

ALGEBRA  
**II**

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# ALGEBRA II

Friday, June 21, 2019 — 1:15 to 4:15 p.m., only

Student Name: \_\_\_\_\_

School Name: \_\_\_\_\_

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice...**

**A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for  
computations.

1 A sociologist reviews randomly selected surveillance videos from a public park over a period of several years and records the amount of time people spent on a smartphone. The statistical procedure the sociologist used is called

- (1) a census
- (2) an experiment
- (3) an observational study
- (4) a sample survey

2 Which statement(s) are true for all real numbers?

$$\text{I } (x - y)^2 = x^2 + y^2$$
$$\text{II } (x + y)^3 = x^3 + 3xy + y^3$$

- (1) I, only
- (2) II, only
- (3) I and II
- (4) neither I nor II

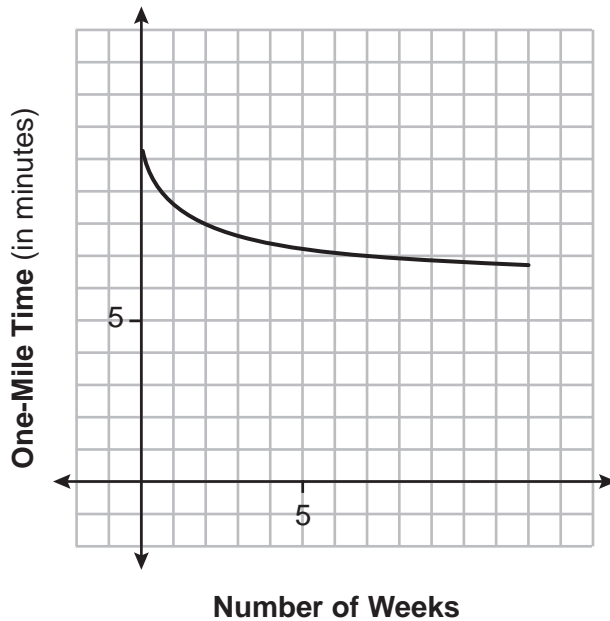
3 What is the solution set of the following system of equations?

$$y = 3x + 6$$
$$y = (x + 4)^2 - 10$$

- (1)  $\{(-5, -9)\}$
- (2)  $\{(5, 21)\}$
- (3)  $\{(0, 6), (-5, -9)\}$
- (4)  $\{(0, 6), (5, 21)\}$

**Use this space for  
computations.**

- 4 Irma initially ran one mile in over ten minutes. She then began a training program to reduce her one-mile time. She recorded her one-mile time once a week for twelve consecutive weeks, as modeled in the graph below.

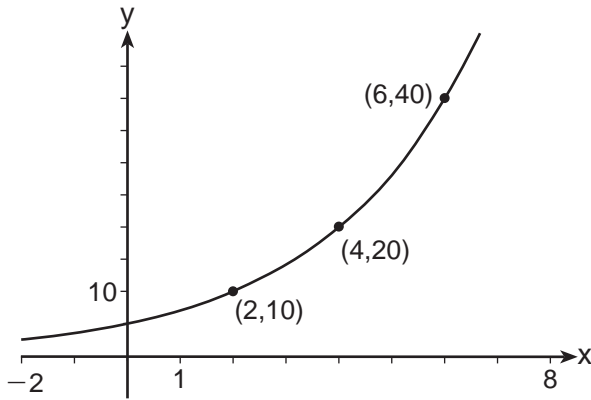


Which statement regarding Irma's one-mile training program is correct?

- (1) Her one-mile speed increased as the number of weeks increased.
  - (2) Her one-mile speed decreased as the number of weeks increased.
  - (3) If the trend continues, she will run under a six-minute mile by week thirteen.
  - (4) She reduced her one-mile time the most between weeks ten and twelve.
- 5 A 7-year lease for office space states that the annual rent is \$85,000 for the first year and will increase by 6% each additional year of the lease. What will the total rent expense be for the entire 7-year lease?
- (1) \$42,809.63
  - (2) \$90,425.53
  - (3) \$595,000.00
  - (4) \$713,476.20

Use this space for computations.

6 The graph of  $y = f(x)$  is shown below.



Which expression defines  $f(x)$ ?

- |              |                                     |
|--------------|-------------------------------------|
| (1) $2x$     | (3) $5\left(2^{\frac{x}{2}}\right)$ |
| (2) $5(2^x)$ | (4) $5(2^{2x})$                     |

7 Given  $P(x) = x^3 - 3x^2 - 2x + 4$ , which statement is true?

- (1)  $(x - 1)$  is a factor because  $P(-1) = 2$ .
- (2)  $(x + 1)$  is a factor because  $P(-1) = 2$ .
- (3)  $(x + 1)$  is a factor because  $P(1) = 0$ .
- (4)  $(x - 1)$  is a factor because  $P(1) = 0$ .

8 For  $x \geq 0$ , which equation is *false*?

- |  |  |
|--|--|
| (1) $\left(\frac{3}{x^2}\right)^2 = \sqrt[4]{x^3}$ | (3) $\left(x^{\frac{3}{2}}\right)^{\frac{1}{2}} = \sqrt[4]{x^3}$ |
| (2) $(x^3)^{\frac{1}{4}} = \sqrt[4]{x^3}$          | (4) $\left(x^{\frac{2}{3}}\right)^2 = \sqrt[3]{x^4}$             |

**Use this space for  
computations.**

**9** What is the inverse of the function  $y = 4x + 5$ ?

(1)  $x = \frac{1}{4}y - \frac{5}{4}$

(3)  $y = 4x - 5$

(2)  $y = \frac{1}{4}x - \frac{5}{4}$

(4)  $y = \frac{1}{4x + 5}$

**10** Which situation could be modeled using a geometric sequence?

- (1) A cell phone company charges \$30.00 per month for 2 gigabytes of data and \$12.50 for each additional gigabyte of data.
- (2) The temperature in your car is  $79^\circ$ . You lower the temperature of your air conditioning by  $2^\circ$  every 3 minutes in order to find a comfortable temperature.
- (3) David's parents have set a limit of 50 minutes per week that he may play online games during the school year. However, they will increase his time by 5% per week for the next ten weeks.
- (4) Sarah has \$100.00 in her piggy bank and saves an additional \$15.00 each week.

**11** The completely factored form of  $n^4 - 9n^2 + 4n^3 - 36n - 12n^2 + 108$  is

(1)  $(n^2 - 9)(n + 6)(n - 2)$

(2)  $(n + 3)(n - 3)(n + 6)(n - 2)$

(3)  $(n - 3)(n - 3)(n + 6)(n - 2)$

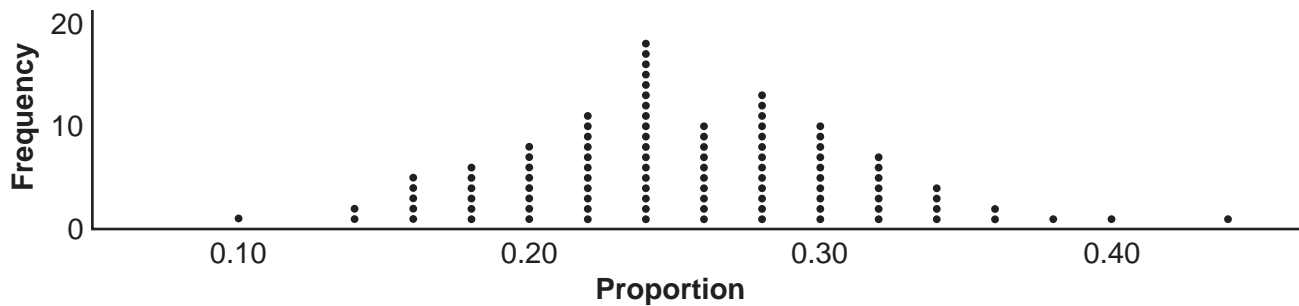
(4)  $(n + 3)(n - 3)(n - 6)(n + 2)$

Use this space for computations.

12 What is the solution when the equation  $wx^2 + w = 0$  is solved for  $x$ , where  $w$  is a positive integer?

- (1)  $-1$  (3)  $6$   
(2)  $0$  (4)  $\pm i$

13 A group of students was trying to determine the proportion of candies in a bag that are blue. The company claims that 24% of candies in bags are blue. A simulation was run 100 times with a sample size of 50, based on the premise that 24% of the candies are blue. The approximately normal results of the simulation are shown in the dot plot below.



The simulation results in a mean of 0.254 and a standard deviation of 0.060. Based on this simulation, what is a plausible interval containing the middle 95% of the data?

- (1) (0.194, 0.314) (3)  $(-0.448, 0.568)$   
(2) (0.134, 0.374) (4) (0.254, 0.374)

**Use this space for computations.**

**14** Selected values for the functions  $f$  and  $g$  are shown in the tables below.

$x$	$f(x)$
-3.12	-4.88
0	-6
1.23	-4.77
8.52	2.53
9.01	3.01

$x$	$g(x)$
-2.01	-1.01
0	0.58
8.52	2.53
13.11	3.01
16.52	3.29

A solution to the equation  $f(x) = g(x)$  is

- (1) 0 (3) 3.01  
(2) 2.53 (4) 8.52

**15** The expression  $6 - (3x - 2i)^2$  is equivalent to

- (1)  $-9x^2 + 12xi + 10$  (3)  $-9x^2 + 10$   
(2)  $9x^2 - 12xi + 2$  (4)  $-9x^2 + 12xi - 4i + 6$

**16** A number, minus twenty times its reciprocal, equals eight.

The number is

- (1) 10 or -2 (3) -10 or -2  
(2) 10 or 2 (4) -10 or 2

**Use this space for  
computations.**

- 17** A savings account,  $S$ , has an initial value of \$50. The account grows at a 2% interest rate compounded  $n$  times per year,  $t$ , according to the function below.

$$S(t) = 50\left(1 + \frac{.02}{n}\right)^{nt}$$

Which statement about the account is correct?

- (1) As the value of  $n$  increases, the amount of interest per year decreases.
  - (2) As the value of  $n$  increases, the value of the account approaches the function  $S(t) = 50e^{0.02t}$ .
  - (3) As the value of  $n$  decreases to one, the amount of interest per year increases.
  - (4) As the value of  $n$  decreases to one, the value of the account approaches the function  $S(t) = 50(1 - 0.02)^t$ .
- 18** There are 400 students in the senior class at Oak Creek High School. All of these students took the SAT. The distribution of their SAT scores is approximately normal. The number of students who scored within 2 standard deviations of the mean is approximately
- |        |         |
|--------|---------|
| (1) 75 | (3) 300 |
| (2) 95 | (4) 380 |

- 19** The solution set for the equation  $b = \sqrt{2b^2 - 64}$  is

- |              |                 |
|--------------|-----------------|
| (1) $\{-8\}$ | (3) $\{\pm 8\}$ |
| (2) $\{8\}$  | (4) $\{ \}$     |



Use this space for computations.

20 Which table best represents an exponential relationship?

x	y
1	8
2	4
3	2
4	1
5	$\frac{1}{2}$

(1)

x	y
8	0
4	1
0	2
-4	3
-8	4

(2)

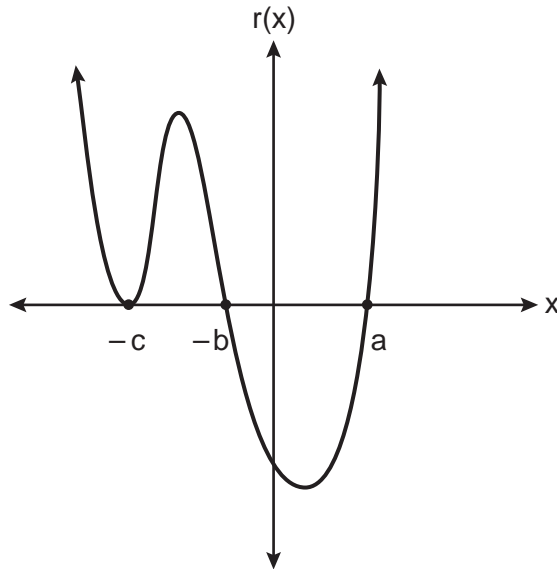
x	y
0	0
1	1
2	4
3	9
4	16

(3)

x	y
1	1
2	8
3	27
4	64
5	125

(4)

21 A sketch of  $r(x)$  is shown below.



An equation for  $r(x)$  could be

(1)  $r(x) = (x - a)(x + b)(x + c)$

(2)  $r(x) = (x + a)(x - b)(x - c)^2$

(3)  $r(x) = (x + a)(x - b)(x - c)$

(4)  $r(x) = (x - a)(x + b)(x + c)^2$

**Use this space for  
computations.**

- 22** The temperature, in degrees Fahrenheit, in Times Square during a day in August can be predicted by the function  $T(x) = 8\sin(0.3x - 3) + 74$ , where  $x$  is the number of hours after midnight. According to this model, the predicted temperature, to the *nearest degree* Fahrenheit, at 7 P.M. is

- (1) 68    (3) 77  
(2) 74    (4) 81

- 23** Consider the system of equations below:

$$\begin{aligned}x + y - z &= 6 \\2x - 3y + 2z &= -19 \\-x + 4y - z &= 17\end{aligned}$$

Which number is *not* the value of any variable in the solution of the system?

- (1)  $-1$     (3) 3  
(2) 2    (4)  $-4$

- 24** Camryn puts \$400 into a savings account that earns 6% annually. The amount in her account can be modeled by  $C(t) = 400(1.06)^t$  where  $t$  is the time in years. Which expression best approximates the amount of money in her account using a weekly growth rate?

- (1)  $400(1.001153846)^t$                           (3)  $400(1.001153846)^{52t}$   
(2)  $400(1.001121184)^t$                           (4)  $400(1.001121184)^{52t}$
-

## Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

Month	Hours of Daylight
Jan.	9.4
Feb.	10.6
March	11.9
April	13.9
May	14.7
June	15.4
July	15.1
Aug.	13.9
Sept.	12.5
Oct.	11.1
Nov.	9.7
Dec.	9.0

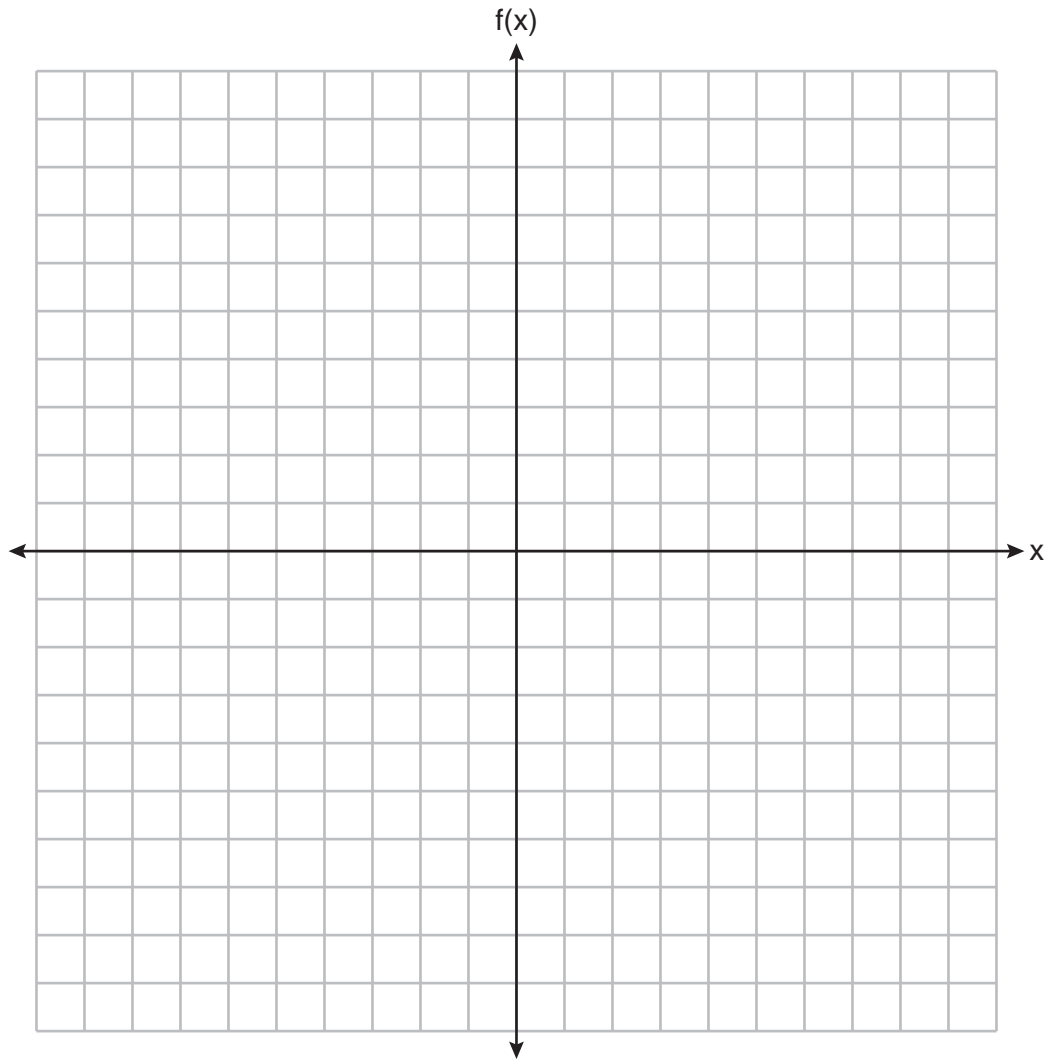
Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

Interpret what this means in the context of the problem.

**26** Algebraically solve for  $x$ :

$$\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$$

27 Graph  $f(x) = \log_2(x + 6)$  on the set of axes below.



**28** Given  $\tan \theta = \frac{7}{24}$ , and  $\theta$  terminates in Quadrant III, determine the value of  $\cos \theta$ .

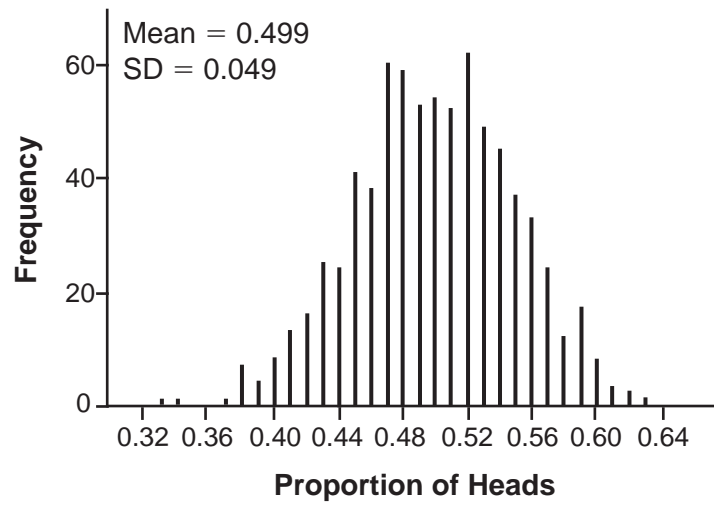
**29** Kenzie believes that for  $x \geq 0$ , the expression  $(\sqrt[7]{x^2})(\sqrt[5]{x^3})$  is equivalent to  $\sqrt[35]{x^6}$ . Is she correct? Justify your response algebraically.

**30** When the function  $p(x)$  is divided by  $x - 1$  the quotient is  $x^2 + 7 + \frac{5}{x - 1}$ . State  $p(x)$  in standard form.

**31** Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .



**32** Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Do the results of the simulation provide strong evidence that Robin's coin is unfair? Explain your answer.

### Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Factor completely over the set of integers:  $16x^4 - 81$

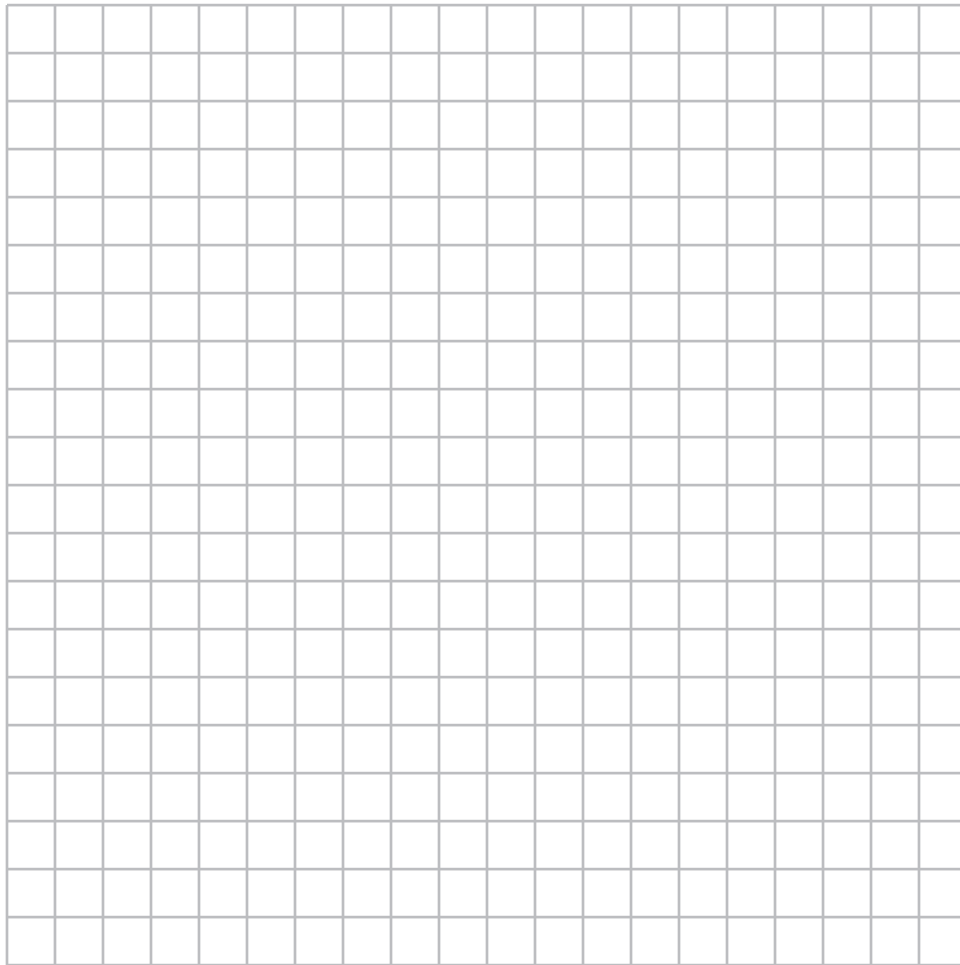
Sara graphed the polynomial  $y = 16x^4 - 81$  and stated “All the roots of  $y = 16x^4 - 81$  are real.” Is Sara correct? Explain your reasoning.

**34** The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount,  $s(t)$ , of 200 grams of this substance that remains after  $t$  years.

Determine algebraically, to the *nearest year*, how long it will take for  $\frac{1}{10}$  of this substance to remain.

**35** Determine an equation for the parabola with focus  $(4, -1)$  and directrix  $y = -5$ .  
(Use of the grid below is optional.)



**36** Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

	<b>Juan Wins</b>	<b>Filipe Wins</b>
<b>Short Practice Time</b>	8	10
<b>Long Practice Time</b>	15	12

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

#### Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

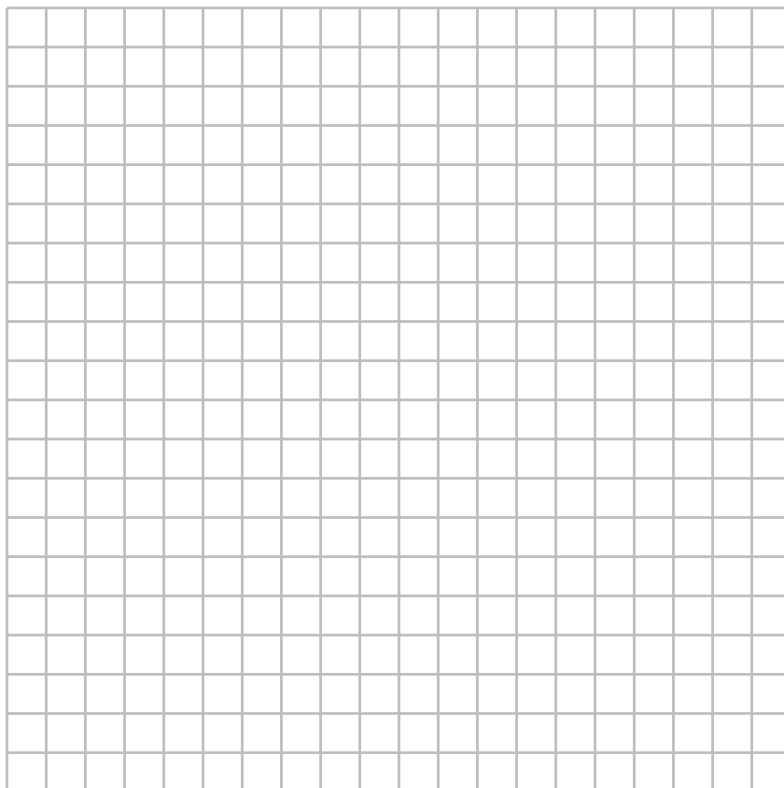
Determine the period of  $f(t)$ .

Interpret what the period represents in this context.

Question 37 is continued on the next page.

**Question 37 continued**

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



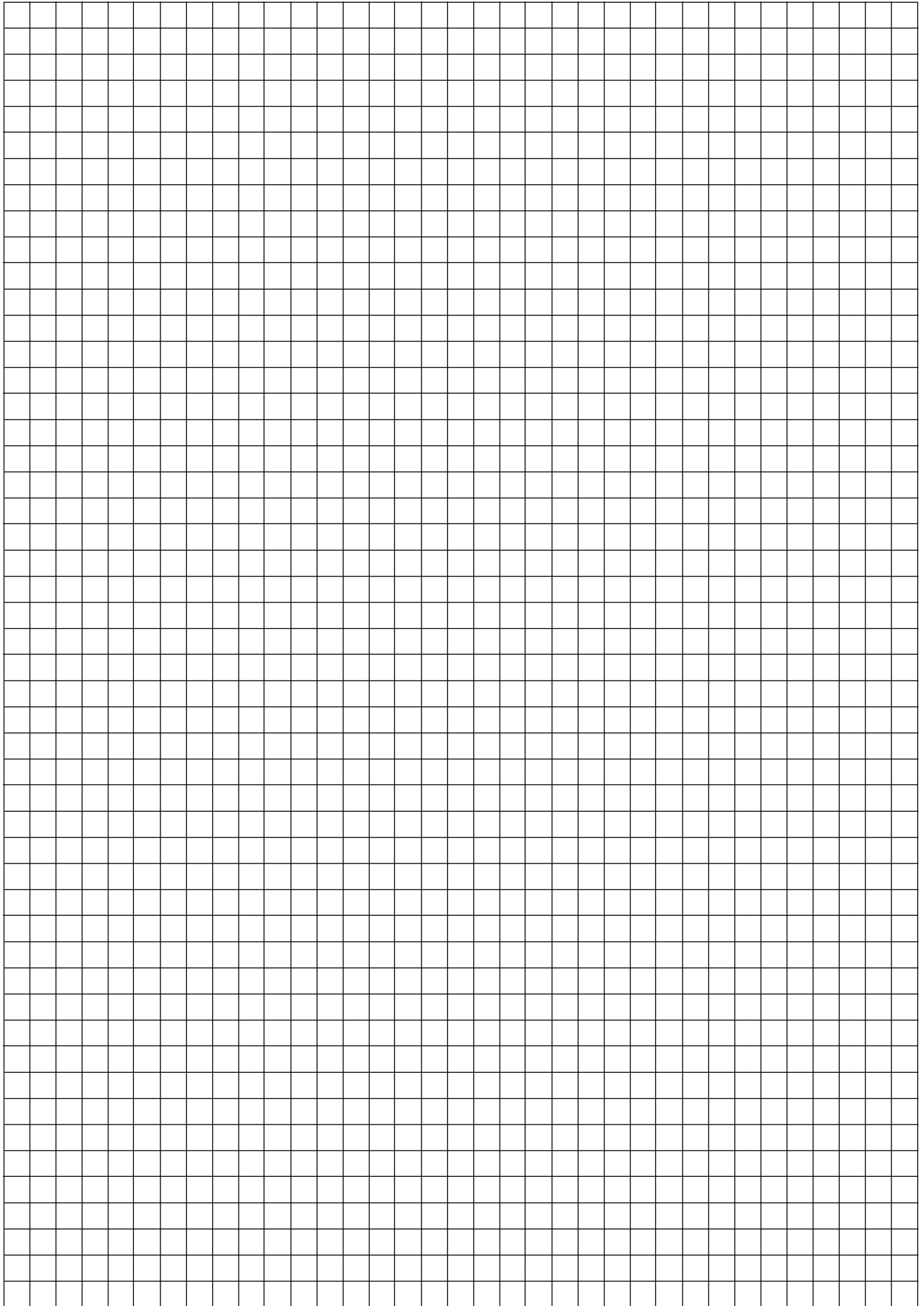
Does the height of the nail ever reach 30 inches above the ground? Justify your answer.





**Scrap Graph Paper — This sheet will *not* be scored.**

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**Scrap Graph Paper — This sheet will *not* be scored.**



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## High School Math Reference Sheet

1 inch = 2.54 centimeters	1 kilometer = 0.62 mile	1 cup = 8 fluid ounces
1 meter = 39.37 inches	1 pound = 16 ounces	1 pint = 2 cups
1 mile = 5280 feet	1 pound = 0.454 kilogram	1 quart = 2 pints
1 mile = 1760 yards	1 kilogram = 2.2 pounds	1 gallon = 4 quarts
1 mile = 1.609 kilometers	1 ton = 2000 pounds	1 gallon = 3.785 liters
		1 liter = 0.264 gallon
		1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian = $\frac{180}{\pi}$ degrees
Degrees	1 degree = $\frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

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**Regents Examination in Algebra II – June 2019**

**Scoring Key: Part I (Multiple-Choice Questions)**

Examination	Date	Question Number	Scoring Key	Question Type	Credit	Weight
Algebra II	June '19	1	3	MC	2	1
Algebra II	June '19	2	4	MC	2	1
Algebra II	June '19	3	3	MC	2	1
Algebra II	June '19	4	1	MC	2	1
Algebra II	June '19	5	4	MC	2	1
Algebra II	June '19	6	3	MC	2	1
Algebra II	June '19	7	4	MC	2	1
Algebra II	June '19	8	1	MC	2	1
Algebra II	June '19	9	2	MC	2	1
Algebra II	June '19	10	3	MC	2	1
Algebra II	June '19	11	2	MC	2	1
Algebra II	June '19	12	4	MC	2	1
Algebra II	June '19	13	2	MC	2	1
Algebra II	June '19	14	4	MC	2	1
Algebra II	June '19	15	1	MC	2	1
Algebra II	June '19	16	1	MC	2	1
Algebra II	June '19	17	2	MC	2	1
Algebra II	June '19	18	4	MC	2	1
Algebra II	June '19	19	2	MC	2	1
Algebra II	June '19	20	1	MC	2	1
Algebra II	June '19	21	4	MC	2	1
Algebra II	June '19	22	3	MC	2	1
Algebra II	June '19	23	2	MC	2	1
Algebra II	June '19	24	4	MC	2	1

**Regents Examination in Algebra II – June 2019**

**Scoring Key: Parts II, III, and IV (Constructed-Response Questions)**

Examination	Date	Question Number	Scoring Key	Question Type	Credit	Weight
Algebra II	June '19	25	-	CR	2	1
Algebra II	June '19	26	-	CR	2	1
Algebra II	June '19	27	-	CR	2	1
Algebra II	June '19	28	-	CR	2	1
Algebra II	June '19	29	-	CR	2	1
Algebra II	June '19	30	-	CR	2	1
Algebra II	June '19	31	-	CR	2	1
Algebra II	June '19	32	-	CR	2	1
Algebra II	June '19	33	-	CR	4	1
Algebra II	June '19	34	-	CR	4	1
Algebra II	June '19	35	-	CR	4	1
Algebra II	June '19	36	-	CR	4	1
Algebra II	June '19	37	-	CR	6	1

Key
MC = Multiple-choice question
CR = Constructed-response question

The chart for determining students' final examination scores for the **June 2019 Regents Examination in Algebra II** will be posted on the Department's web site at: <http://www.p12.nysed.gov/assessment/> on the day of the examination. Conversion charts provided for the previous administrations of the Regents Examination in Algebra II must NOT be used to determine students' final scores for this administration.

# FOR TEACHERS ONLY

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

## ALGEBRA II

Friday, June 21, 2019 — 1:15 p.m. to 4:15 p.m., only

### RATING GUIDE

Updated information regarding the rating of this examination may be posted on the New York State Education Department's web site during the rating period. Check this web site at: <http://www.p12.nysed.gov/assessment/> and select the link "Scoring Information" for any recently posted information regarding this examination. This site should be checked before the rating process for this examination begins and several times throughout the Regents Examination period.

The Department is providing supplemental scoring guidance, the "Model Response Set," for the Regents Examination in Algebra II. This guidance is intended to be part of the scorer training. Schools are encouraged to incorporate the Model Response Sets into the scorer training or to use them as additional information during scoring. While not reflective of all scenarios, the model responses selected for the Model Response Set illustrate how less common student responses to constructed-response questions may be scored. The Model Response Set will be available on the Department's web site at <http://www.nysedregents.org/algebratwo/>.

## Mechanics of Rating

The following procedures are to be followed for scoring student answer papers for the Regents Examination in Algebra II. More detailed information about scoring is provided in the publication *Information Booklet for Scoring the Regents Examination in Algebra II*.

Do *not* attempt to correct the student's work by making insertions or changes of any kind. In scoring the constructed-response questions, use check marks to indicate student errors. Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Each student's answer paper is to be scored by a minimum of three mathematics teachers. No one teacher is to score more than approximately one-third of the constructed-response questions on a student's paper. Teachers may not score their own students' answer papers. On the student's separate answer sheet, for each question, record the number of credits earned and the teacher's assigned rater/scorer letter.

**Schools are not permitted to rescore any of the constructed-response questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.**

Raters should record the student's scores for all questions and the total raw score on the student's separate answer sheet. Then the student's total raw score should be converted to a scale score by using the conversion chart that will be posted on the Department's web site at: <http://www.p12.nysed.gov/assessment/> by Friday, June 21, 2019. Because scale scores corresponding to raw scores in the conversion chart may change from one administration to another, it is crucial that, for each administration, the conversion chart provided for that administration be used to determine the student's final score. The student's scale score should be entered in the box provided on the student's separate answer sheet. The scale score is the student's final examination score.

# General Rules for Applying Mathematics Rubrics

## I. General Principles for Rating

The rubrics for the constructed-response questions on the Regents Examination in Algebra II are designed to provide a systematic, consistent method for awarding credit. The rubrics are not to be considered all-inclusive; it is impossible to anticipate all the different methods that students might use to solve a given problem. Each response must be rated carefully using the teacher's professional judgment and knowledge of mathematics; all calculations must be checked. The specific rubrics for each question must be applied consistently to all responses. In cases that are not specifically addressed in the rubrics, raters must follow the general rating guidelines in the publication *Information Booklet for Scoring the Regents Examination in Algebra II*, use their own professional judgment, confer with other mathematics teachers, and/or contact the State Education Department for guidance. During each Regents Examination administration period, rating questions may be referred directly to the Education Department. The contact numbers are sent to all schools before each administration period.

## II. Full-Credit Responses

A full-credit response provides a complete and correct answer to all parts of the question. Sufficient work is shown to enable the rater to determine how the student arrived at the correct answer.

When the rubric for the full-credit response includes one or more examples of an acceptable method for solving the question (usually introduced by the phrase “such as”), it does not mean that there are no additional acceptable methods of arriving at the correct answer. Unless otherwise specified, mathematically correct alternative solutions should be awarded credit. The only exceptions are those questions that specify the type of solution that must be used; e.g., an algebraic solution or a graphic solution. A correct solution using a method other than the one specified is awarded half the credit of a correct solution using the specified method.

## III. Appropriate Work

*Full-Credit Responses:* The directions in the examination booklet for all the constructed-response questions state: “Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.” The student has the responsibility of providing the correct answer **and** showing how that answer was obtained. The student must “construct” the response; the teacher should not have to search through a group of seemingly random calculations scribbled on the student paper to ascertain what method the student may have used.

*Responses With Errors:* Rubrics that state “Appropriate work is shown, but...” are intended to be used with solutions that show an essentially complete response to the question but contain certain types of errors, whether computational, rounding, graphing, or conceptual. If the response is incomplete; i.e., an equation is written but not solved or an equation is solved but not all of the parts of the question are answered, appropriate work has **not** been shown. Other rubrics address incomplete responses.

## IV. Multiple Errors

*Computational Errors, Graphing Errors, and Rounding Errors:* Each of these types of errors results in a 1-credit deduction. Any combination of two of these types of errors results in a 2-credit deduction. No more than 2 credits should be deducted for such mechanical errors in a 4-credit question and no more than 3 credits should be deducted in a 6-credit question. The teacher must carefully review the student's work to determine what errors were made and what type of errors they were.

*Conceptual Errors:* A conceptual error involves a more serious lack of knowledge or procedure. Examples of conceptual errors include using the incorrect formula for the area of a figure, choosing the incorrect trigonometric function, or multiplying the exponents instead of adding them when multiplying terms with exponents.

If a response shows repeated occurrences of the same conceptual error, the student should not be penalized twice. If the same conceptual error is repeated in responses to other questions, credit should be deducted in each response.

For 4- and 6-credit questions, if a response shows one conceptual error and one computational, graphing, or rounding error, the teacher must award credit that takes into account both errors. Refer to the rubric for specific scoring guidelines.



## Part II

For each question, use the specific criteria to award a maximum of 2 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

- (25) [2] 1.5, and a correct interpretation is written.
- [1] Appropriate work is shown, but one computational error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- or*
- [1] 1.5, but the interpretation is incomplete, incorrect, or missing.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- (26) [2]  $\{-2,7\}$ , and correct algebraic work is shown.
- [1] Appropriate work is shown, but one computational error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- or*
- [1] A correct quadratic equation in standard form is written, but no further correct work is shown.
- or*
- [1]  $\{-2,7\}$ , but a method other than algebraic is used.
- or*
- [1]  $\{-2,7\}$ , but no work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- (27) [2] A correct graph is drawn.
- [1] One graphing error is made.
- or*
- [1] One conceptual error is made.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

- (28) [2]  $-\frac{24}{25}$  or equivalent, and correct work is shown.
- [1] Appropriate work is shown, but one computational error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- or*
- [1]  $-\frac{24}{25}$ , but no work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- 
- (29) [2] A correct justification indicating a negative response is given.
- [1] Appropriate work is shown, but one computational error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- or*
- [1] An incomplete justification is given.
- [0] No, but no justification is given.
- or*
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
- 
- (30) [2]  $x^3 - x^2 + 7x - 2$  is stated and correct work is shown.
- [1] Appropriate work is shown, but one computational error is made.
- or*
- [1] Appropriate work is shown, but one conceptual error is made.
- or*
- [1]  $x^3 - x^2 + 7x - 2$ , but no work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
-

(31) **[2]**  $a_1 = 6$   
 $a_n = \frac{3}{2}a_{n-1}$  or equivalent is written.

**[1]** Appropriate work is shown, but one computational error is made.

**or**

**[1]** Appropriate work is shown, but one conceptual error is made.

**or**

**[1]**  $a_1 = 6$  or  $a_n = \frac{3}{2}a_{n-1}$  is written, but no further correct work is shown.

**[0]** A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(32) **[2]** No or not unfair, and a correct explanation is written.

**[1]** Appropriate work is shown, but one computational error is made.

**or**

**[1]** Appropriate work is shown, but one conceptual error is made.

**or**

**[1]** No or not unfair, but the explanation is incomplete.

**[0]** No or not unfair, but the explanation is missing or incorrect.

**or**

**[0]** A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

### Part III

For each question, use the specific criteria to award a maximum of 4 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(33) [4]  $(4x^2 + 9)(2x + 3)(2x - 3)$  and correct work is shown, and a correct explanation indicating a negative response is written.

[3] Appropriate work is shown, but one computational, factoring, or simplification error is made.

*or*

[3] Appropriate work is shown, but the explanation is incomplete.

[2] Appropriate work is shown, but two or more computational, factoring, or simplification errors are made.

*or*

[2] Appropriate work is shown, but one conceptual error is made.

