The student created an incorrect equation for $F(x)$.

Question 36

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

$$y = a(b)^x$$

$$A = Pe^{rt}$$

Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now.

$$P_x = 500(0.97)^x$$

$$F_x = 200e^{0.02x}$$

State the solution to the equation $P(x) = F(x)$, rounded to the *nearest year*. Interpret the meaning of this value within the given context.

x	Y_1	Y_2
15	316.63	269.97
16	307.13	275.43
17	297.91	280.99
18	288.98	286.98
19	280.31	292.46

18 years

Score 3: The student did not interpret the meaning of 18 years.

Question 36

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now.

$$P(x) = 500e^{-.03x}$$

$$F(x) = 200e^{.02x}$$

State the solution to the equation $P(x) = F(x)$, rounded to the *nearest year*. Interpret the meaning of this value within the given context.

The solution to the equation is 18.3, which means that at 18.3 years, the populations will have the same amount, and then the tree population will continue decreasing and the flamingo population will continue increasing.

Score 2: The student wrote an incorrect equation for $P(x)$ and rounded the solution to $P(x) = F(x)$ incorrectly.

Question 36

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now.

$$P(x) = 500(0.97)^x$$

$$f(x) = 200(1.02)^x$$

State the solution to the equation $P(x) = F(x)$, rounded to the nearest year. Interpret the meaning of this value within the given context.

$$P(x) = f(x)$$

$$\frac{500(0.97)^x}{200} = \frac{200(1.02)^x}{200}$$

$$2.5(0.97)^x = (1.02)^x$$

$$x \frac{\log(2.5)}{\log 1.02} = x \frac{\log 1.02}{\log 1.02}$$

$$x = 44.73$$

Score 1: The student received one credit for correctly writing $P(x)$.

Question 36

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now.

$$A = P(1+r)^t \quad \text{palm trees} \quad \frac{x}{365}$$
$$A = 500(1 - 0.03)^{\frac{x}{365}}$$

flamingos

$$A = 500e^{(0.02)\frac{x}{365}}$$

Part

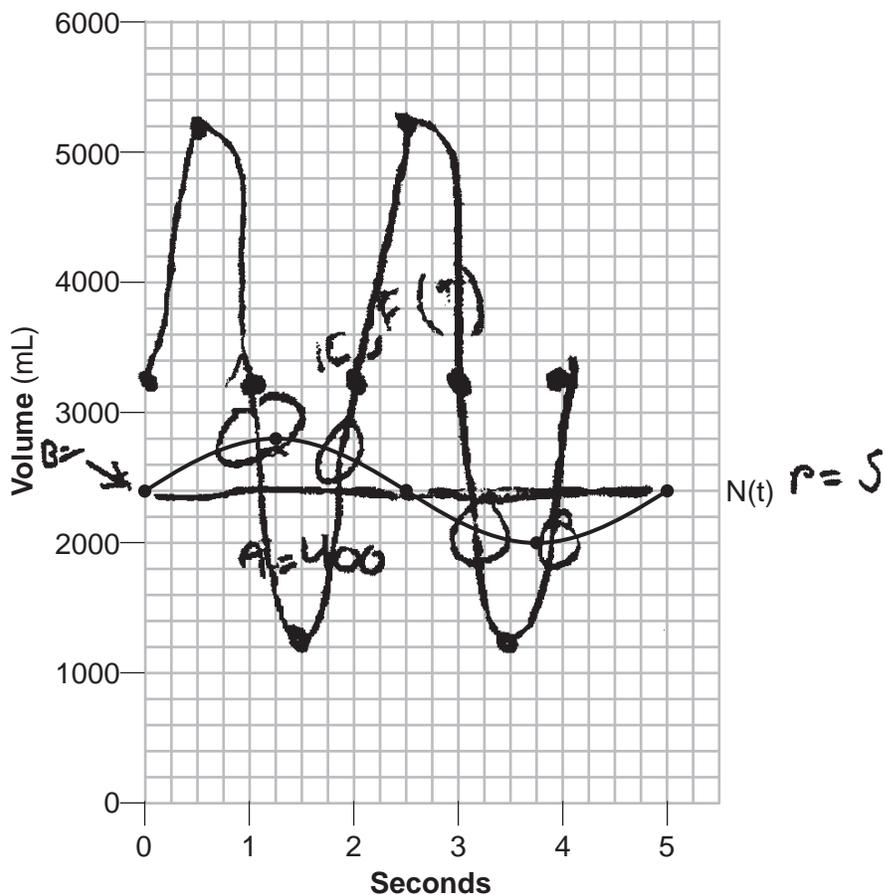
State the solution to the equation $P(x) = F(x)$, rounded to the *nearest year*. Interpret the meaning of this value within the given context.

The x -coordinate of the solution to equation $P(x) = F(x)$ is at 0. The meaning of the value in this context is that it is the point where they intersect, thus they are the same.

Score 0: The student did not show enough correct work to receive any credit.

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$A = 400$
 $B = \frac{2\pi}{P} = \frac{2\pi}{3}$
 $C = 2400$
 $N(t) = 400 \sin\left(\frac{2\pi}{3}t\right) + C$

Question 37 is continued on the next page.