*or*

[2] Appropriate work is shown to find  $(4x^2 + 9)(2x + 3)(2x - 3)$ , but no further correct work is shown.

*or*

[2] A correct explanation indicating a negative response is written, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual error and one computational, factoring, or simplification error are made.

*or*

[1]  $(4x^2 + 9)(4x^2 - 9)$ , but no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

- (34) [4]  $s(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{15}}$  or equivalent, 50 and correct algebraic work is shown.
- [3] Appropriate work is shown, but one computational, notation, or rounding error is made.
- or**
- [3]  $s(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{15}}$  and 50, but a method other than algebraic is used.
- [2] Appropriate work is shown, but two or more computational, notation, or rounding errors are made.
- or**
- [2] Appropriate work is shown, but one conceptual error is made.
- or**
- [2]  $s(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{15}}$  is written, but no further correct work is shown.
- or**
- [2] Appropriate work is shown to find 50, but no further correct work is shown.
- [1] Appropriate work is shown, but one conceptual error and one computational, notation, or rounding error are made.
- or**
- [1] 50, but no work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(35) [4]  $y + 3 = \frac{1}{8}(x - 4)^2$ , or an equivalent equation, and correct work is shown.

[3] Appropriate work is shown, but one computational or graphing error is made.

[2] Appropriate work is shown, but two or more computational or graphing errors are made.

**or**

[2] Appropriate work is shown, but one conceptual error is made.

[1] Appropriate work is shown, but one conceptual error and one computational or graphing error are made.

**or**

[1]  $y + 3 = \frac{1}{8}(x - 4)^2$ , but no work is shown.

**or**

[1] The vertex,  $(4, -3)$ , was correctly determined, but no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

- (36) [4]  $\frac{12}{27}$  or equivalent and correct work is shown, not independent and a correct justification is given.
- [3] Appropriate work is shown, but one computational or simplification error is made.
- [2] Appropriate work is shown, but two or more computational or simplification errors are made.
- or**
- [2] Appropriate work is shown, but one conceptual error is made.
- or**
- [2] Appropriate work is shown to find  $\frac{12}{27}$ , but no further correct work is shown.
- or**
- [2] Not independent and a correct justification is given, but no further correct work is shown.
- [1] Appropriate work is shown, but one conceptual error and one computational or simplification error are made.
- or**
- [1]  $\frac{12}{27}$ , but no work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.
-

## Part IV

For each question, use the specific criteria to award a maximum of 6 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

- (37) [6] 2.5, a correct interpretation is written, a correct graph is drawn, and a correct justification indicating a negative response is given.
- [5] Appropriate work is shown, but one computational, graphing, or rounding error is made.
- or*
- [5] 2.5, a correct interpretation is written, and a correct graph is drawn, but an incomplete justification is written.
- [4] Appropriate work is shown, but two computational, graphing, or rounding errors are made.
- or*
- [4] Appropriate work is shown, but one conceptual error is made.
- [3] Appropriate work is shown, but three or more computational or graphing errors are made.
- or*
- [3] Appropriate work is shown, but one conceptual error and one computational or graphing error are made.
- [2] Appropriate work is shown, but two conceptual errors are made.
- or*
- [2] Appropriate work is shown, but one conceptual error and two or more computational or graphing errors are made.
- or*
- [2] 2.5 and a correct interpretation is written, but no further correct work is shown.
- or*
- [2] A correct graph is drawn, but no further correct work is shown.
- or*
- [2] A correct justification indicating a negative response is given, but no further correct work is shown.



[1] 2.5 or a correct interpretation is written, but no further correct work is shown.

*or*

[1] Appropriate work is shown, but two conceptual errors and one computational or graphing error are made.

[0] No, but no further correct work is shown.

*or*

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

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**Map to the Learning Standards  
Algebra II  
June 2019**

<b>Question</b>	<b>Type</b>	<b>Credits</b>	<b>Cluster</b>
1	Multiple Choice	2	S-IC.B
2	Multiple Choice	2	A-APR.C
3	Multiple Choice	2	A-REI.C
4	Multiple Choice	2	F-IF.B
5	Multiple Choice	2	A-SSE.B
6	Multiple Choice	2	F-IF.C
7	Multiple Choice	2	A-APR.B
8	Multiple Choice	2	N-RN.A
9	Multiple Choice	2	F-BF.B
10	Multiple Choice	2	F-BF.A
11	Multiple Choice	2	A-SSE.A
12	Multiple Choice	2	A-REI.B
13	Multiple Choice	2	S-IC.B
14	Multiple Choice	2	A-REI.D
15	Multiple Choice	2	N-CN.A
16	Multiple Choice	2	A-CED.A
17	Multiple Choice	2	F-LE.B
18	Multiple Choice	2	S-ID.A
19	Multiple Choice	2	A-REI.A
20	Multiple Choice	2	F-LE.A

21	Multiple Choice	2	A-APR.B
22	Multiple Choice	2	S-ID.B
23	Multiple Choice	2	A-REI.C
24	Multiple Choice	2	A-SSE.B
25	Constructed Response	2	F-IF.B
26	Constructed Response	2	A-REI.A
27	Constructed Response	2	F-IF.C
28	Constructed Response	2	F-TF.C
29	Constructed Response	2	N-RN.A
30	Constructed Response	2	A-APR.D
31	Constructed Response	2	F-BF.A
32	Constructed Response	2	S-IC.A
33	Constructed Response	4	A-SSE.A
34	Constructed Response	4	F-BF.A
35	Constructed Response	4	G-GPE.A
36	Constructed Response	4	S-CP.A
37	Constructed Response	6	F-IF.B

**Regents Examination in Algebra II**  
**June 2019**  
**Chart for Converting Total Test Raw Scores to**  
**Final Examination Scores (Scale Scores)**

**The *Chart for Determining the Final Examination Score for the June 2019 Regents Examination in Algebra II* will be posted on the Department’s web site at: <http://www.p12.nysed.gov/assessment/> by Friday, June 21, 2019. Conversion charts provided for previous administrations of the Regents Examination in Algebra II must NOT be used to determine students’ final scores for this administration.**

**Online Submission of Teacher Evaluations of the Test to the Department**

Suggestions and feedback from teachers provide an important contribution to the test development process. The Department provides an online evaluation form for State assessments. It contains spaces for teachers to respond to several specific questions and to make suggestions. Instructions for completing the evaluation form are as follows:

1. Go to <http://www.forms2.nysed.gov/emsc/osa/exameval/reexameval.cfm>.
2. Select the test title.
3. Complete the required demographic fields.
4. Complete each evaluation question and provide comments in the space provided.
5. Click the SUBMIT button at the bottom of the page to submit the completed form.

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION

# ALGEBRA II

Friday, June 21, 2019 — 1:15 to 4:15 p.m., only

## MODEL RESPONSE SET

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**Question 25**

25 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

Month	Hours of Daylight
Jan.	9.4
Feb.	10.6
March	11.9
April	13.9
May	14.7
June	15.4
July	15.1
Aug.	13.9
Sept.	12.5
Oct.	11.1
Nov.	9.7
Dec.	9.0

Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

1.5

Interpret what this means in the context of the problem.

It means that over the course from January to April, the average rate of change of the number of hours of daylight is 1.5.

**Score 2:** The student gave a complete and correct response.

**Question 25**

25 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

	Month	Hours of Daylight
1	Jan.	9.4
2	Feb.	10.6
3	March	11.9
4	April	13.9
	May	14.7
	June	15.4
	July	15.1
	Aug.	13.9
	Sept.	12.5
	Oct.	11.1
	Nov.	9.7
	Dec.	9.0

Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

$$\frac{\Delta y}{\Delta x} = \frac{13.9 - 9.4}{4 - 1} = 1.5$$

Interpret what this means in the context of the problem.

*On average, the number of hours of daylight increased 1.5 hours per month from January - April.*

**Score 2:** The student gave a complete and correct response.

**Question 25**

25 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

Month	Hours of Daylight
Jan.	9.4
Feb.	10.6
March	11.9
April	13.9
May	14.7
June	15.4
July	15.1
Aug.	13.9
Sept.	12.5
Oct.	11.1
Nov.	9.7
Dec.	9.0

Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

$$\frac{13.9 - 9.4}{4 - 1} = \frac{4.5}{3} = \boxed{1.5}$$

Interpret what this means in the context of the problem.

On average, the temperature increased by 1.5 degrees every month.

**Score 1:** The student gave an incorrect interpretation.



**Question 25**

25 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

Month	Hours of Daylight
Jan.	9.4
Feb.	10.6
March	11.9
April	13.9
May	14.7
June	15.4
July	15.1
Aug.	13.9
Sept.	12.5
Oct.	11.1
Nov.	9.7
Dec.	9.0

Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

$$\frac{\Delta y}{\Delta x} = \frac{4.5}{3} = 1.5 \text{ hrs/month}$$

Interpret what this means in the context of the problem.

Every month from January to April, there are 1.5 more hours of daylight

**Score 1:** The student gave an incomplete interpretation.

**Question 25**

25 The table below shows the number of hours of daylight on the first day of each month in Rochester, NY.

Month	Hours of Daylight
Jan.	9.4
Feb.	10.6
March	11.9
April	13.9
May	14.7
June	15.4
July	15.1
Aug.	13.9
Sept.	12.5
Oct.	11.1
Nov.	9.7
Dec.	9.0

Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st?

Jan  $\rightarrow$  Apr.

$$9.4 - 13.9 = \boxed{4.5}$$

Interpret what this means in the context of the problem.

A means from January to April, the number of daylight hours increases by 4.5

**Score 0:** The student found an incorrect average rate of change and wrote an incomplete interpretation.

Question 26

26 Algebraically solve for  $x$ :

$$\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$$

$$\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$$

$$\begin{array}{l} \frac{(7)(x+1)}{(2x)(x+1)} - \frac{4x}{(2x)(x+1)} = \frac{1}{4} \\ - \frac{2 \cdot 2x}{(x+1) \cdot 2x} \\ \hline \frac{7x+7-4x}{(2x)(x+1)} = \frac{1}{4} \end{array}$$

$$\frac{3x+7}{(2x)(x+1)} \neq \frac{1}{4}$$

$$(2x)(x+1) = (4)(3x+7)$$

$$2x^2 + 2x = 12x + 28$$

$$-12x \quad -12x$$

$$2x^2 - 10x = 28$$

$$-28 \quad -28$$

$$2x^2 - 10x - 28 = 0$$

$$2(x^2 - 5x - 14) = 0$$

$$2(x+2)(x-7) = 0 \rightarrow x = -2, x = 7$$

$$\begin{array}{r} -14 \\ 2 \overline{) 14} \\ \underline{14} \\ 0 \end{array}$$

Score 2: The student gave a complete and correct response.

Question 26

26 Algebraically solve for x:

$$\frac{28}{2x} - \frac{2}{x+1} = \frac{4}{4}$$

*(Handwritten annotations: circled terms include  $(x+1) \cdot 4$ ,  $22x(x+1) \cdot 4$ , and  $2x(x+1) \cdot 4$ )*

$$28(x+1) - 10x = 2x(x+1)$$

$$28x + 28 - 10x = 2x^2 + 2x$$

LCD:  $2x(x+1) \cdot 4$

$$12x + 28 = 2x^2 + 2x$$

$$0 = 2x^2 - 12x + 2x - 28$$

$$0 = 2x^2 - 10x - 28$$

$$(2x+4)(x-7) = 0$$

$$2x+4=0 \quad | \quad x-7=0$$

$$\frac{2x}{2} = \frac{-4}{2}$$

$$x = -2$$

$$x = 7$$

**Score 2:** The student gave a complete and correct response.

Question 26

26 Algebraically solve for x:

$$\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$$

$$\frac{7}{-4} = \frac{2}{-1} = \frac{1}{4}$$

$$\frac{7}{2x(x+1)} - \frac{2}{x+1(2x)} = \frac{1}{4}$$

$$\frac{7x+7}{2x^2+2x} - \frac{4x}{2x^2+2x} = \frac{1}{4}$$

$$\frac{3x+7}{2x^2+2x} \times \frac{1}{4}$$

$$\begin{array}{r} -56 \\ -14 + 4 \end{array}$$

$$4(3x+7) = 2x^2+2x$$

$$\begin{array}{r} 12x + 28 = 2x^2 + 2x \\ -2x \qquad -2x \end{array}$$

$$10x + 28 = 2x^2$$

$$2x^2 - 10x - 28 = 0$$

$$2x^2 + 4x - 14x - 28 = 0$$

$$2x(x+2) - 14(x+2) = 0$$

$$\begin{array}{r} 2x - 14 = 0 \quad x + 2 = 0 \\ +14 \quad +14 \quad -2 \quad -2 \end{array}$$

$$x = 7 \quad \frac{2x}{2} = \frac{14}{2} \quad x = -2$$

$x = \{7\}$   
-2 is  
extraneous  
root

**Score 1:** The student incorrectly identified -2 as an extraneous root.

Question 26

26 Algebraically solve for x:

$$(x+1) \frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$$

$$\frac{7x+7-4x}{2x(x+1)} = \frac{1}{4}$$

$$2x(x+1) = 28x + 28 - 16x$$

$$2x^2 + 2x = 12x + 28$$
$$-12x - 28$$

$$2x^2 - 10x - 28 = 0$$

$$2(x^2 - 5x - 14) = 0$$

$$2(x+2)(x-7) = 0$$

$$2x + 2 = 0$$
$$\frac{2x + 2}{2} = \frac{0}{2}$$
$$x + 1 = 0$$
$$x = -1$$

x = 7

$$\frac{2x}{2} = \frac{-2}{2}$$

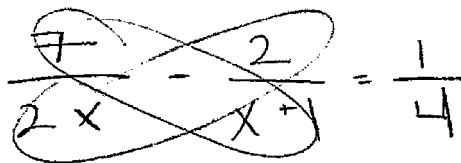
~~x = -1~~  
undefined

**Score 1:** The student made a computational error by not distributing the 2 correctly.

Question 26

26 Algebraically solve for x:

$$\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$$



The student has written the equation  $\frac{7}{2x} - \frac{2}{x+1} = \frac{1}{4}$  but has circled the 7 and 2 in the numerators and the 2x and x+1 in the denominators, with lines crossing through them, indicating a conceptual error in identifying the common denominator.

$$7x + 7 - 4x = \frac{1}{4}$$

$$3x + 7 = \frac{1}{4}$$
$$-7 \quad -7$$

$$? \quad 3x = \frac{6.75}{3}$$

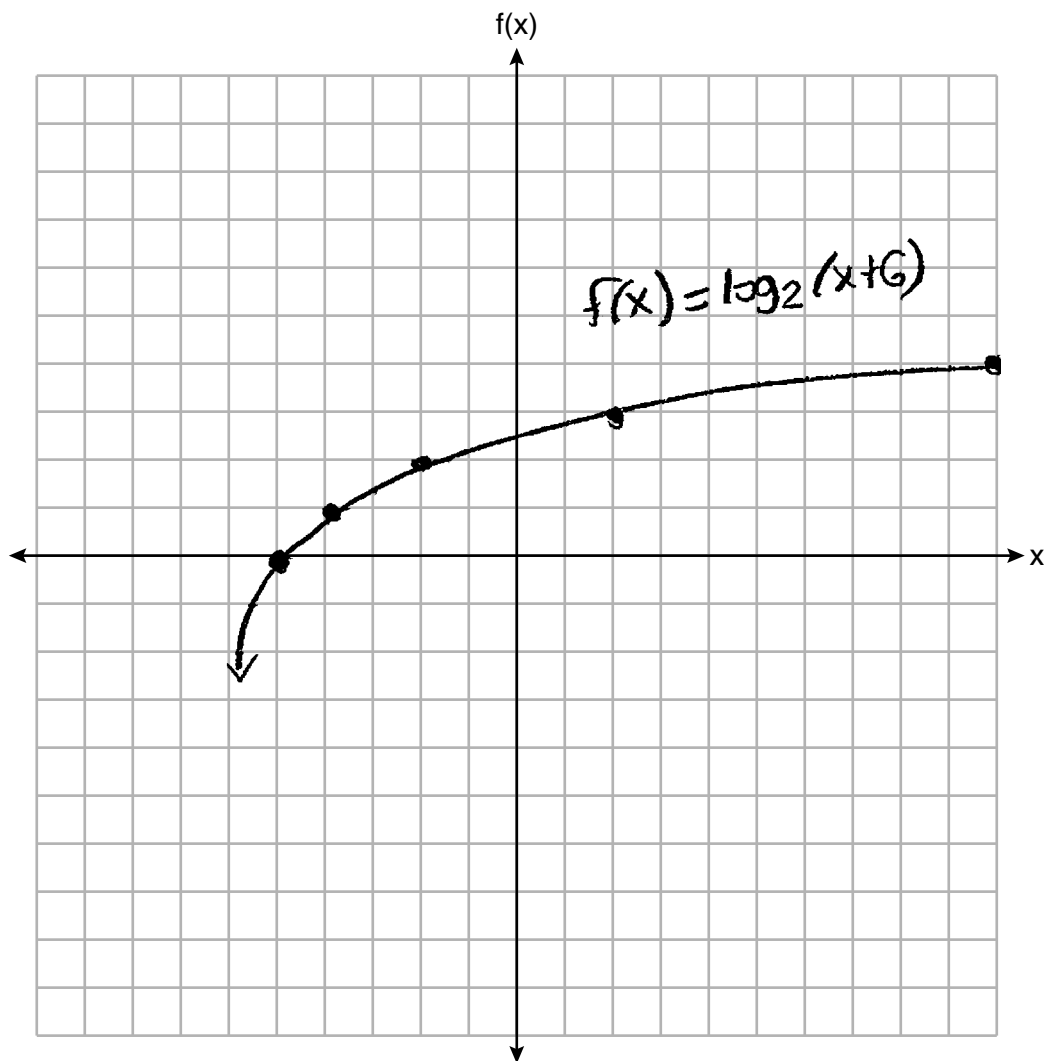
$$x = 2.25$$

$$x = \frac{9}{4}$$

**Score 0:** The student made a conceptual error and a computational error.

Question 27

27 Graph  $f(x) = \log_2(x + 6)$  on the set of axes below.

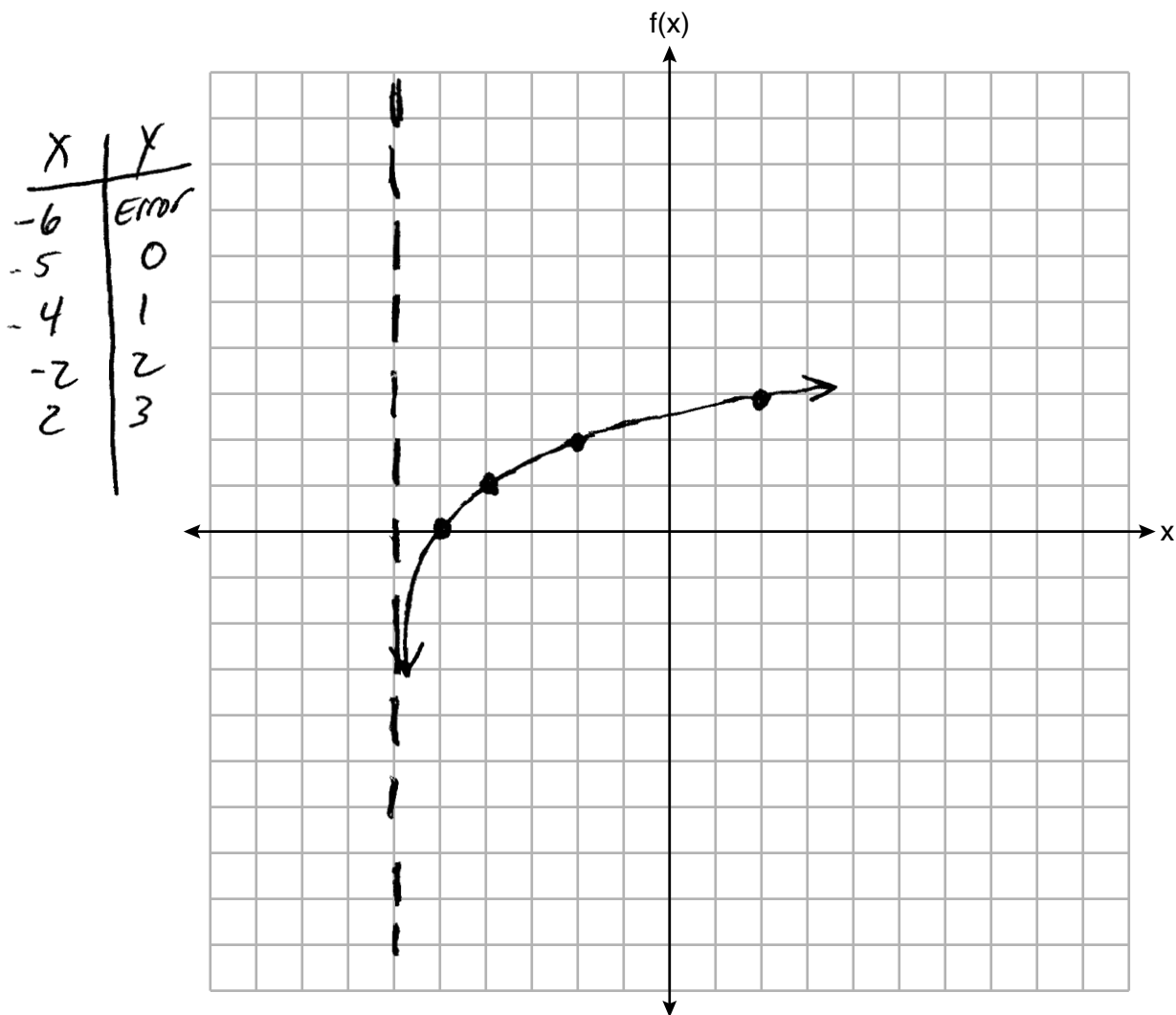


**Score 2:** The student gave a complete and correct response.



Question 27

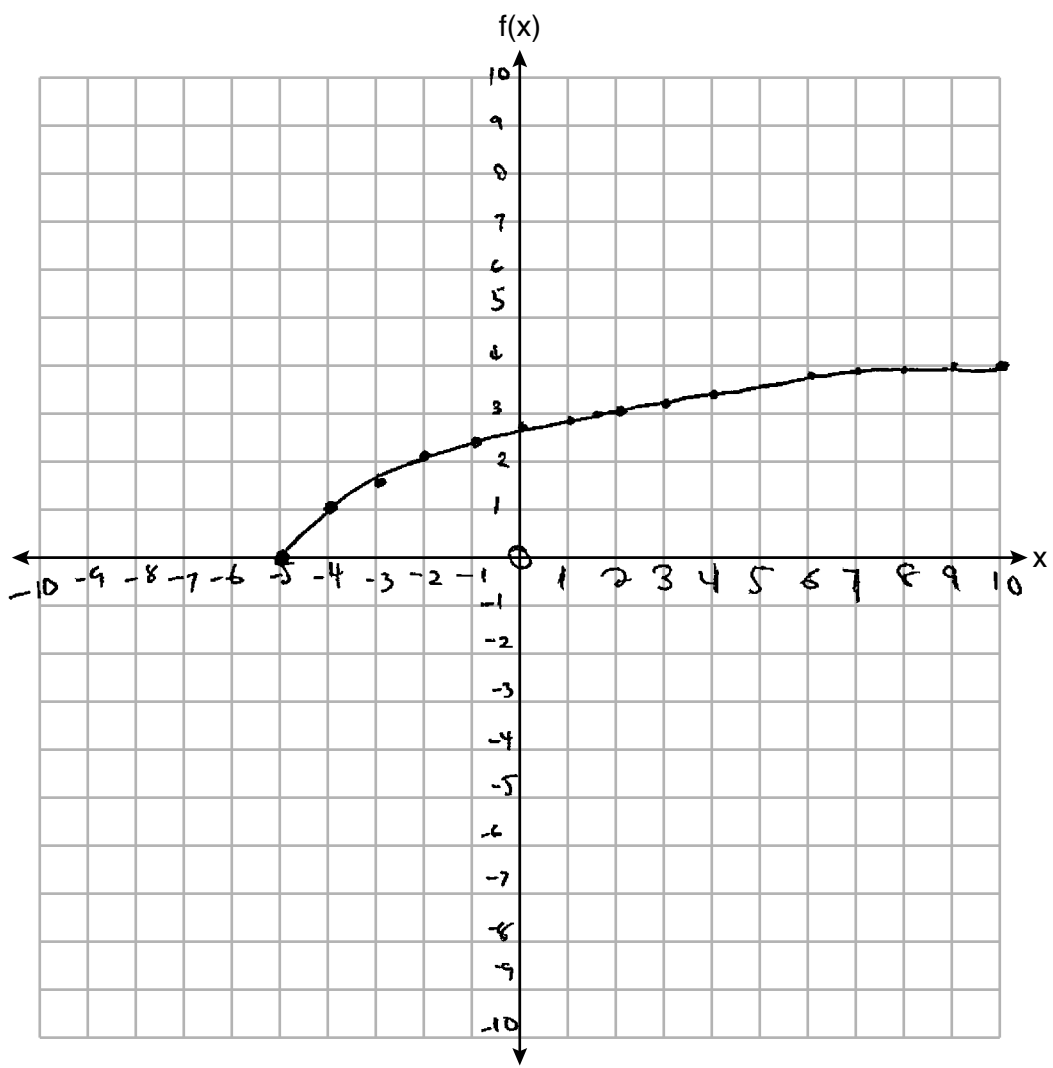
27 Graph  $f(x) = \log_2(x + 6)$  on the set of axes below.



**Score 2:** The student gave a complete and correct response.

Question 27

27 Graph  $f(x) = \log_2(x + 6)$  on the set of axes below.



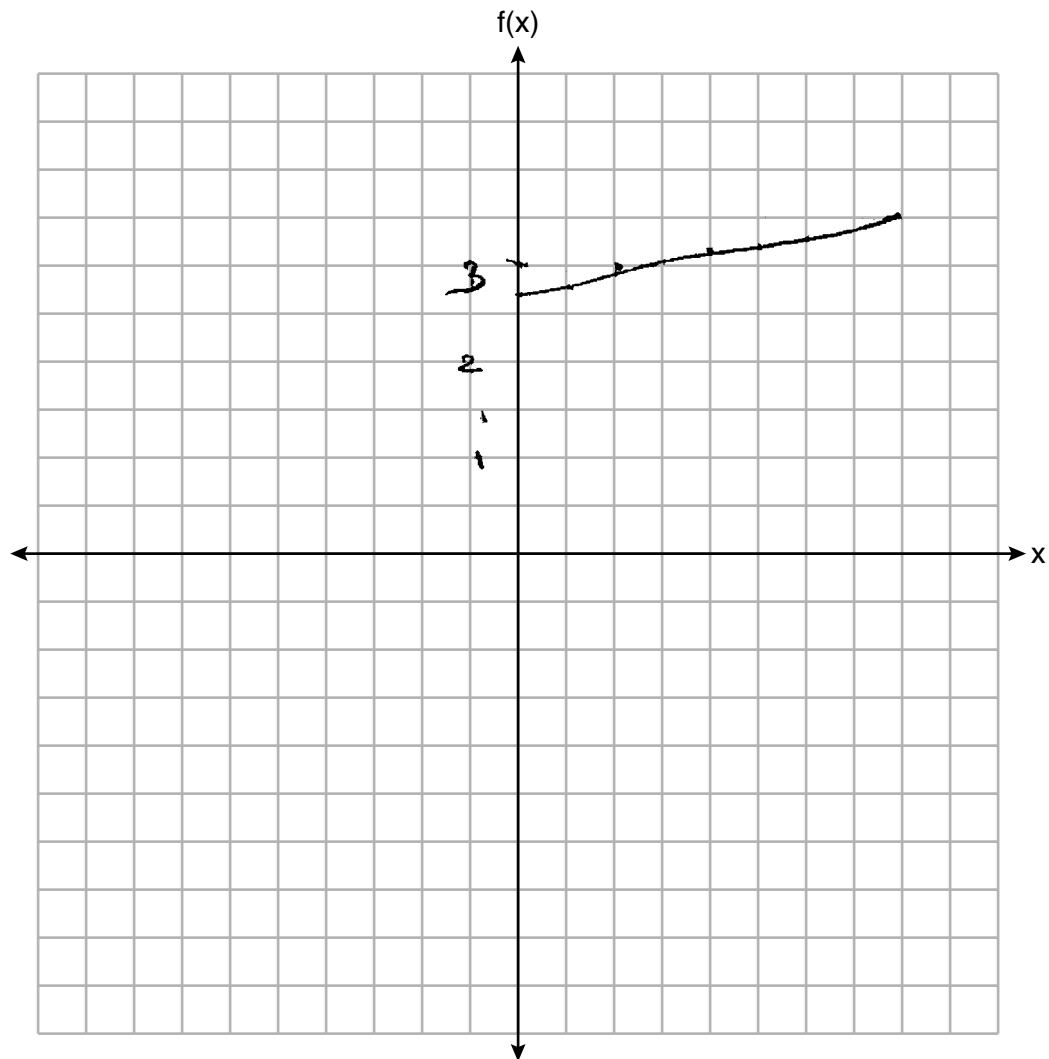
x	y
0	3
-4	0
-5	0

**Score 1:** The student made an error graphing the end behavior as  $x \rightarrow -6$ .

Question 27

27 Graph  $f(x) = \log_2(x + 6)$  on the set of axes below.

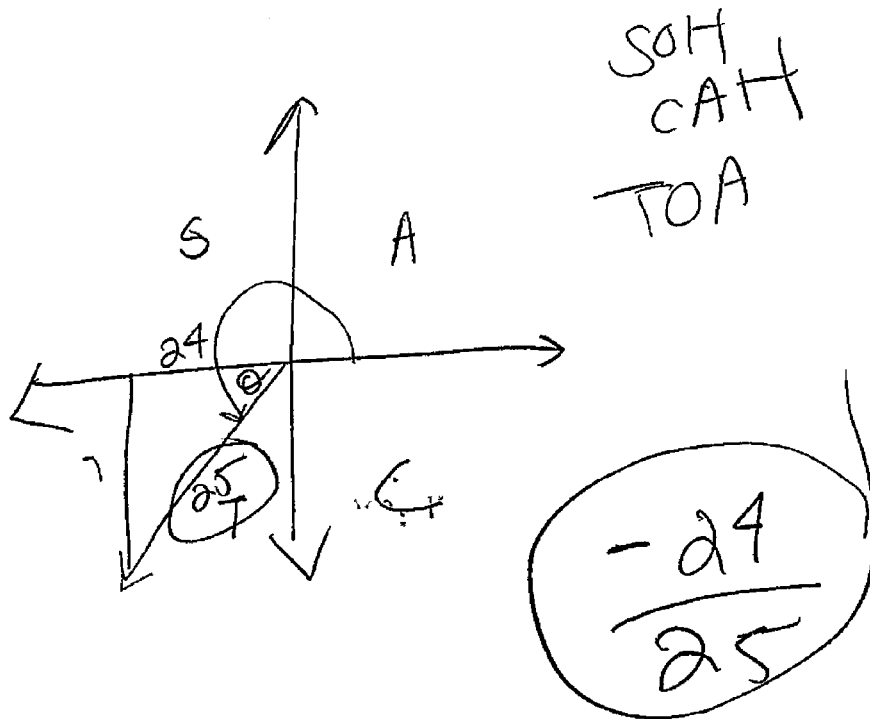
$x$	$y$
0	2.5849
1	2.8073
2	3
3	3.1699
4	3.3219
5	3.4594
6	3.5849



**Score 0:** The student made multiple graphing errors.

Question 28

28 Given  $\tan \theta = \frac{7}{24}$ , and  $\theta$  terminates in Quadrant III, determine the value of  $\cos \theta$ .



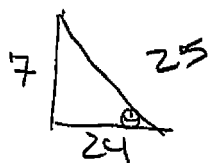
**Score 2:** The student gave a complete and correct response.

Question 28

28 Given  $\tan \theta = \frac{7}{24}$ , and  $\theta$  terminates in Quadrant III, determine the value of  $\cos \theta$ .

Sohcahtoa

S A  
T C



$$7^2 + 24^2 = x^2$$

$$49 + 576 = x^2$$

$$\sqrt{625} = \sqrt{x^2}$$

$$x = 25$$

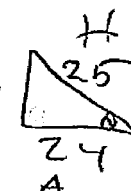
$$-\frac{24}{25}$$

**Score 2:** The student gave a complete and correct response.

Question 28

28 Given  $\tan \theta = \frac{7}{24}$ , and  $\theta$  terminates in Quadrant III, determine the value of  $\cos \theta$ .

SOH CAHTOA

$$\cos \theta = \frac{A}{H} = \frac{24}{25}$$


or

$$a^2 + b^2 = c^2$$
$$7^2 + 24^2 = x^2$$
$$49 + 576 = x^2$$
$$\sqrt{625} = x^2$$
$$x = 25$$

**Score 1:** The student did not consider the quadrant.

Question 28

28 Given  $\tan \theta = \frac{7}{24}$ , and  $\theta$  terminates in Quadrant III, determine the value of  $\cos \theta$ .

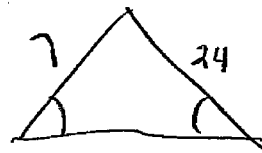
$$y = mx + b$$

$$\tan(t_1, t_1)$$

$$\cos(t_1, t_1)$$

$$\tan \theta = \frac{7}{24}$$

SO  
H CA TO



C  
H

Tan =

$$\cos \theta = \frac{23}{24}$$

$$a^2 + b^2 = c^2$$

$$7^2 = 24^2$$

$$49 = 576$$

$$-49 \quad -49$$

$$b^2 = 527$$

$$b = 23$$

**Score 0:** The student did not show enough correct work to receive any credit.

**Question 29**

- 29 Kenzie believes that for  $x \geq 0$ , the expression  $(\sqrt[7]{x^2})(\sqrt[5]{x^3})$  is equivalent to  $\sqrt[35]{x^6}$ . Is she correct? Justify your response algebraically.

$$\left(x^{\frac{2}{7}}\right)\left(x^{\frac{3}{5}}\right) = x^{\frac{31}{35}} = \sqrt[35]{x^{31}}$$

she is not correct because when you convert the expression into radical form and multiply, add the exponents, the answer should be  $\sqrt[35]{x^{31}}$

**Score 2:** The student gave a complete and correct response.



**Question 29**

29 Kenzie believes that for  $x \geq 0$ , the expression  $(\sqrt[7]{x^2})(\sqrt[5]{x^3})$  is equivalent to  $\sqrt[35]{x^6}$ . Is she correct? Justify your response algebraically.

$$\begin{aligned} & X^{2/7} \cdot X^{3/5} & \frac{2}{7} + \frac{3}{5} = \frac{31}{35} \\ & X^{31/35} \\ & \sqrt[35]{X^{31}} \neq \sqrt[35]{X^6} \end{aligned}$$

**Score 2:** The student gave a complete and correct response. It is indicated that Kenzie is incorrect.

**Question 29**

29 Kenzie believes that for  $x \geq 0$ , the expression  $(\sqrt[7]{x^2})(\sqrt[5]{x^3})$  is equivalent to  $\sqrt[35]{x^6}$ . Is she correct? Justify your response algebraically.

$$(x)^{\frac{6}{35}}$$

$$(\sqrt[7]{x^2})(\sqrt[5]{x^3})$$
$$((x^2)^{\frac{1}{7}})((x^3)^{\frac{1}{5}})$$

~~(x)~~

~~(x)~~  $(x^{\frac{2}{7}})(x^{\frac{3}{5}})$

$$(x)^{\frac{6}{35}}$$

yes she is correct

**Score 1:** The student applied exponent properties incorrectly.

**Question 29**

- 29 Kenzie believes that for  $x \geq 0$ , the expression  $(\sqrt[7]{x^2})(\sqrt[5]{x^3})$  is equivalent to  $\sqrt[35]{x^6}$ . Is she correct? Justify your response algebraically.

$$(\sqrt[7]{(2)^2})(\sqrt[5]{(2)^3}) = 1.84767919$$

$$\sqrt[35]{(2)^6} = 1.126173081$$

NO, when plugging in a tester they are not the same

**Score 1:** The student used a method other than algebraic by showing a contradiction.

---

**Question 29**

---

29 Kenzie believes that for  $x \geq 0$ , the expression  $(\sqrt[7]{x^2})(\sqrt[5]{x^3})$  is equivalent to  $\sqrt[35]{x^6}$ . Is she correct? Justify your response algebraically.

$$\sqrt[7]{x^2} = (x^2)^{\frac{1}{7}}$$
$$\sqrt[5]{x^3} = (x^3)^{\frac{1}{5}}$$

$$(x^2)^{\frac{1}{7}} \cdot (x^3)^{\frac{1}{5}} = x^{6 \frac{35}{4}} = (\sqrt[35]{x^6})^4$$

**Score 0:** The student made multiple errors.

**Question 30**

30 When the function  $p(x)$  is divided by  $x - 1$  the quotient is  $x^2 + 7 + \frac{5}{x - 1}$ . State  $p(x)$  in standard form.

$$\frac{p(x)}{x-1} = x^2 + 7 + \frac{5}{x-1}$$
$$x^2(x-1) + 7(x-1) + \left(\frac{5}{x-1}\right)(x-1)$$
$$x^3 - x^2 + 7x - 7 + 5$$
$$p(x) = x^3 - x^2 + 7x - 2$$

**Score 2:** The student gave a complete and correct response.

**Question 30**

30 When the function  $p(x)$  is divided by  $x - 1$  the quotient is  $x^2 + 7 + \frac{5}{x - 1}$ . State  $p(x)$  in standard form.

$$\begin{array}{r} x^2 + 7 \\ \overline{) x - 1} \\ -x^2 - 7 \\ \hline x^3 + 7x \\ \overline{) x^3 - x^2 + 7x - 7} \quad +5 \\ x^3 - x^2 + 7x - 2 \end{array}$$

**Score 2:** The student gave a complete and correct response.

**Question 30**

30 When the function  $p(x)$  is divided by  $x - 1$  the quotient is  $x^2 + 7 + \frac{5}{x - 1}$ . State  $p(x)$  in standard form.

$$\frac{p(x)}{x-1} = x^2 + 7 + \frac{5}{x-1}$$

$$\frac{(x-1)(x^2+7)}{(x-1)} + \frac{5}{x-1}$$

$$\frac{x^3 - x^2 + 7x - 7 + 5}{x-1}$$

$$\frac{x^3 - x^2 + 7x - 2}{x-1}$$

$$p(x) = x^3 - x^2 + 7x - 2$$

**Score 2:** The student gave a complete and correct response.

---

**Question 30**

---

30 When the function  $p(x)$  is divided by  $x - 1$  the quotient is  $x^2 + 7 + \frac{5}{x - 1}$ . State  $p(x)$  in standard form.

$$x^3 - x^2 + 7x - 7 + 5x - 5$$

$$x^3 - x^2 + 12x - 12$$

---

**Score 1:** The student incorrectly distributed the  $x - 1$  to the rational term.



Question 30

30 When the function  $p(x)$  is divided by  $x - 1$  the quotient is  $x^2 + 7 + \frac{5}{x - 1}$ . State  $p(x)$  in standard form.

$$x^3 - x^2 + 7x - 7$$

$$x-1(x^2) = x^3 - x^2$$
$$x-1(7) = 7x - 7$$

$$\begin{array}{r} x^2 + 7 \\ x-1 \overline{) x^3 - x^2 + 7x - 7} \\ \underline{-x^3 + x^2} \phantom{+ 7x - 7} \\ -x - \phantom{7} \end{array}$$

**Score 1:** The student excluded the remainder.

---

**Question 30**

---

30 When the function  $p(x)$  is divided by  $x - 1$  the quotient is  $x^2 + 7 + \frac{5}{x - 1}$ . State  $p(x)$  in standard form.

$$\begin{array}{r} x^3 + x^2 \\ \underline{x^2 + 7x - 7} \quad \frac{5}{x-1} \\ x^3 + x^2 + 7x - 7 + 5 \end{array}$$

$$x^3 + x^2 + 7x - 7 + 5$$

**Score 0:** The student made an error distributing the  $x^2$  and did not state  $p(x)$  in standard form.

---

**Question 31**

---

31 Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .

$$a_1 = 6$$

$$a_n = a_{n-1} \cdot 1.5$$

check

$$a_2 = a_{2-1} \cdot 1.5$$

$$a_2 = a_1 \cdot 1.5$$

$$a_2 = 6 \cdot 1.5$$

$$a_2 = 9$$

✓

**Score 2:** The student gave a complete and correct response.

Question 31

$\frac{9 \times 3}{8 \times 2} = \frac{27}{4}$

31 Write a recursive formula for the sequence 6, 9, 13.5, 20.25, ...

$$a_1 = 6$$

$$a_n = a_{n-1} \cdot k$$

$$r = \frac{3}{2}$$

$$a_n = a_{n-1} \cdot \frac{3}{2}$$

$$a_1 = 6$$

**Score 2:** The student gave a complete and correct response.

---

**Question 31**

---

31 Write a recursive formula for the sequence  $6, 9, 13.5, 20.25, \dots$

$$a_1 = 6$$
$$a_n = a_1 \left(\frac{3}{2}\right)^{n-1}$$

---

**Score 1:** The student received credit for writing  $a_1 = 6$ .

---

**Question 31**

---

**31** Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .

$$a_n = 1.5(a_{n-1})$$

---

**Score 1:** The student did not write the initial term.

Question 31

31 Write a recursive formula for the sequence 6, 9, 13.5, 20.25, ...

$\begin{array}{ccc} \triangle & \triangle & \triangle \\ 1.5 & 1.5 & 1.5 \end{array}$

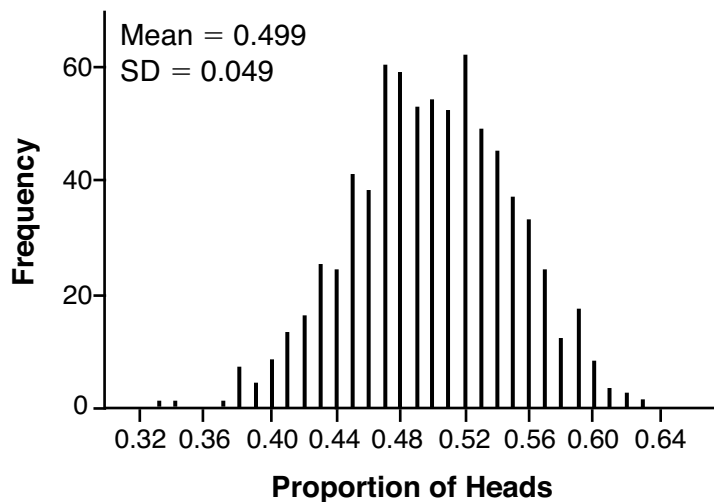
$$\begin{array}{|l} a_n = 6 \\ a_n = a_1 + 1.5 \end{array}$$

$6, 9, 13.5, 20.25$   
 $\begin{array}{ccc} \vee & \vee & \vee \\ 1.5 & 1.5 & 1.5 \end{array}$

**Score 0:** The student did not show enough correct work to receive any credit.

**Question 32**

**32** Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Do the results of the simulation provide strong evidence that Robin's coin is unfair? Explain your answer.

$$\text{Robin's coin} = \frac{43}{100} = .43$$

$$.499 \pm 2(.049) \rightarrow (.401, .597)$$

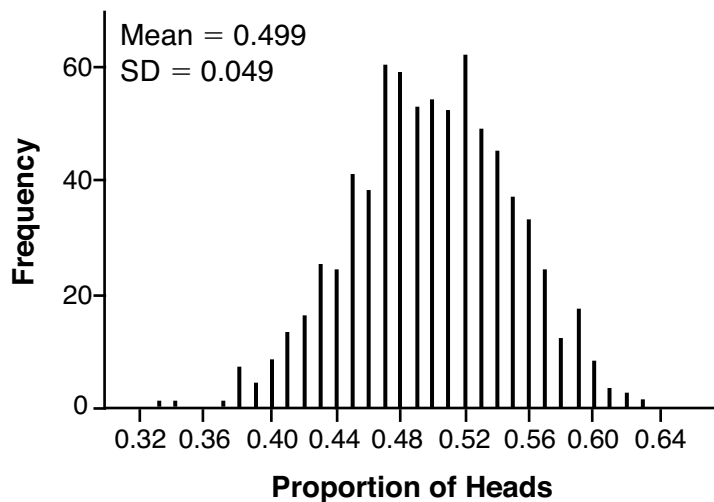
Since .43 is within the interval of (.401, .597) her coin is likely not unfair.

**Score 2:** The student gave a complete and correct response.

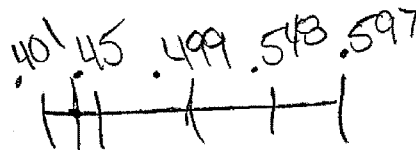


### Question 32

32 Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Do the results of the simulation provide strong evidence that Robin's coin is unfair? Explain your answer.



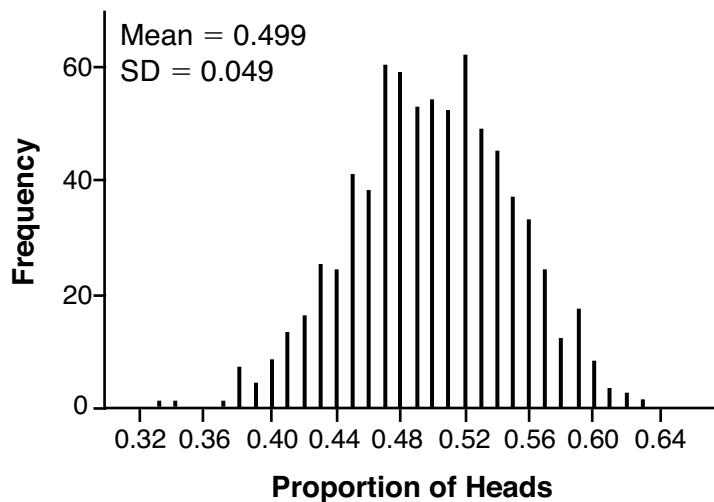
$$\frac{43}{100} = .43$$

NO because .43 falls inside the 95% / 2 standard deviation.

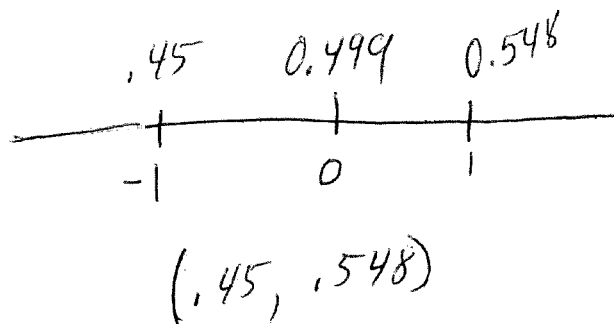
**Score 2:** The student gave a complete and correct response.

### Question 32

32 Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Do the results of the simulation provide strong evidence that Robin's coin is unfair? Explain your answer.

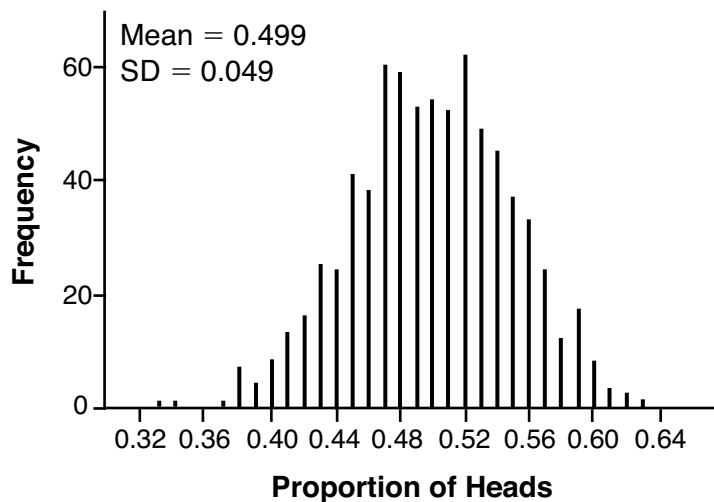


.43 is outside the interval so the simulation provides strong evidence Robin's coin is unfair.

**Score 1:** The student gave a correct explanation based on an inappropriate interval.

### Question 32

**32** Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



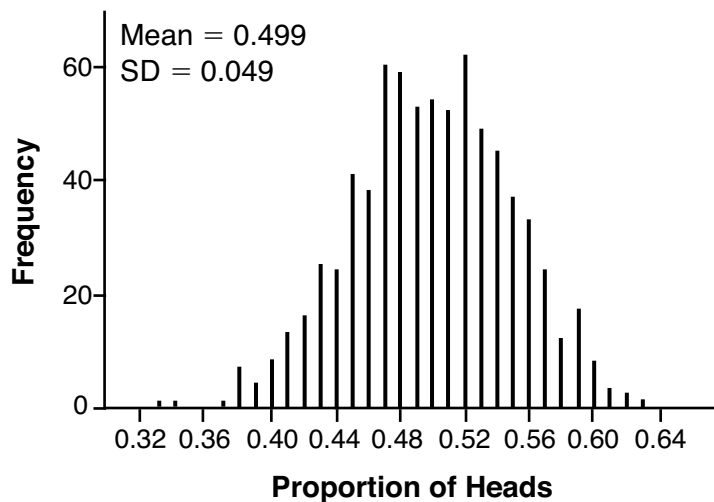
Do the results of the simulation provide strong evidence that Robin's coin is unfair? Explain your answer.

No, .43 is within 2 standard deviations from the mean.

**Score 1:** The student gave an explanation, but provided no statistical evidence.

### Question 32

32 Robin flips a coin 100 times. It lands heads up 43 times, and she wonders if the coin is unfair. She runs a computer simulation of 750 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Do the results of the simulation provide strong evidence that Robin's coin is unfair? Explain your answer.

Yes, her coin is more than 1 standard deviation away. Although it isn't more than 1.5 deviations, it is still much less than the mean.

**Score 0:** The student did not show enough correct statistical evidence to receive any credit.

Question 33

33 Factor completely over the set of integers:  $16x^4 - 81$

$$16x^4 - 81$$
$$(4x^2 + 9)(4x^2 - 9)$$
$$(4x^2 + 9)(2x + 3)(2x - 3)$$

Sara graphed the polynomial  $y = 16x^4 - 81$  and stated "All the roots of  $y = 16x^4 - 81$  are real."  
Is Sara correct? Explain your reasoning.

$$~~a = 16 \quad b = 0 \quad c = -81~~$$

$$4x^2 + 9 = 0 \quad 2x + 3 = 0 \quad 2x - 3 = 0$$
$$\begin{array}{r} 4x^2 + 9 = 0 \\ -9 \quad -9 \\ \hline 4x^2 = -9 \\ \sqrt{4x^2} = \sqrt{-9} \end{array}$$

NO, When you make mini equations,  
 $4x^2 + 9 = 0$  can be solved for  $x$ , but  
your answer is an imaginary number,  
meaning not all roots of  $y = 16x^4 - 81 = 0$  are  
real.

**Score 4:** The student gave a complete and correct response.

Question 33

33 Factor completely over the set of integers:  $16x^4 - 81$

$$(4x^2 - 9)(4x^2 + 9)$$

$$(2x - 3)(2x + 3)(4x^2 + 9)$$

Sara graphed the polynomial  $y = 16x^4 - 81$  and stated "All the roots of  $y = 16x^4 - 81$  are real." Is Sara correct? Explain your reasoning.

$(2x - 3)$	$(2x + 3)$	$(4x^2 + 9)$
$2x - 3 = 0$	$2x + 3 = 0$	$4x^2 + 9 = 0$
$2x = 3$	$2x = -3$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$x = \frac{3}{2}$	$x = -\frac{3}{2}$	$b^2 - 4ac = 0^2 - 4(4)(9) = -144$

Sara is incorrect. Although 2 of the roots are real, we know at least one root is nonreal because, when using the quadratic formula to determine the roots for the factor  $4x^2 + 9$ , the discriminant is negative.

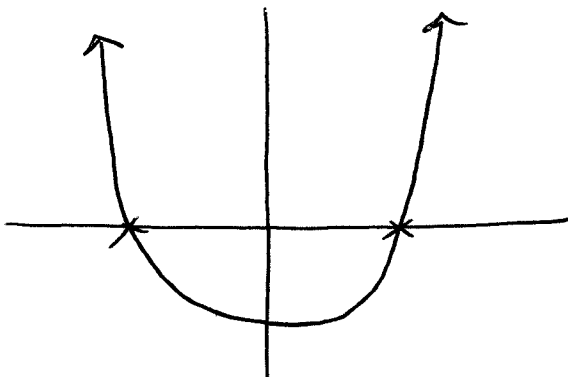
**Score 4:** The student gave a complete and correct response.

**Question 33**

33 Factor completely over the set of integers:  $16x^4 - 81$

$$(4x^2 - 9)(4x^2 + 9)$$
$$(2x - 3)(2x + 3)(4x^2 + 9)$$

Sara graphed the polynomial  $y = 16x^4 - 81$  and stated "All the roots of  $y = 16x^4 - 81$  are real."  
Is Sara correct? Explain your reasoning.



No because the graph only crosses the x axis two times meaning only 2 real roots not 4.

**Score 4:** The student gave a complete and correct response.

**Question 33**

33 Factor completely over the set of integers:  $16x^4 - 81$

$$\begin{aligned} & 16x^4 - 81 \\ & (4x^2 - 9)(4x^2 + 9) \qquad \frac{16}{1, 2, 4, 8, 16} \\ & (2x + 3)(2x - 3)(2x + 3)(2x + 3) \\ & (2x + 3)^3(2x - 3) \end{aligned}$$

Sara graphed the polynomial  $y = 16x^4 - 81$  and stated "All the roots of  $y = 16x^4 - 81$  are real." Is Sara correct? Explain your reasoning.

NO. Because it has 4 possible zeros, but only crosses the x-axis twice. (Graphing calculator)

**Score 3:** The student made one factoring error.



Question 33

33 Factor completely over the set of integers:  $16x^4 - 81$

$$(4x^2 - 9)(4x^2 + 9)$$

$$\begin{array}{r} 4x^2 + 9 = 0 \\ +9 \quad +9 \\ \hline 4x^2 = 9 \\ \frac{4}{4} \quad \frac{9}{4} \\ \sqrt{x^2} = \sqrt{\frac{9}{4}} = x = \pm \frac{3}{2} \end{array}$$

$$\begin{array}{r} 4x^2 + 9 = 0 \\ -9 \quad -9 \\ \hline 4x^2 = -9 \\ \frac{4}{4} \quad \frac{-9}{4} \\ \sqrt{x^2} = \sqrt{\frac{-9}{4}} \end{array}$$

$x = \pm \frac{3}{2}i$

Sara graphed the polynomial  $y = 16x^4 - 81$  and stated "All the roots of  $y = 16x^4 - 81$  are real." Is Sara correct? Explain your reasoning.

Not all of the roots of  $16x^4 - 81$  are real, because if it would be real it wouldn't had an imaginary number

**Score 3:** The student did not factor completely.

**Question 33**

33 Factor completely over the set of integers:  $16x^4 - 81$

$$(4x^2 - 9)(4x^2 + 9)$$
$$(2x + 3)(2x - 3)(2x + 3i)(2x - 3i)$$

Sara graphed the polynomial  $y = 16x^4 - 81$  and stated "All the roots of  $y = 16x^4 - 81$  are real."  
Is Sara correct? Explain your reasoning.

$$\text{Roots: } -\frac{3}{2}, \frac{3}{2}, \frac{3i}{2}, -\frac{3i}{2}$$

No, b/c  $\frac{3i}{2}$  and  $-\frac{3i}{2}$  are imaginary #'s

**Score 3:** The student did not factor over the set of integers.

**Question 33**

33 Factor completely over the set of integers:  $16x^4 - 81$

$$(4x^2+9)(2x+3)(2x-3) \quad \boxed{16x^4-81}$$
$$(4x^2+9) \quad (4x^2-9)$$
$$2x+3 \quad 2x-3$$

Sara graphed the polynomial  $y = 16x^4 - 81$  and stated "All the roots of  $y = 16x^4 - 81$  are real." Is Sara correct? Explain your reasoning.

Sara is incorrect because if you plug into  $y =$  and go to 2nd graph to the table and scroll up you can see there are some unreal roots.

-17	1.34e6
-16	1.05e6
-15	809919
-14	614575

**Score 2:** The student only received credit for factoring completely.

**Question 33**

33 Factor completely over the set of integers:  $16x^4 - 81$

$$\begin{array}{c} 16x^4 - 81 \\ (4x^2 - 9)(4x^2 + 9) \\ \uparrow \quad \uparrow \quad \uparrow \\ (2x - 3)(2x + 3) \end{array}$$

Sara graphed the polynomial  $y = 16x^4 - 81$  and stated "All the roots of  $y = 16x^4 - 81$  are real."  
Is Sara correct? Explain your reasoning.

$$\begin{array}{l} (2x-3)(2x+3) = 0 \\ 2x-3=0 \quad 2x+3=0 \\ 2x=3 \quad 2x=-3 \\ x=\frac{3}{2} \quad x=-\frac{3}{2} \end{array}$$

No, because 1.5 is a real number  
because -1.5 is not a real number  
because it is negative

**Score 1:** The student made one factoring error and gave an incorrect explanation.

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**Question 33**

---

33 Factor completely over the set of integers:  $16x^4 - 81$

$$16x^4 - 81$$

Sara graphed the polynomial  $y = 16x^4 - 81$  and stated “All the roots of  $y = 16x^4 - 81$  are real.” Is Sara correct? Explain your reasoning.

Sarah is incorrect because some of the roots are imaginary.

---

**Score 0:** The student's explanation was not sufficient to receive any credit.

**Question 34**

34 The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount,  $s(t)$ , of 200 grams of this substance that remains after  $t$  years.

$$s(t) = 200 \left(\frac{1}{2}\right)^{\frac{t}{15}}$$

Determine algebraically, to the nearest year, how long it will take for  $\frac{1}{10}$  of this substance to remain.

$$\frac{20}{200} = \frac{200}{200} \left(\frac{1}{2}\right)^{\frac{t}{15}}$$

$$\frac{1}{10} = \left(\frac{1}{2}\right)^{\frac{t}{15}}$$

$$\log\left(\frac{1}{10}\right) = \frac{t}{15} \log\left(\frac{1}{2}\right)$$

$$3.32193 = \frac{t}{15}$$

$$49.8289 = t$$

50. years

**Score 4:** The student gave a complete and correct response.

**Question 34**

34 The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount,  $s(t)$ , of 200 grams of this substance that remains after  $t$  years.

$$s(t) = 200 \left(\frac{1}{2}\right)^{t/15}$$

Determine algebraically, to the *nearest year*, how long it will take for  $\frac{1}{10}$  of this substance to remain.

$$\frac{1}{10} = 200 \left(\frac{1}{2}\right)^{t/15}$$

$$\frac{1}{2000} = \left(\frac{1}{2}\right)^{t/15}$$

$$\log\left(\frac{1}{2000}\right) = \frac{t}{15} \log\left(\frac{1}{2}\right)$$

$$t = 164 \text{ years}$$

**Score 3:** The student made an error assuming that  $\frac{1}{10}$  of a gram of the substance remained.

Question 34

34 The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount,  $s(t)$ , of 200 grams of this substance that remains after  $t$  years.

$$s(t) = 200(.5)^t$$

Determine algebraically, to the nearest year, how long it will take for  $\frac{1}{10}$  of this substance to remain.

$$s(t) = 200(0.5)^t$$

$$\frac{1}{10} \cdot 200 = 20$$

$$\frac{20}{200} = \frac{200(0.5)^t}{200}$$

$$.1 = (0.5)^t$$

$$\frac{\log .1}{\log 0.5} = \frac{t \log 0.5}{\log 0.5}$$

$$t = 3.32 \rightarrow \text{3 years}$$

$$3.32 \cdot 15 = 49.82$$

$$49.82 \rightarrow 50$$

50 years

**Score 3:** The student made an error writing the equation for  $s(t)$ , assuming  $t$  was the number of half-lives.



**Question 34**

34 The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount,  $s(t)$ , of 200 grams of this substance that remains after  $t$  years.

$$s(t) = 200\left(\frac{1}{2}\right)^t$$

Determine algebraically, to the *nearest year*, how long it will take for  $\frac{1}{10}$  of this substance to remain.

$$\frac{\frac{1}{10}}{200} = \frac{200\left(\frac{1}{2}\right)^t}{200}$$

$$\log \frac{1}{2000} = \log \frac{1}{2}^t$$

$$\frac{\log \frac{1}{2000}}{\log \frac{1}{2}} = \frac{t \log \frac{1}{2}}{\log \frac{1}{2}}$$

$$t = 10.96578428$$

11 years

**Score 2:** The student wrote an incorrect equation and made an error assuming  $\frac{1}{10}$  of a gram of the substance remained.