Score 6: The student gave a complete and correct response.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

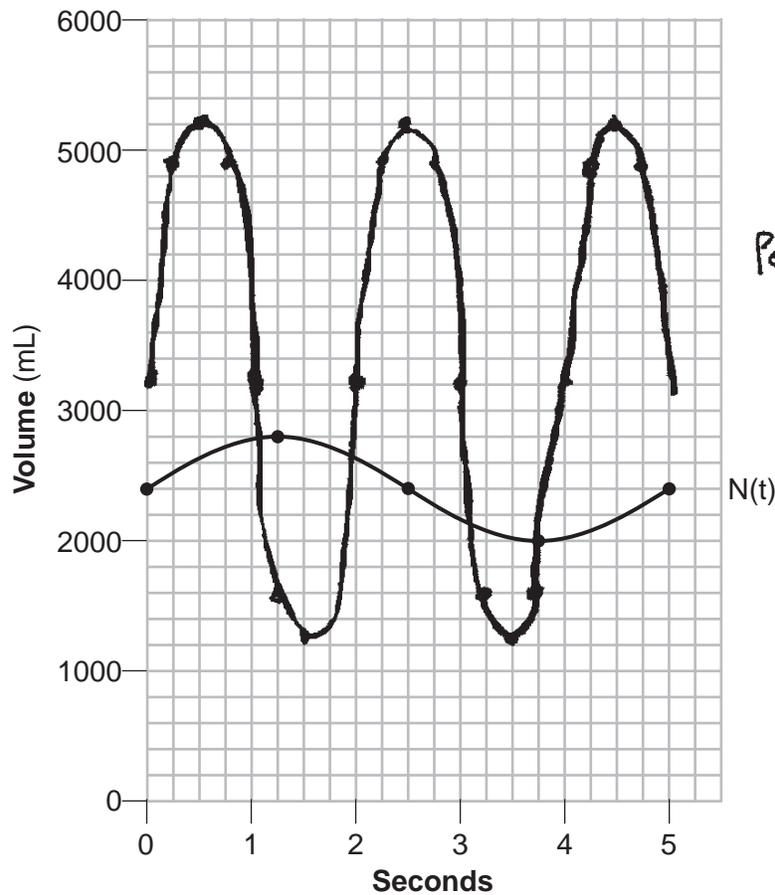
$$P = \frac{B}{8} = 2 \quad B = 3200$$
$$A = 2000 \quad :$$

How many times during the 5-second interval will $N(t) = E(t)$?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + 2400$$

Question 37 is continued on the next page.

Score 6: The student gave a complete and correct response.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

$$x = \frac{2\pi}{\pi}$$

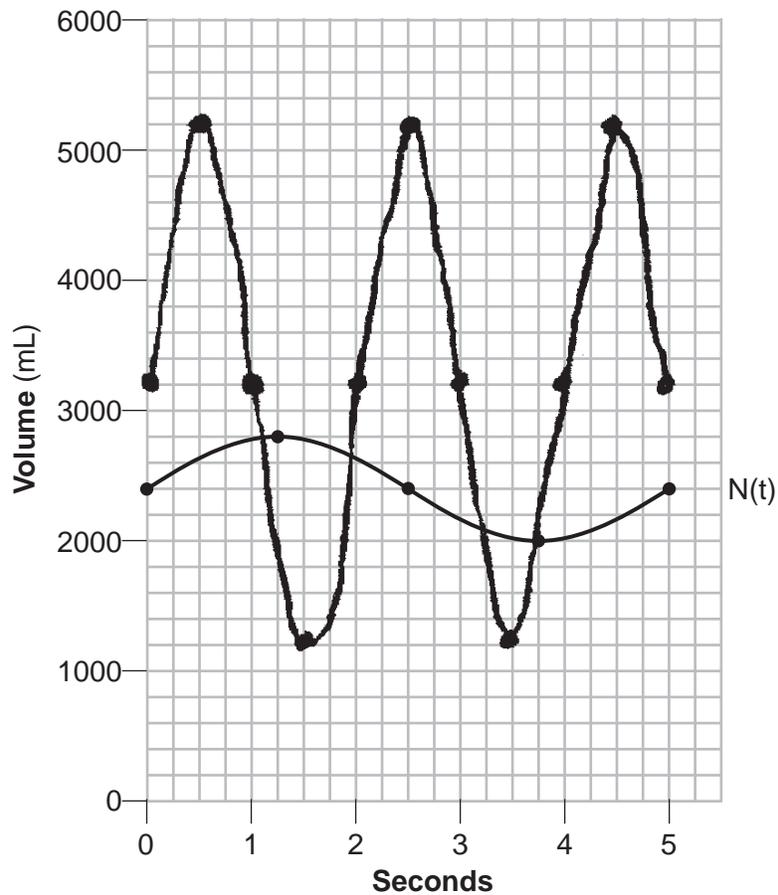
$$x = 2$$

How many times during the 5-second interval will $N(t) = E(t)$?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$N(t) = 400 \sin\left(\frac{\pi}{4} t\right) + 2400$$

Question 37 is continued on the next page.

Score 5: The student incorrectly found the value for B .

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

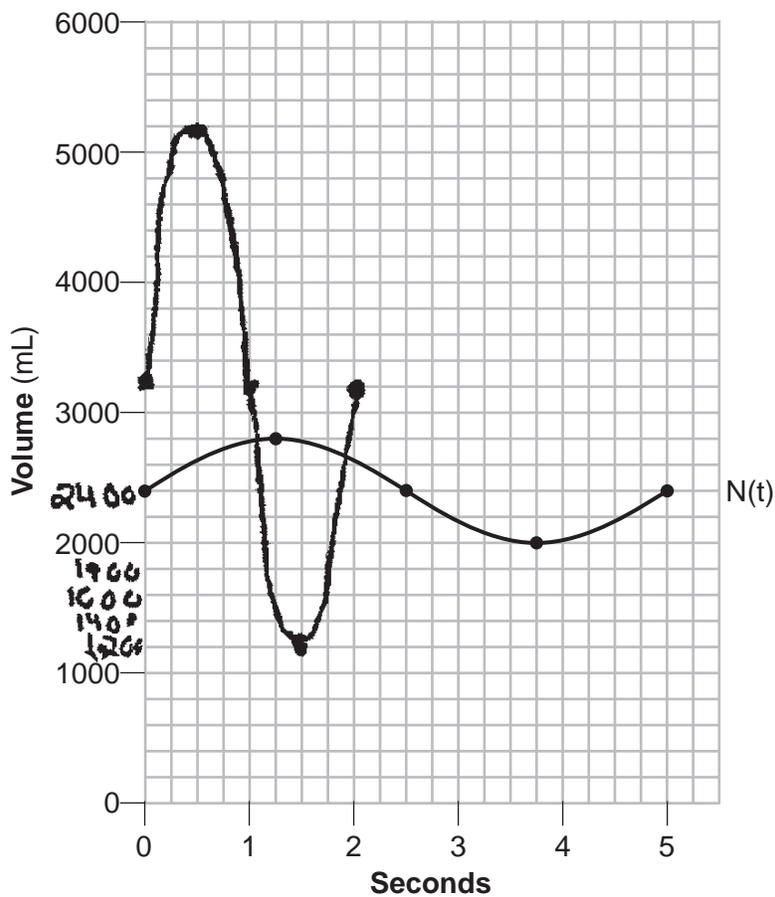
Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$\begin{aligned}
 Bp &= 2\pi \\
 B(5) &= 2\pi \\
 B &= \frac{2\pi}{5}
 \end{aligned}$$

$$\begin{aligned}
 C &= 2400 \\
 A &= 400
 \end{aligned}$$

$$N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + 2400$$

Question 37 is continued on the next page.

Score 5: The student did not find all the intersections within the 5 second interval.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

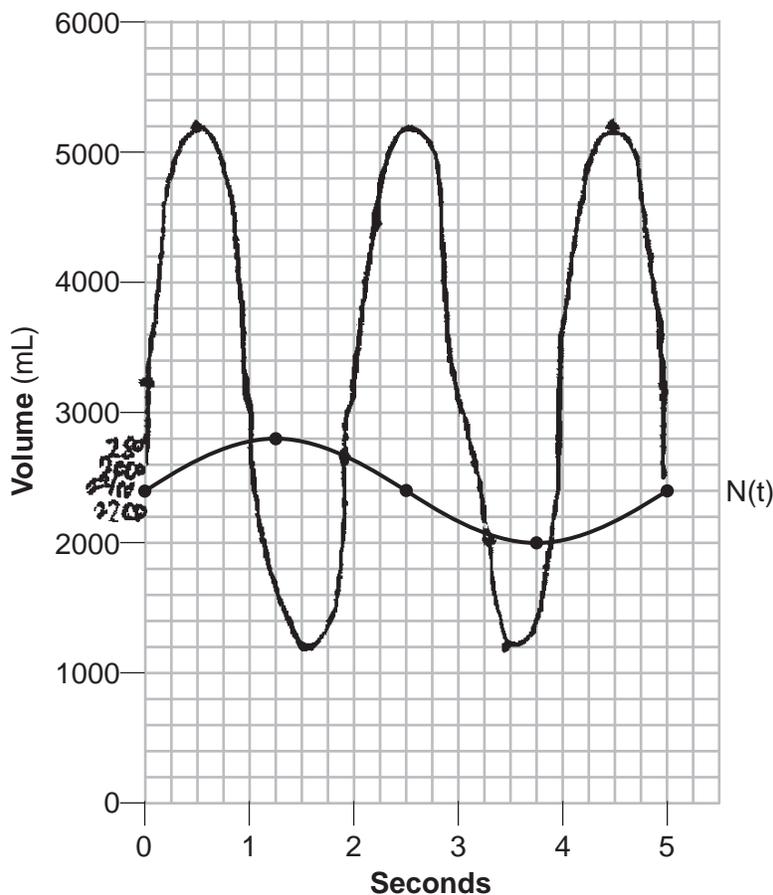
$$\begin{array}{l} Bp = 2\pi \\ \pi p = 2\pi \end{array} \quad p = 2 \text{ sec}$$

How many times during the 5-second interval will $N(t) = E(t)$?

2

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$\frac{2800 - 2000}{2} = 400$$

amp
period = 5

$$N(t) = 400 \sin(Bt) + C$$

$$N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + C$$

$B = \frac{2\pi}{5}$ midline = 2400

$$N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + 2400$$

Question 37 is continued on the next page.