**Question 34**

34 The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount,  $s(t)$ , of 200 grams of this substance that remains after  $t$  years.

$$200$$
$$100 = \frac{200}{200}(1-r)^{15}$$
$$200(1-r)^t$$
$$200(1-.045)^t$$

Determine algebraically, to the *nearest year*, how long it will take for  $\frac{1}{10}$  of this substance to remain.

$$15(1-r)^5$$
$$r = .04516$$
$$\frac{20}{200} = \frac{200(1-.045)^x}{200}$$
$$.1 = (.955)^x$$

$$x = 50$$

It will take 50 years to only have  $\frac{1}{10}$  of the substance to remain.

**Score 1:** The student received no credit for the first part and showed incomplete algebraic work on the second part.

---

**Question 34**

---

**34** The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount,  $s(t)$ , of 200 grams of this substance that remains after  $t$  years.

$$s(t) = 200\left(\frac{1}{2}\right)^{15t}$$

Determine algebraically, to the nearest year, how long it will take for  $\frac{1}{10}$  of this substance to remain.

$$s(t) = 200\left(\frac{1}{2}\right)^{15\left(\frac{1}{10}\right)} \rightarrow 71 \text{ years}$$

---

**Score 1:** The student received 1 credit for the equation.

**Question 34**

34 The half-life of a radioactive substance is 15 years.

Write an equation that can be used to determine the amount,  $s(t)$ , of 200 grams of this substance that remains after  $t$  years.

$$S(t) = 200(1 - .15)^t$$

Determine algebraically, to the *nearest year*, how long it will take for  $\frac{1}{10}$  of this substance to remain.

~~$$\frac{200}{200} = \frac{200}{200}$$~~

$$\frac{200}{10} = 20$$

$$= 200(.85)^t$$

$$\frac{20g}{200} = \frac{200(.85)^t}{200}$$

$$.1 = .85^t$$

$$\log .1^t = \log .85$$

13 years

**Score 0:** The student did not show enough correct work to receive any credit.

Question 35

35 Determine an equation for the parabola with focus (4, -1) and directrix  $y = -5$ .  
(Use of the grid below is optional.)

$$\begin{array}{l} x \\ 4 \end{array} \begin{array}{|c|c|} \hline x^2 & -4x \\ \hline -4x & 16 \\ \hline \end{array} \quad \begin{array}{l} y \\ 1 \end{array} \begin{array}{|c|c|} \hline y^2 & y \\ \hline y & 1 \\ \hline \end{array}$$

$$\begin{array}{l} 4 \\ 5 \end{array} \begin{array}{|c|c|} \hline y^2 & 5y \\ \hline 5y & 25 \\ \hline \end{array}$$

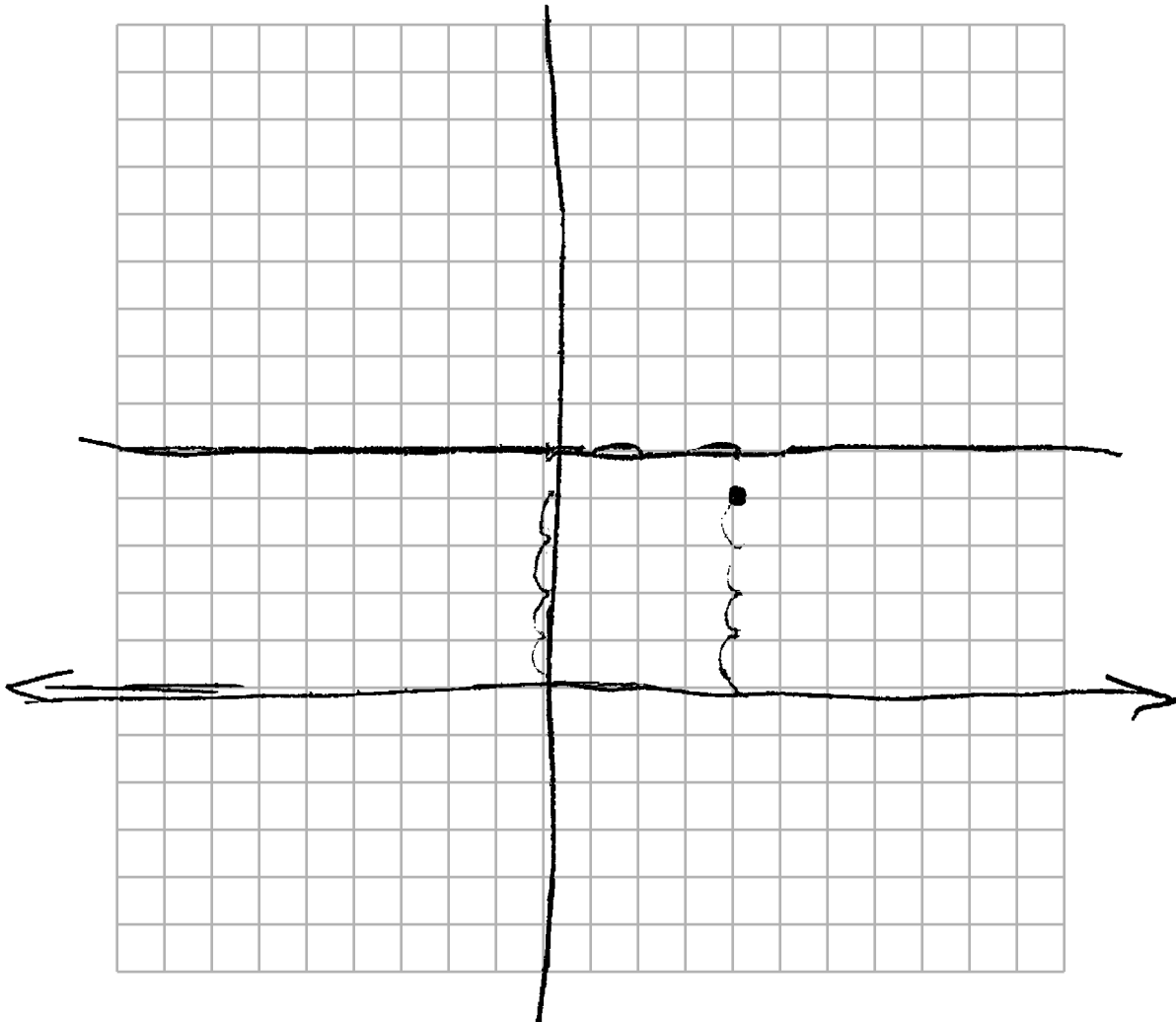
$$\left( \sqrt{(x-4)^2 + (y+1)^2} \right)^2 = (y+5)^2$$

$$x^2 - 8x + 16 + y^2 + 2y + 1 = y^2 + 10y + 25$$

$$x^2 - 8x + 16 - 24 = 8y$$

$$\frac{x^2 - 8x - 8}{8} = \frac{8y}{8}$$

$$y = \frac{1}{8}x^2 - x - 1$$



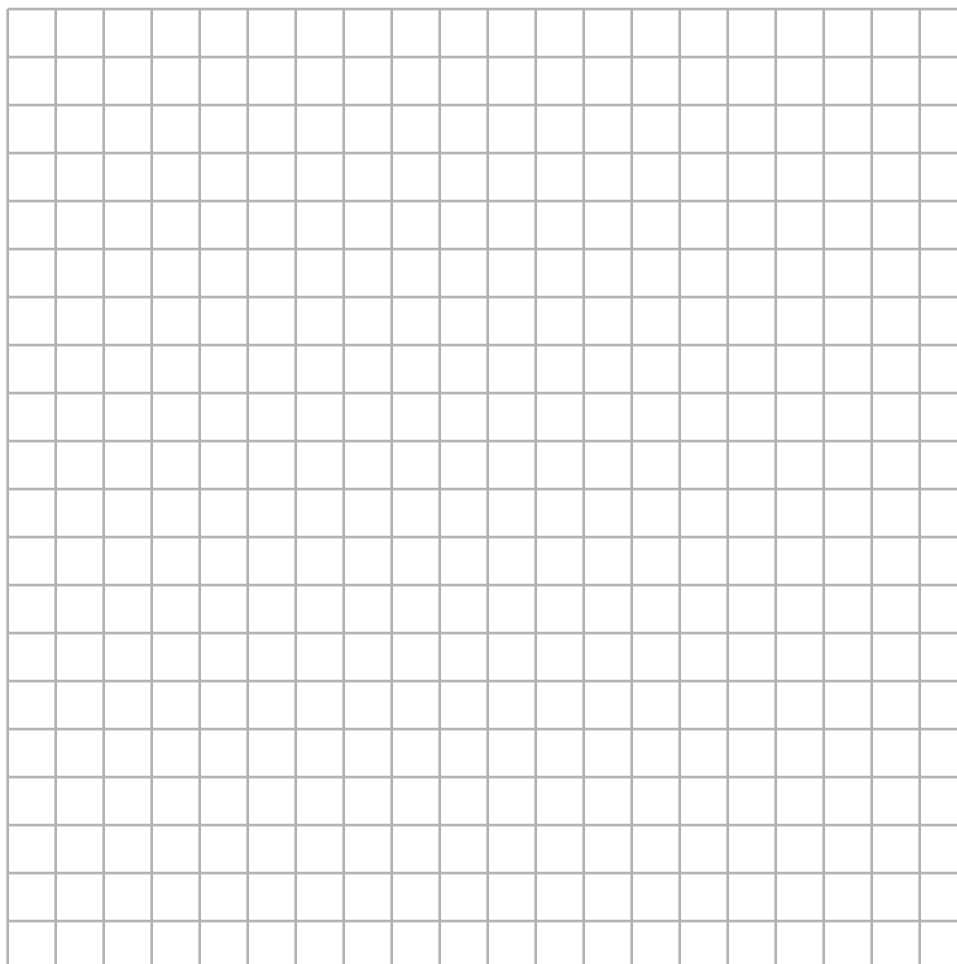
Score 4: The student gave a complete and correct response.

**Question 35**

**35** Determine an equation for the parabola with focus  $(4, -1)$  and directrix  $y = -5$ .  
(Use of the grid below is optional.)

$$y = \frac{1}{2(-1-(-5))} (x-4)^2 + \frac{-1+(-5)}{2}$$

$$y = \frac{1}{8} (x-4)^2 - 3$$

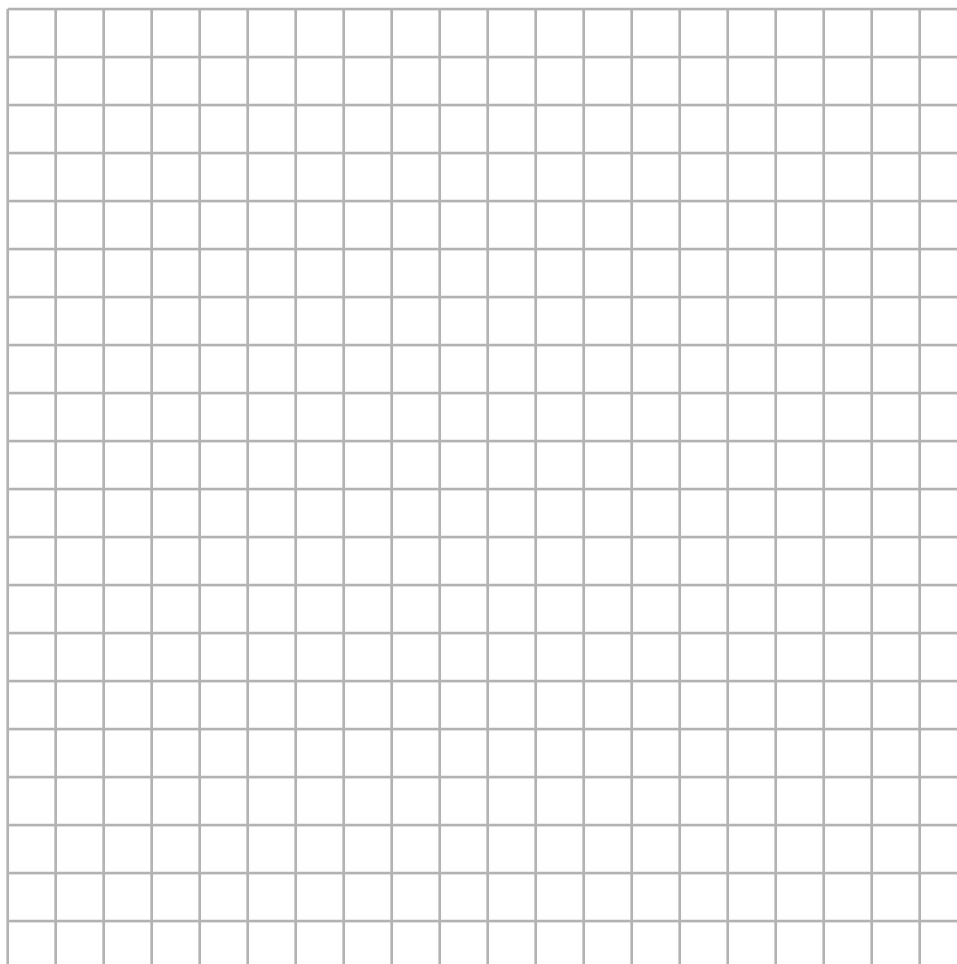


**Score 4:** The student gave a complete and correct response.

Question 35

35 Determine an equation for the parabola with focus  $(4, -1)$  and directrix  $y = -5$ .  
(Use of the grid below is optional.)

$$\begin{aligned} \text{Vertex } x &= \frac{-5 + (-1)}{2} = -3 \\ y &= -3 \\ x &= 4 \\ \text{Vertex } (x, y) &= (4, -3) \\ y &= \frac{1}{4p}(x-4)^2 - 3 \\ y &= -\frac{1}{8}(x-4)^2 - 3 \end{aligned}$$

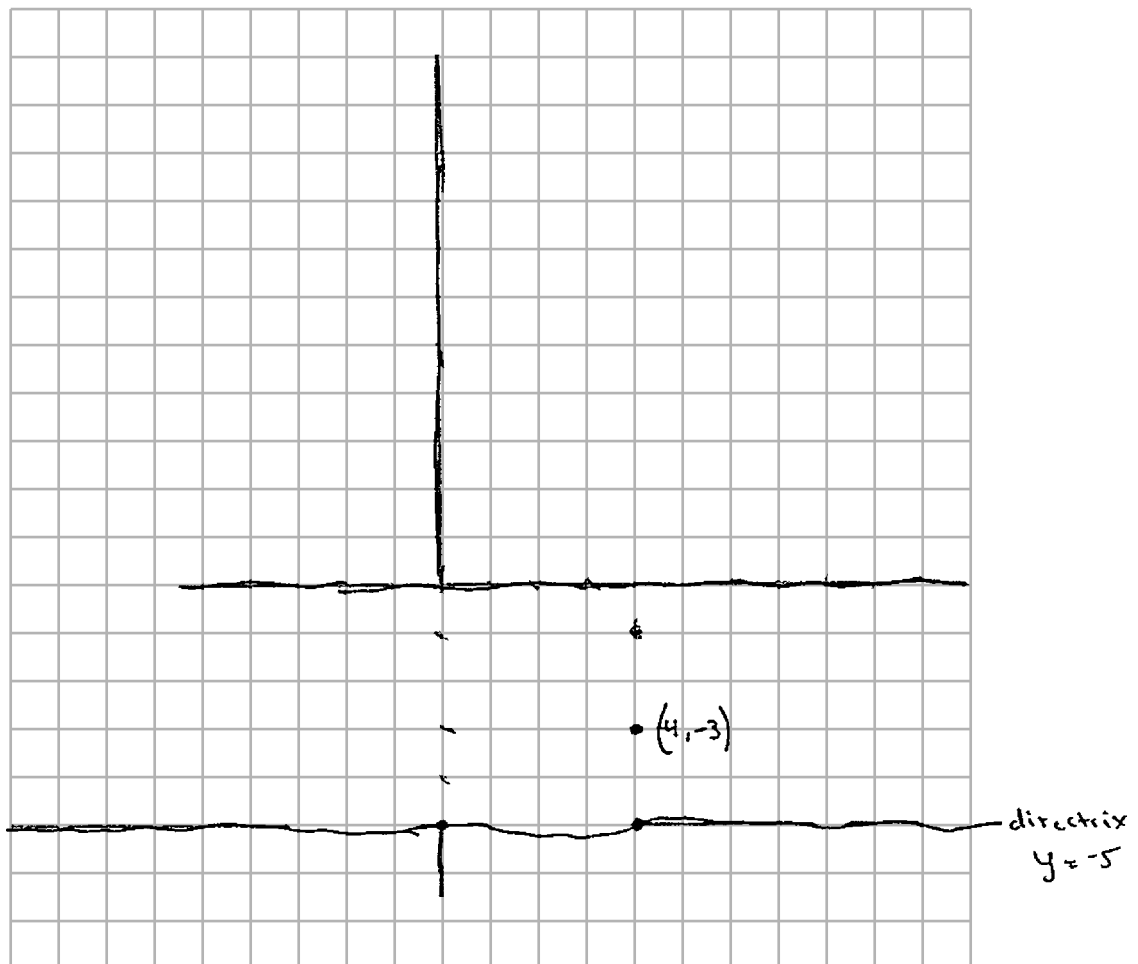


**Score 3:** The student used an incorrect value for  $p$ .

**Question 35**

**35** Determine an equation for the parabola with focus  $(4, -1)$  and directrix  $y = -5$ .  
(Use of the grid below is optional.)

$$y = (x - 4)^2 - 3$$



**Score 2:** The student correctly found the vertex and received 1 credit for the equation.



Question 35

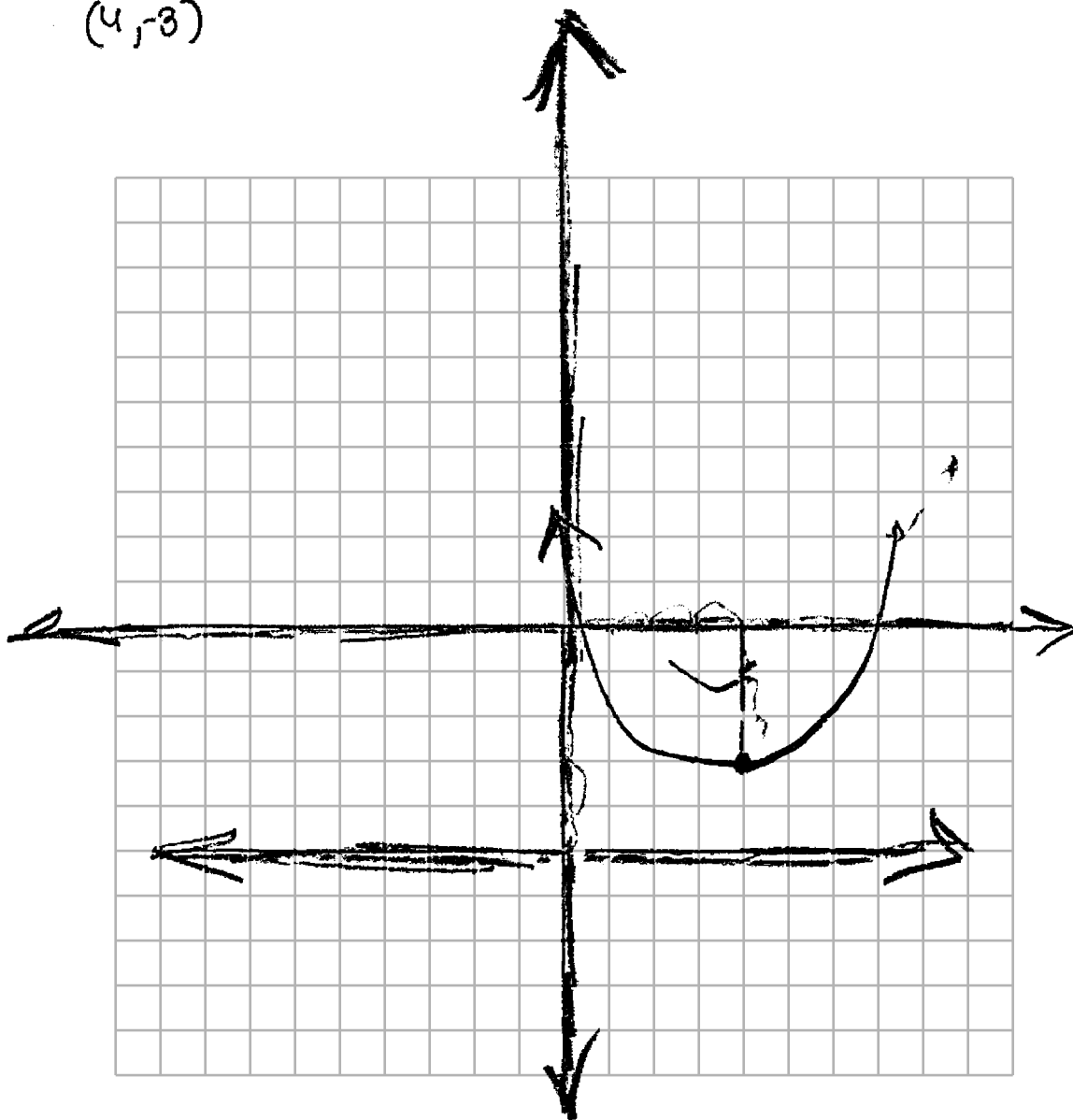
35 Determine an equation for the parabola with focus  $(4, -1)$  and directrix  $y = -5$ .  
(Use of the grid below is optional.)