Score 5: The student made a notation error.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

Graph *at least one cycle* of $E(t)$ on the same grid as $N(t)$.

$$\text{period} = \frac{2\pi}{\pi} = 2$$
$$\text{amplitude: } 2000 \quad \text{mid line} = 3200$$

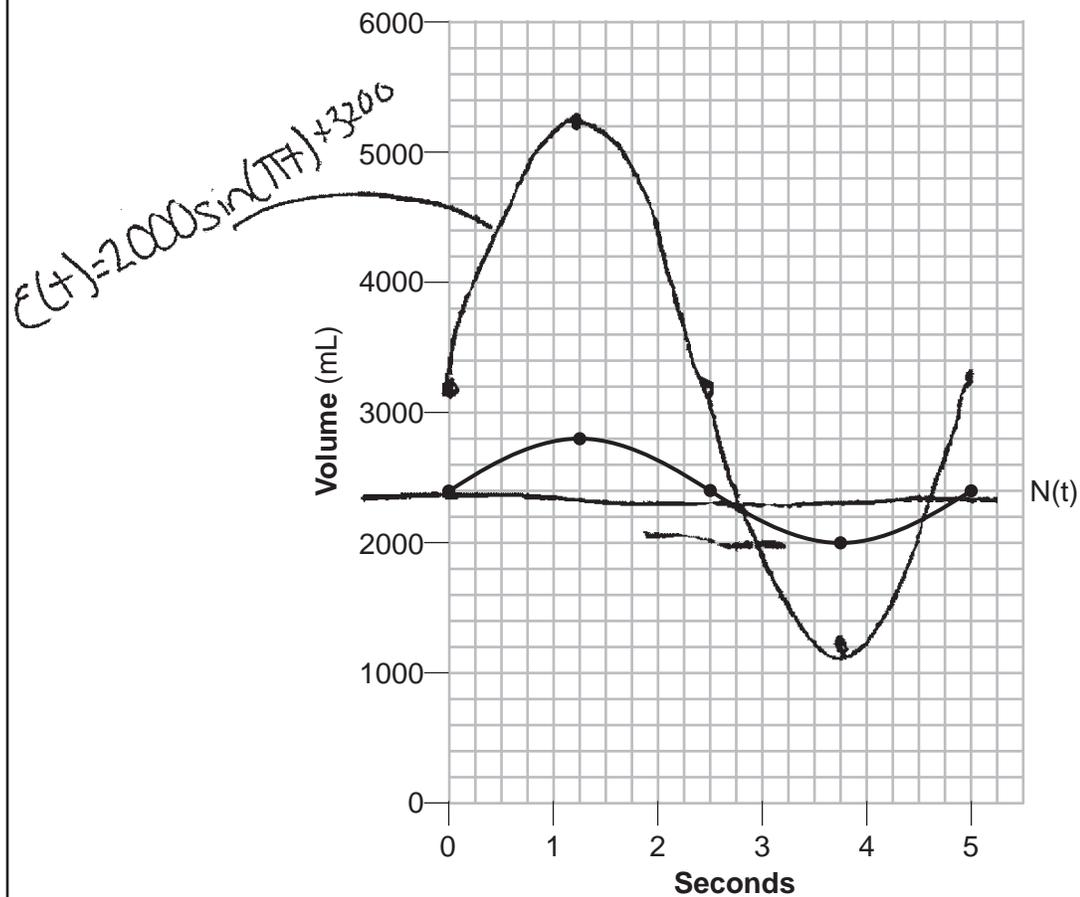
x	y
0	3200
$\frac{1}{2}$	5200
1	3200
$\frac{3}{2}$	1200
2	3200
$\frac{5}{2}$	5200

How many times during the 5-second interval will $N(t) = E(t)$?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$N(t) = 4000 \sin(\pi t) + 2400$$

Question 37 is continued on the next page.

Score 4: The student did not find the correct value for B and did not graph the correct period.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

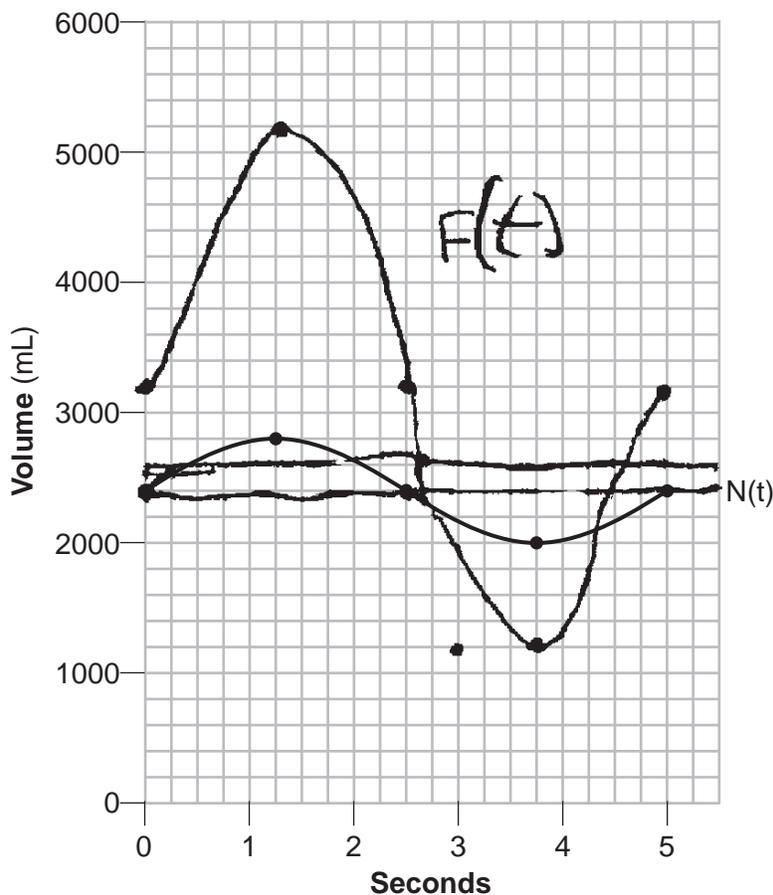
Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

2

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$N(t) = A \sin(Bt) + C$$

$$\frac{5}{1} = \frac{2\pi}{b}$$

$$\frac{5}{5} = \frac{2\pi}{5}$$

$$4100 \sin\left(\frac{2\pi}{5}t\right) + 2100$$

Question 37 is continued on the next page.

Score 4: The student did not write an equation and did not graph the correct period for $E(t)$.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

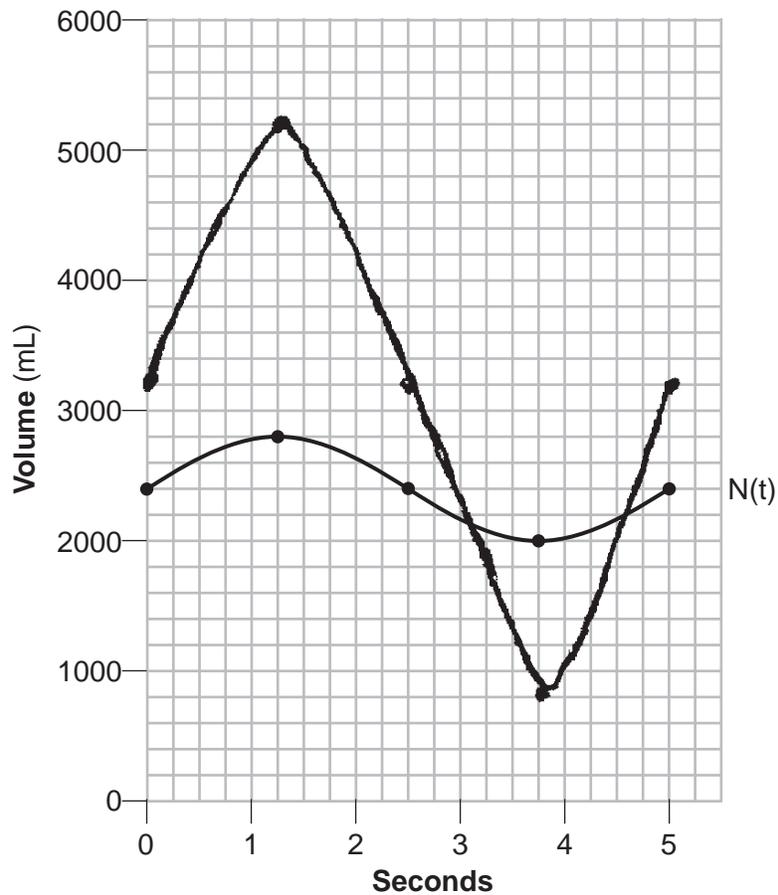
Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

2

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

In five seconds $N(t) = 400 \sin(\pi t) + 2400$

Question 37 is continued on the next page.

Score 3: The student did not find the correct value for B , and made two graphing errors.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

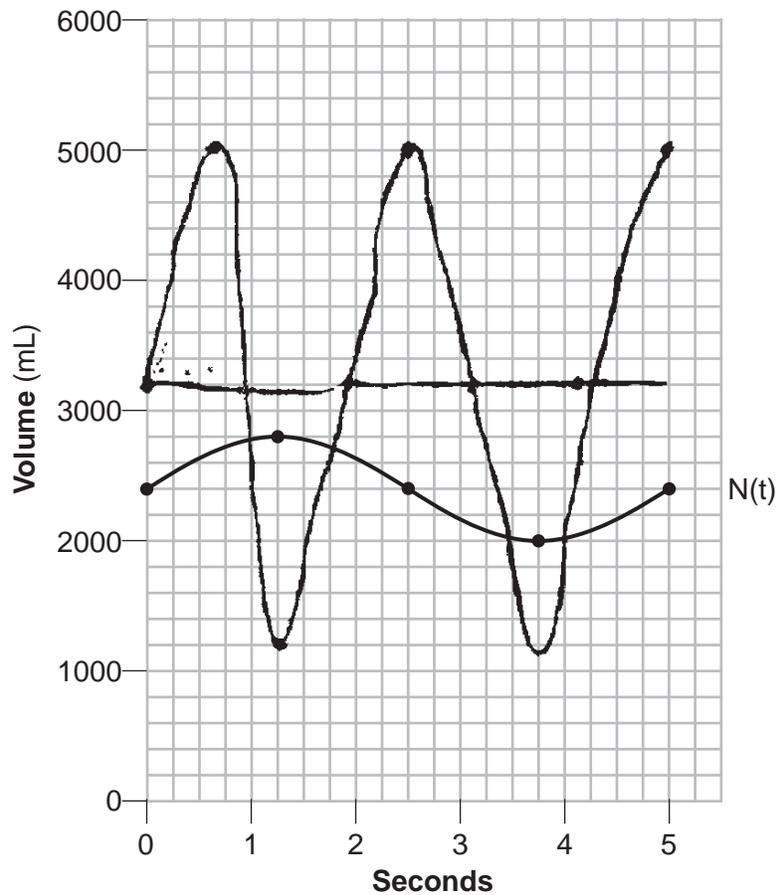
Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

~~two~~ two locations.