$$y = \frac{1}{4(p)}(x-h) + k$$

$$y = \frac{1}{8}(x-4) - 3$$

$(h, k)$

$(4, -3)$

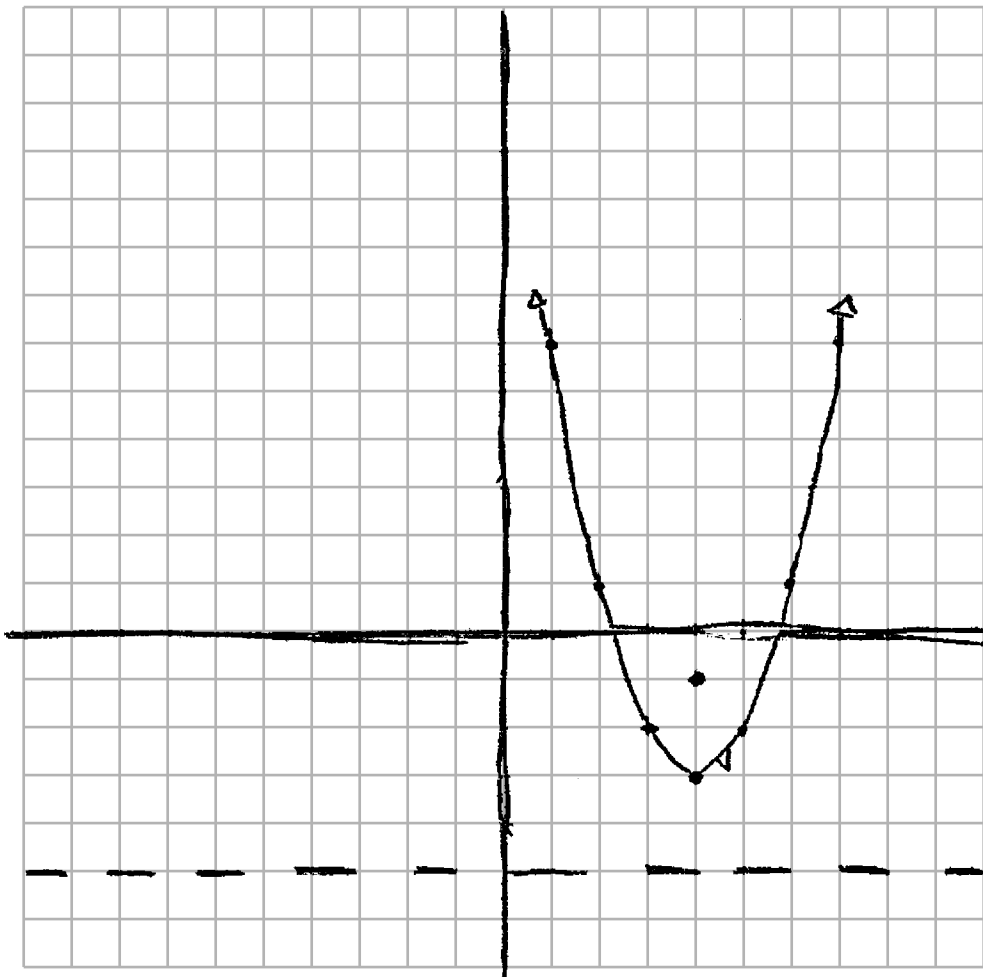


**Score 2:** The student correctly found the vertex and received 1 credit for the equation.

Question 35

- 35 Determine an equation for the parabola with focus  $(4, -1)$  and directrix  $y = -5$ .  
(Use of the grid below is optional.)

$$f(x) = (x^2 - 4) - 3$$



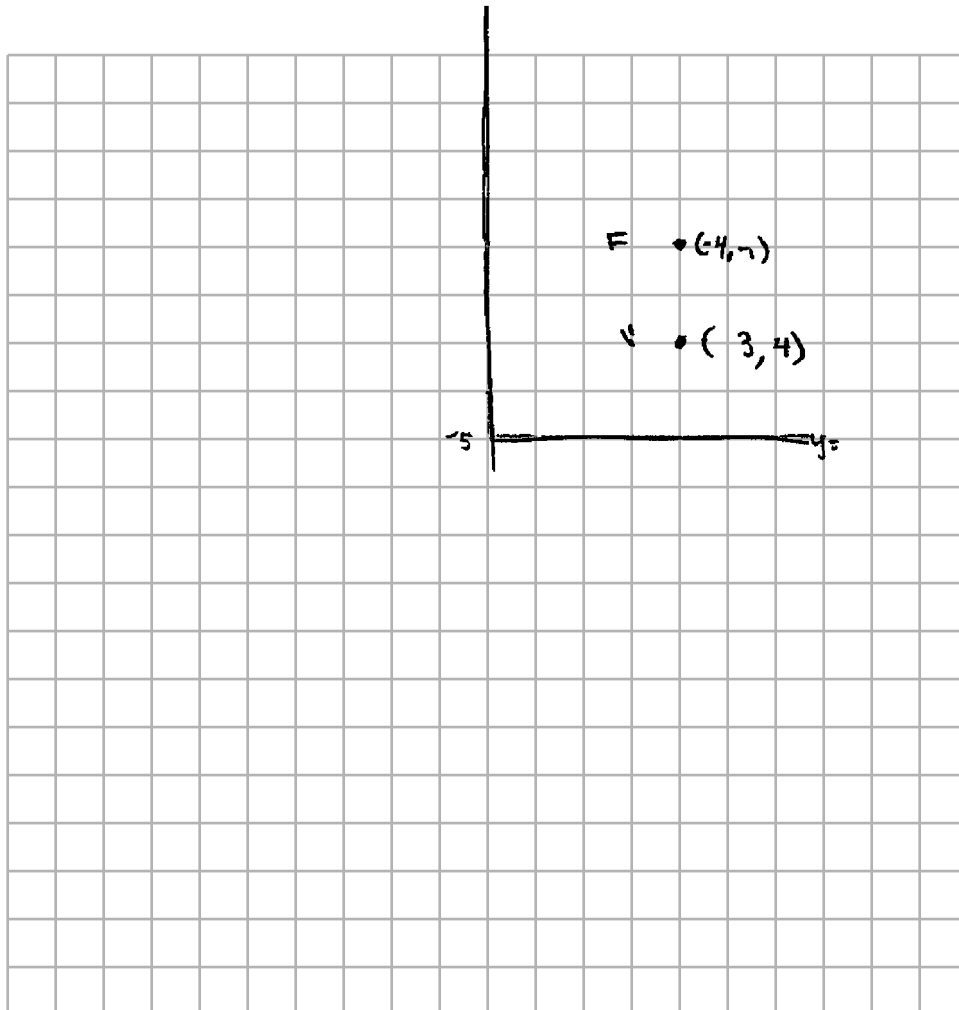
**Score 1:** The student correctly found the vertex, but made multiple errors writing the equation.

Question 35

35 Determine an equation for the parabola with focus  $(4, -1)$  and directrix  $y = -5$ .  
(Use of the grid below is optional.)

$$y = \frac{1}{8}(x-4) - 5$$

$$\frac{1}{4p^2} =$$
$$p=2 \quad \frac{1}{8}$$



**Score 0:** The student did not show enough correct work to receive any credit.

Question 36

36 Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

	Juan Wins	Filipe Wins
Short Practice Time	8	10
Long Practice Time	15	12

$22$   
 $+23$   
 $45$

$27$

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

$$P(F|L) = \frac{12}{27}$$

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

$$P(F|L) \neq P(F)$$

$$\frac{12}{27} \neq \frac{22}{45}$$

$$.44 \neq .488$$

not independent

**Score 4:** The student gave a complete and correct response.

**Question 36**

**36** Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

	Juan Wins	Filipe Wins	
Short Practice Time	8	10	18
Long Practice Time	15	12	27
	23	22	45

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

$$P(F|L) = \frac{12}{27}$$

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

$$P(F \text{ AND } L) \stackrel{?}{=} P(F) \cdot P(L)$$

$$\frac{12}{45} \stackrel{?}{=} \frac{22}{45} \cdot \frac{27}{45}$$

$$.2667 \neq .2933$$

No, the two events are Not Independent

**Score 4:** The student gave a complete and correct response.

Question 36

36 Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

		J	F	
		Juan Wins	Filipe Wins	
S	Short Practice Time	8	10	18
S <sup>c</sup>	Long Practice Time	15	12	27
		23	22	45

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

$$P(F|S^c) = \frac{P(F \cap S^c)}{P(S^c)} = \frac{12/45}{27/45} = 0.160 \quad (\text{or } 16\%)$$

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

$$P(F \cap S^c) \stackrel{?}{=} P(F) \cdot P(S^c)$$

$$\frac{12}{45} \stackrel{?}{=} \left(\frac{22}{45}\right) \left(\frac{27}{45}\right)$$

$$0.267 \neq 0.293$$

not independent

**Score 3:** The student made a computational error finding  $p(f|s^c)$ .

**Question 36**

**36** Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

	Juan Wins	Filipe Wins
Short Practice Time	8	10
Long Practice Time	15	12
	23	22

18  
 27  
 45

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

$$P(F|L) = \frac{12}{27}$$

$$= .44$$

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

$$P(A) = P(A|B)$$

$$P(F) \stackrel{?}{=} P(F|B)$$

$$\frac{22}{45} = .44$$

$$.48 \neq .44$$

not independent

**Score 3:** The student made an error rounding to 0.48.

**Question 36**

36 Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

	Juan Wins	Filipe Wins
Short Practice Time	8	10
Long Practice Time	15	12

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

$$\frac{12}{15+12} \rightarrow \boxed{\frac{12}{27}}$$

Determine whether or not the two events "Filipe wins" and "long practice time" are independent. Justify your answer.

The events of "Filipe wins" and "long practice" are dependent on one another because as "long practices" are done, the less times the event of "Filipe wins".

**Score 2:** The student only received credit for the first part.



**Question 36**

36 Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

	Juan Wins	Filipe Wins
Short Practice Time	8	10
Long Practice Time	15	12

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

$$\frac{12}{27}$$

Determine whether or not the two events “Filipe wins” and “long practice time” are independent. Justify your answer.

$$\frac{22}{45} = \frac{18}{27}$$

NO because they are different %

**Score 1:** The student received one credit for  $\frac{12}{27}$ .

Question 36

36 Juan and Filipe practice at the driving range before playing golf. The number of wins and corresponding practice times for each player are shown in the table below.

	Juan Wins	Filipe Wins
Short Practice Time	8	10
Long Practice Time	15	12 + 1

Handwritten calculations and annotations:

- Under "Short Practice Time":  $8 + 10 = 18$  (circled),  $18 + 1 = 19$  (circled)
- Under "Long Practice Time":  $15 + 12 = 27$  (circled),  $27 + 1 = 28$  (circled)
- Below the table:  $23$  (circled) under Juan Wins,  $22$  (circled) under Filipe Wins

Given that the practice time was long, determine the exact probability that Filipe wins the next match.

Probability that Filipe wins the next match is  $12/28$

Determine whether or not the two events "Filipe wins" and "long practice time" are independent. Justify your answer.

$$12/27 = .444 \approx 44\%$$

They're independent to each other because they don't have the same proportion, it's only a 44% between them.

**Score 0:** The student did not show enough correct work to receive any credit.

### Question 37

37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

$$\begin{aligned}B P &= 2\pi & B &= .8\pi \\(P) \cdot .8\pi &= 2\pi \\P &= 2.5\end{aligned}$$

Interpret what the period represents in this context.

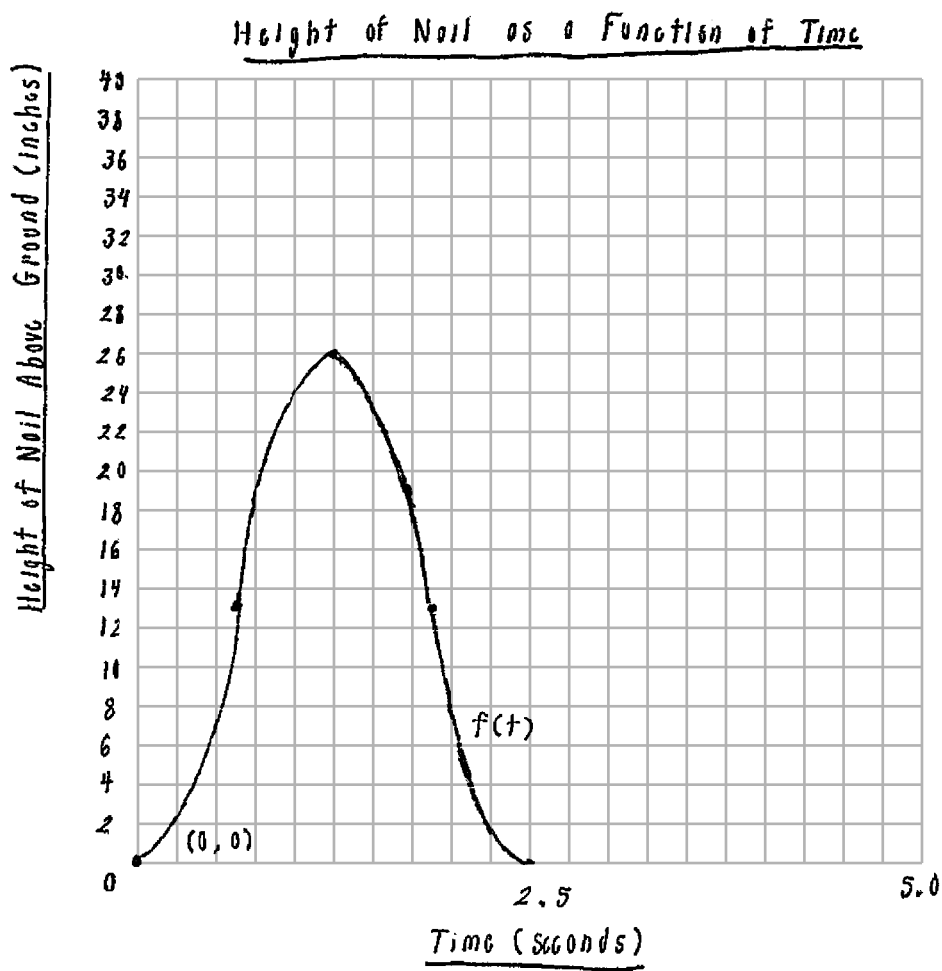
The period of  $f(t)$  represents the amount of time it would take the tire to spin one full rotation

**Question 37 is continued on the next page.**

**Score 6:** The student gave a complete and correct response.

Question 37 continued.

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No because at its max height, the tire can only reach 26 feet, as proven through adding the absolute value of a (13) to  $d$  (the midline, 13).

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**Question 37**

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**37** Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

$$\begin{aligned} \text{period} &= \frac{2\pi}{B} \\ &= \frac{2\pi}{0.8\pi} \\ &= \underline{\underline{2.5 \text{ seconds}}} \end{aligned}$$

Interpret what the period represents in this context.

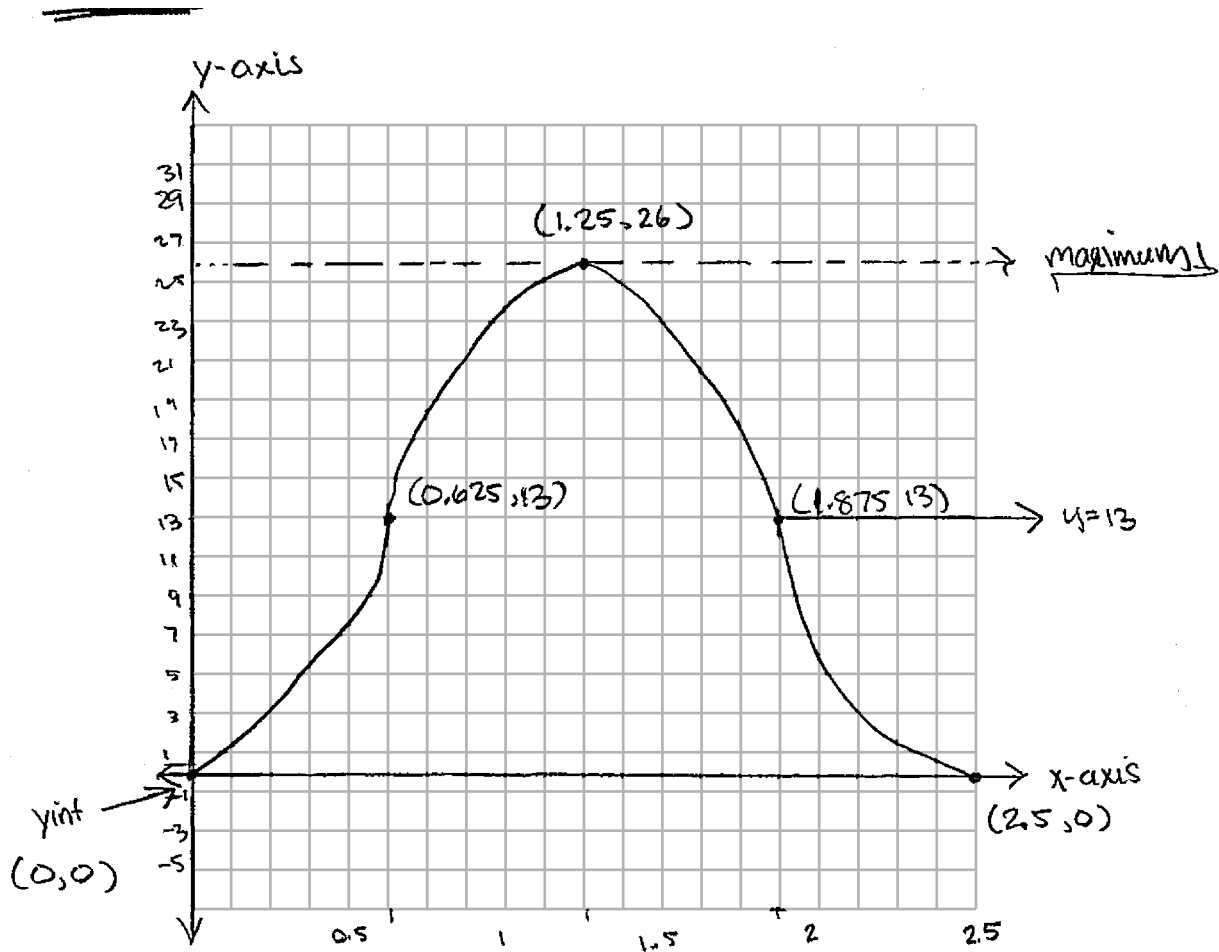
period is the time it takes for the nail to make one rotation

**Question 37 is continued on the next page.**

**Score 6:** The student gave a complete and correct response.

Question 37 continued.

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No, the maximum of the sinusoidal curve is 26.

**Question 37**

**37** Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

period is 2.5,

Interpret what the period represents in this context.