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$N(t) = 400 \sin\left(\frac{2\pi}{5}x\right) + 2400$$

Question 37 is continued on the next page.

Score 3: The student did not graph the correct amplitude or period and made a notation error.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

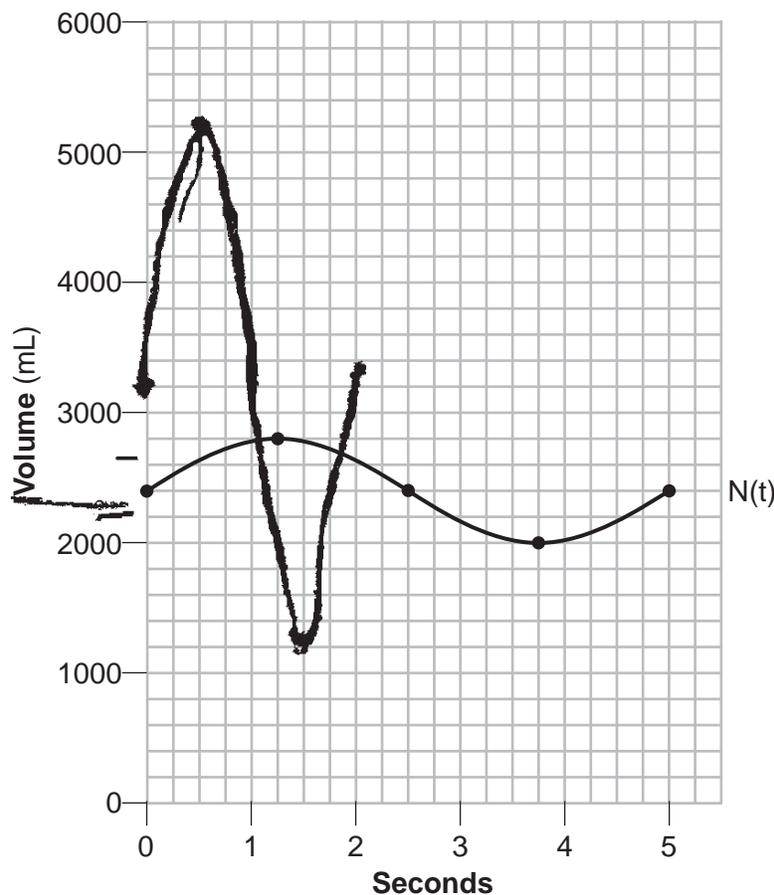
Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$N(t) = 400 \sin\left(\frac{5}{2\pi} t\right) + 2200$$

Question 37 is continued on the next page.

Score 3: The student did not find the correct values of B and C for $N(t)$ and did not graph the correct midline for $E(t)$.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

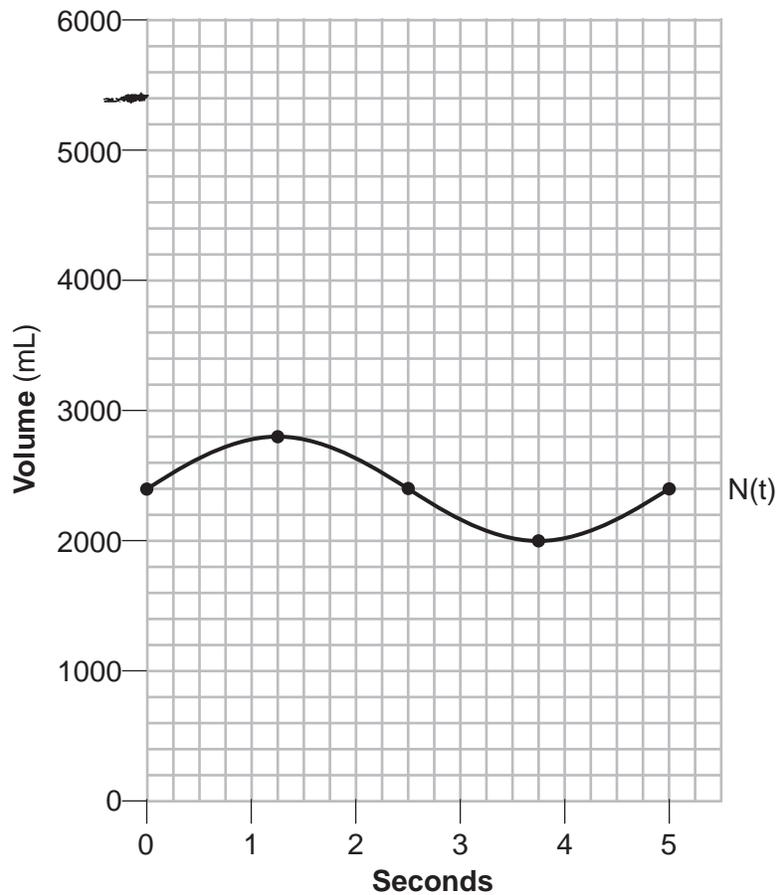
Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

4 locations

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$\begin{aligned} A &= 400 \\ B &= 5 \\ C &= \end{aligned}$$

$$N(t) = 400 \sin\left(\frac{2\pi}{5}t\right) + 2200$$

Question 37 is continued on the next page.

Score 2: The student earned one point for the $N(t)$ equation and one point for the number of intersections.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

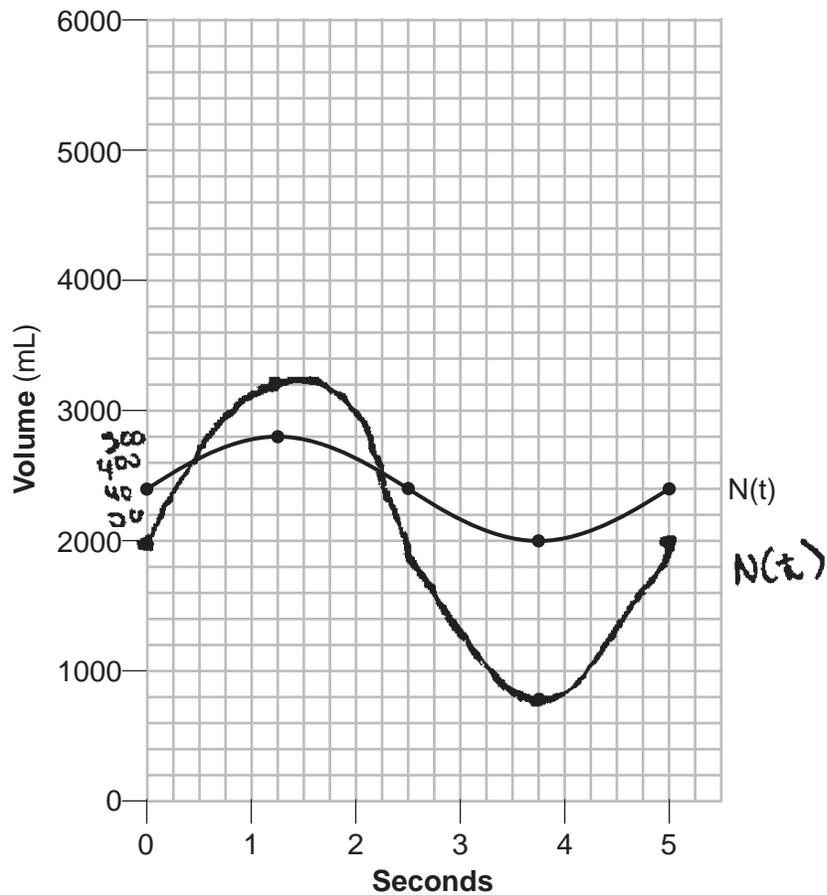
$$p = \frac{2\pi}{T} \quad 2$$

How many times during the 5-second interval will $N(t) = E(t)$?

4 times

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$N(t) = 2000 \sin(\pi t) + 2800$$

Question 37 is continued on the next page.

Score 1: The student found the correct number of intersections based on their graph.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

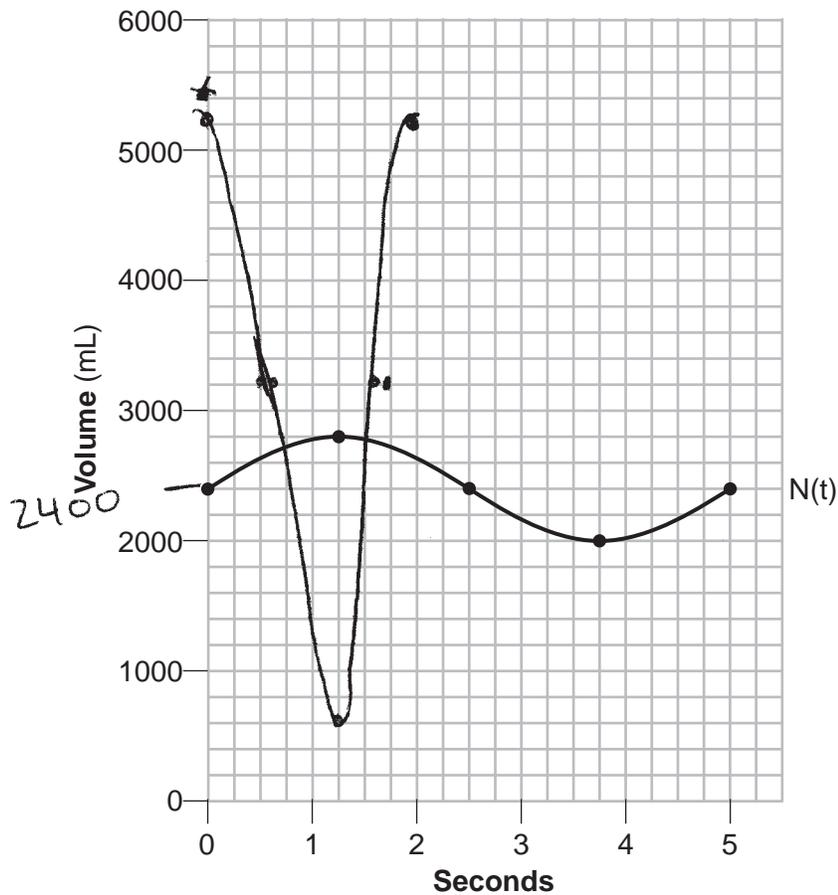
Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

Two (2)

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$N(t) = 2400 \sin\left(\frac{2\pi}{5}t\right) + 400$$

$$p = 5 \quad \frac{2\pi}{p}$$

$$B = ?$$

Question 37 is continued on the next page.

Score 1: The student found the correct number of intersections based on their graph.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

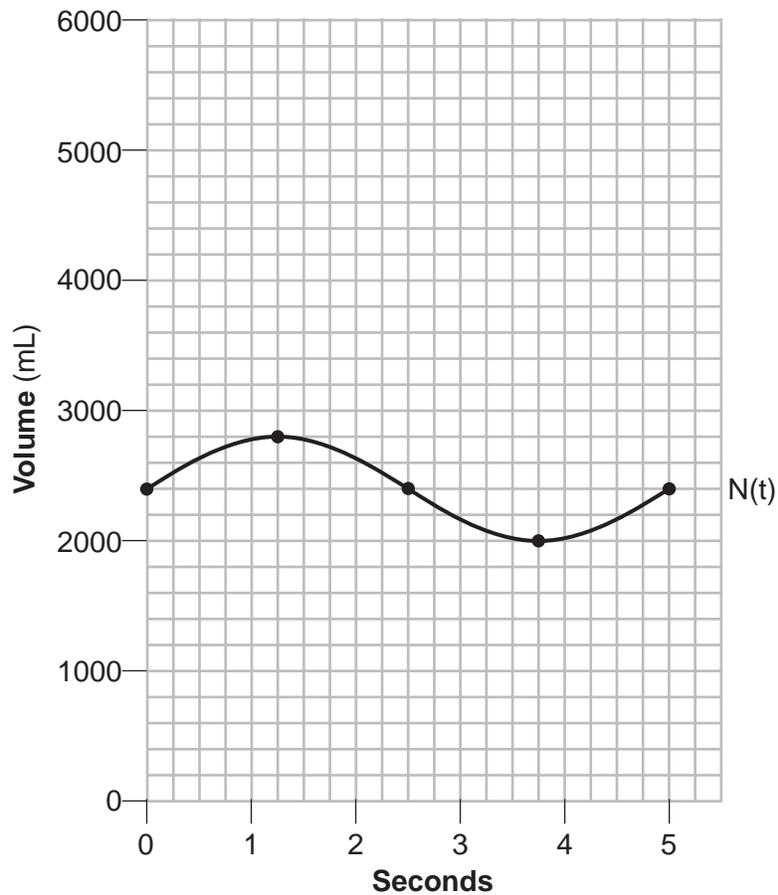
Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

4

Question 37

37 The volume of air in an average lung during breathing can be modeled by the graph below.



Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

~~$C \cos A = e$~~
 $2400 \cos(\frac{\pi}{5}x) = N(x)$

Question 37 is continued on the next page.

Score 0: The student did not show enough correct work to receive any credit.

Question 37

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

How many times during the 5-second interval will $N(t) = E(t)$?

One time $N(t) = E(t)$ will
intersect at the same
x and y

Regents Examination in Algebra II – June 2023

Chart for Converting Total Test Raw Scores to Final Exam Scores (Scale Scores)

(Use for the June 2023 exam only.)

Raw Score	Scale Score	Performance Level	Raw Score	Scale Score	Performance Level	Raw Score	Scale Score	Performance Level
86	100	5	57	81	4	28	65	3
85	99	5	56	81	4	27	64	2
84	98	5	55	80	4	26	63	2
83	97	5	54	80	4	25	62	2
82	96	5	53	79	4	24	61	2
81	95	5	52	79	4	23	59	2
80	94	5	51	79	4	22	57	2
79	93	5	50	78	4	21	55	2
78	92	5	49	78	4	20	54	1
77	91	5	48	78	4	19	53	1
76	91	5	47	77	3	18	51	1
75	90	5	46	77	3	17	49	1
74	89	5	45	77	3	16	47	1
73	89	5	44	76	3	15	45	1
72	88	5	43	76	3	14	43	1
71	87	5	42	75	3	13	41	1
70	87	5	41	75	3	12	38	1
69	86	5	40	74	3	11	36	1
68	86	5	39	74	3	10	33	1
67	86	5	38	73	3	9	30	1
66	85	5	37	73	3	8	28	1
65	84	4	36	72	3	7	25	1
64	84	4	35	71	3	6	22	1
63	83	4	34	71	3	5	18	1
62	83	4	33	70	3	4	15	1
61	82	4	32	69	3	3	11	1
60	82	4	31	68	3	2	8	1
59	82	4	30	67	3	1	4	1
58	81	4	29	66	3	0	0	1

To determine the student’s final examination score (scale score), find the student’s total test raw score in the column labeled “Raw Score” and then locate the scale score that corresponds to that raw score. The scale score is the student’s final examination score. Enter this score in the space labeled “Scale Score” on the student’s answer sheet.

Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Because scale scores corresponding to raw scores in the conversion chart change from one administration to another, it is crucial that for each administration the conversion chart provided for that administration be used to determine the student’s final score. The chart above is usable only for this administration of the Regents Examination in Algebra II.