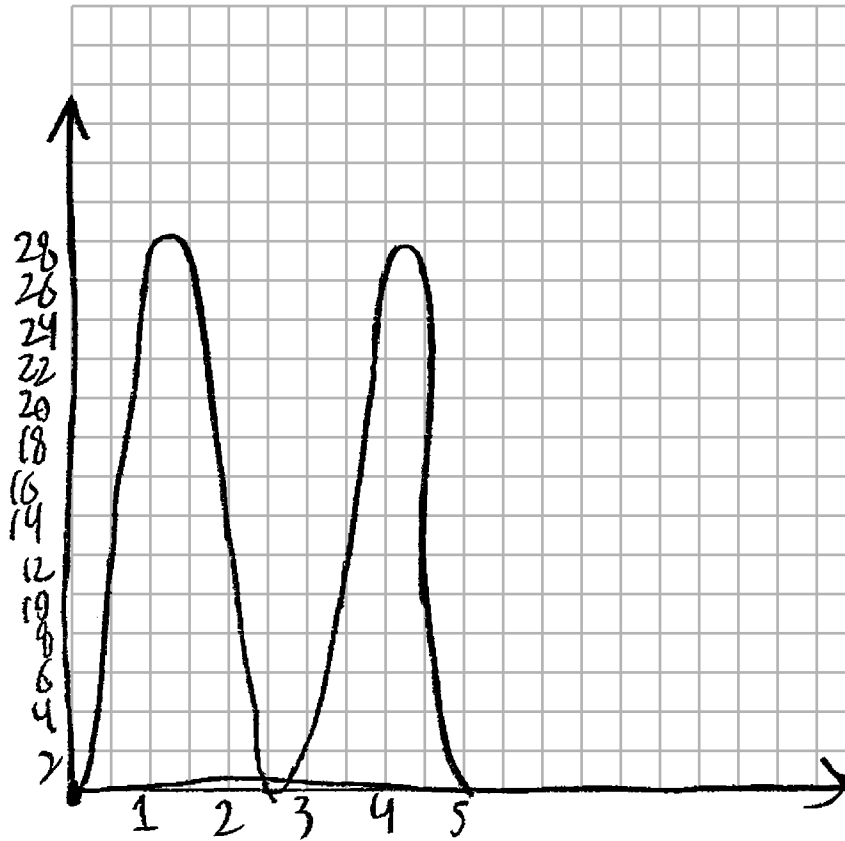
It takes 2.5 seconds for the nail  
to do a full revolution on the tire

**Question 37 is continued on the next page.**

**Score 5:** The student made one graphing error.

Question 37 continued.

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No, it peaks at 28 inches



**Question 37**

**37** Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

$$\frac{P(0.8\pi)}{0.8\pi} = \frac{2\pi}{0.8\pi}$$
$$P = 2.5$$
$$P_b = 2$$

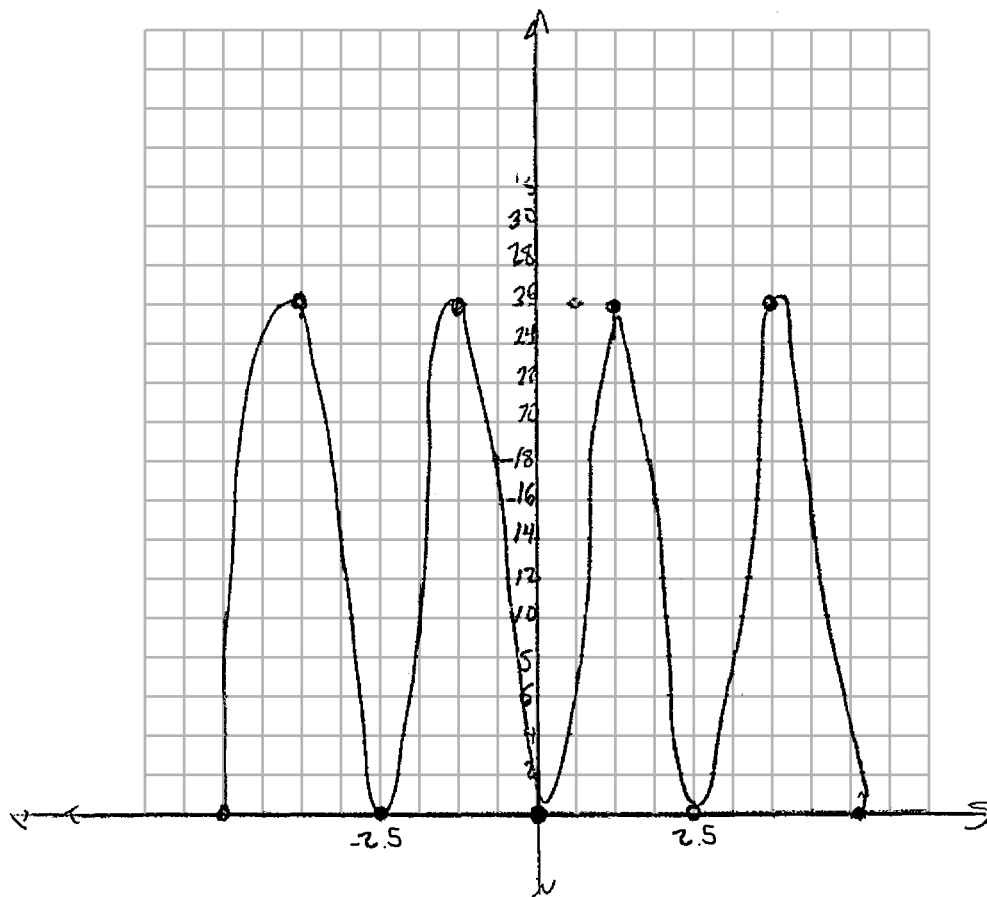
Interpret what the period represents in this context.

**Question 37 is continued on the next page.**

**Score 4:** The student did not interpret the period and gave an incomplete justification in the last part.

Question 37 continued.

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No it does not because the cosine graphs amplitude is 13 and the midline is 13

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**Question 37**

---

**37** Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

$$\frac{2\pi}{0.8\pi} = 2.5 \text{ seconds,}$$

Interpret what the period represents in this context.

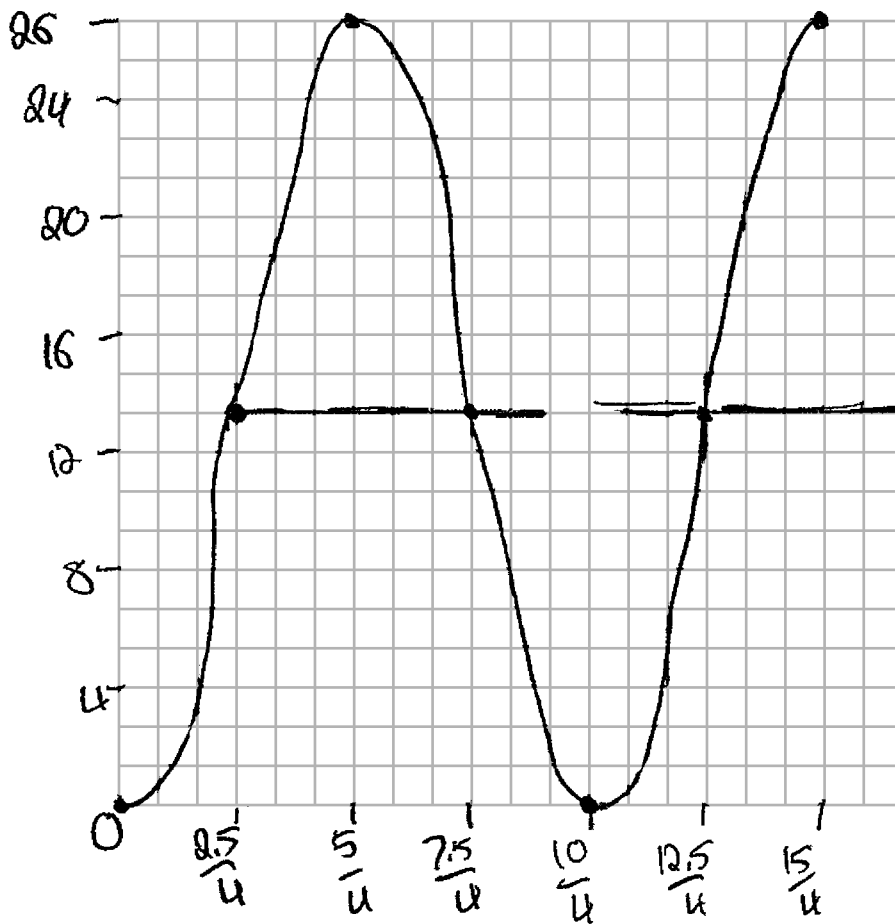
It takes 2.5 seconds  
for the nail to go  
from highpoint back to  
highpoint.

**Question 37 is continued on the next page.**

**Score 3:** The student made a labeling error on the graph and did not answer the last part.

Question 37 continued.

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



$$\begin{aligned}
 a &= -13 \\
 b &= .8\pi \\
 \text{mid} &= 13 \\
 \text{period} &= 2.5 \\
 \text{interval} &= \frac{2.5}{4} \\
 \text{Max} &= 26 \\
 \text{mid} &= 0
 \end{aligned}$$

Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

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**Question 37**

---

**37** Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

The period is 0.8 second, this represents how many seconds it takes for the nail to reach the original height it came stuck at.

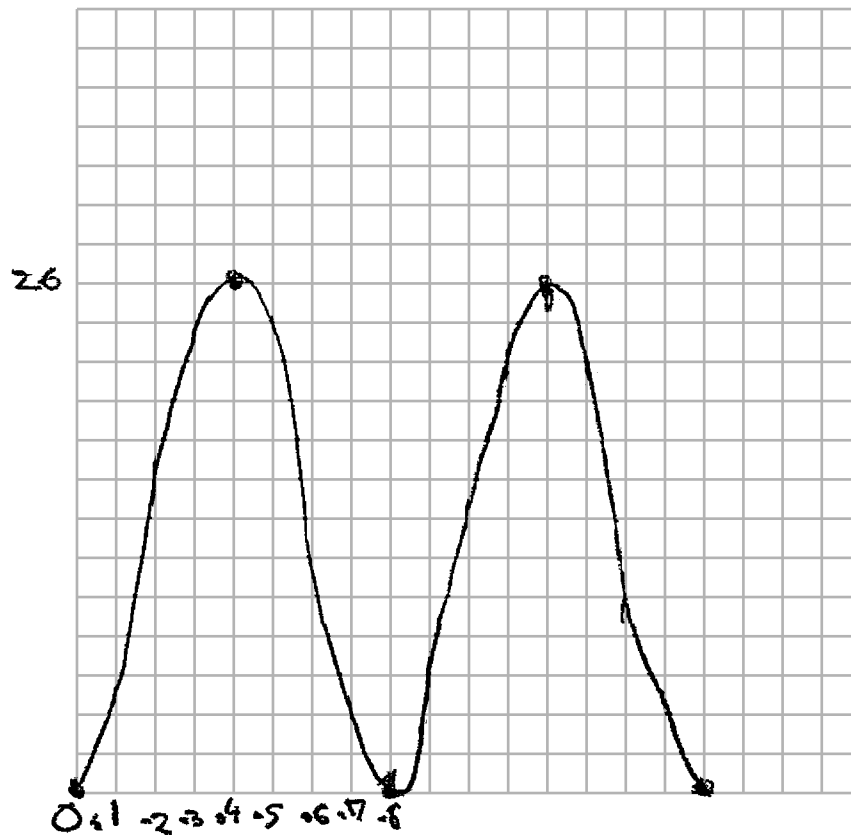
Interpret what the period represents in this context.

**Question 37 is continued on the next page.**

**Score 3:** The student gave a correct interpretation based on an incorrect period and received full credit for the graph.

**Question 37 continued.**

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

**Question 37**

37 Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

$$0.8\pi(p) = 2\pi$$
$$p = 2.5$$
$$\frac{2\pi}{1} \times \frac{5}{4\pi} = \frac{5}{2}$$

Interpret what the period represents in this context.

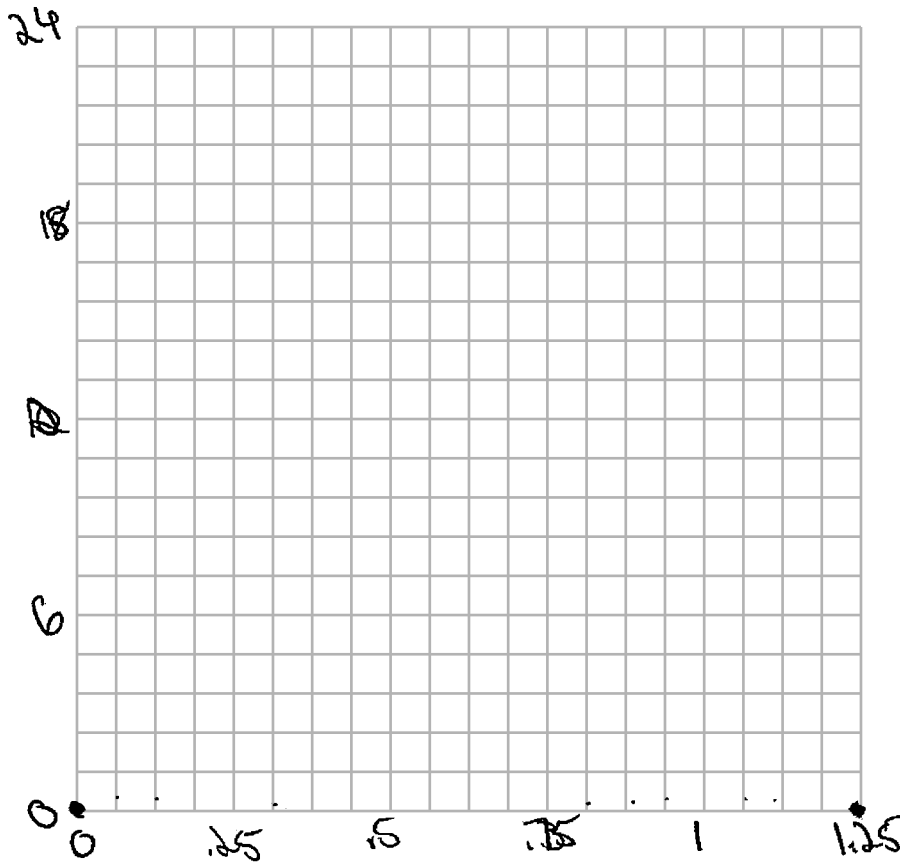
If takes 2.5  
seconds for the  
nail to complete  
1 full rotation

**Question 37 is continued on the next page.**

**Score 2:** The student received credit for the period and the interpretation.

**Question 37 continued.**

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.



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**Question 37**

---

**37** Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

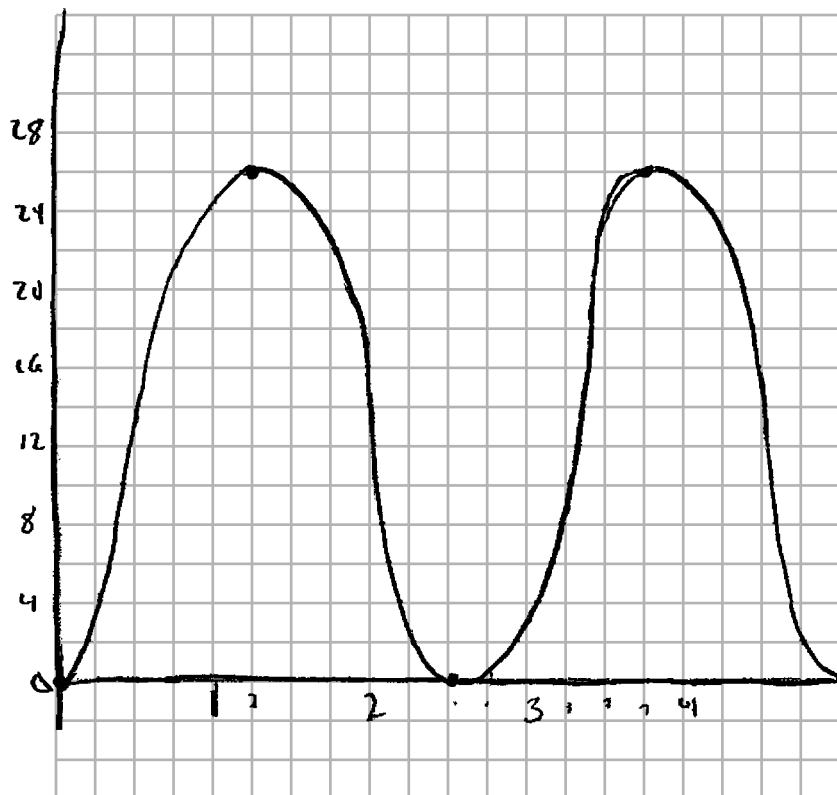
Interpret what the period represents in this context.

**Question 37 is continued on the next page.**

**Score 2:** The student drew a correct graph.

Question 37 continued.

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No, the nail would not be short enough to go in the fire it would just tip over

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**Question 37**

---

**37** Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

$$\text{Period} = \frac{2\pi}{\text{frequency}}$$

$$\begin{aligned} \text{Period} &= \frac{2\pi}{0.8\pi} \\ &= 2.5 \end{aligned}$$

Interpret what the period represents in this context.

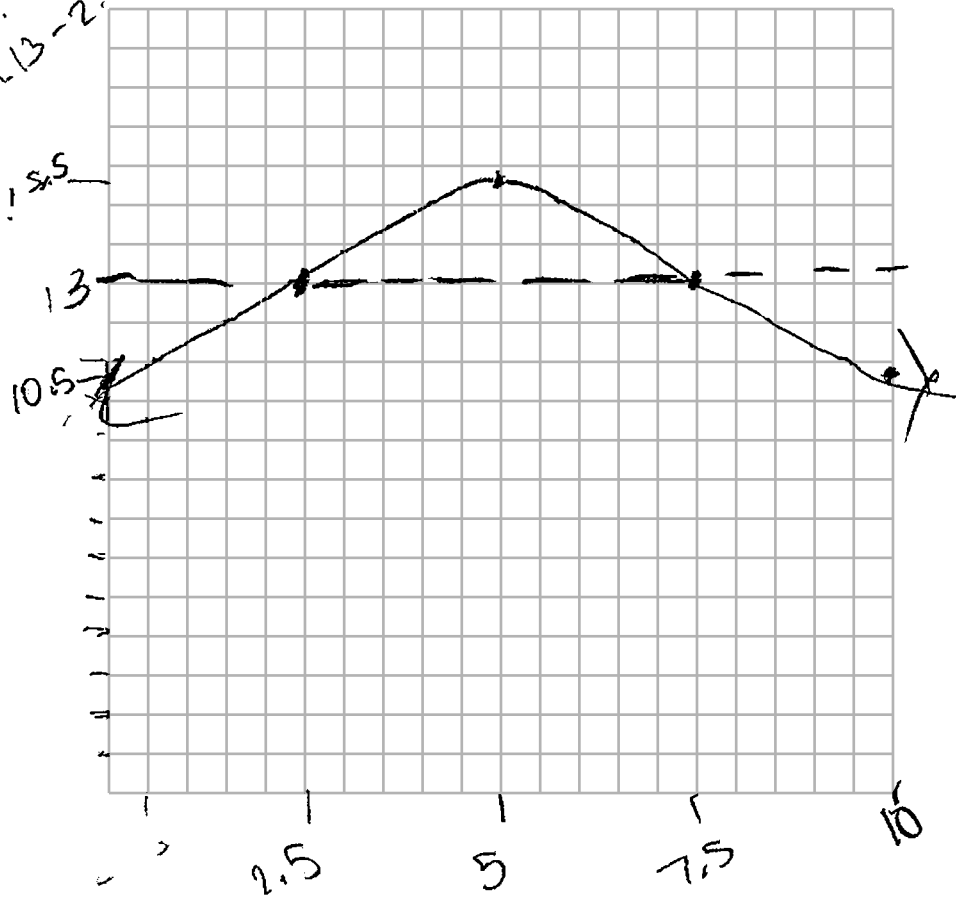
**Question 37 is continued on the next page.**

**Score 1:** The student received credit for correctly finding the period.

Question 37 continued.

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.

max:  $13 + 2.5 = 15.5$   
min:  $13 - 2.5 = 10.5$



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

NO

---

**Question 37**

---

**37** Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

Interpret what the period represents in this context.

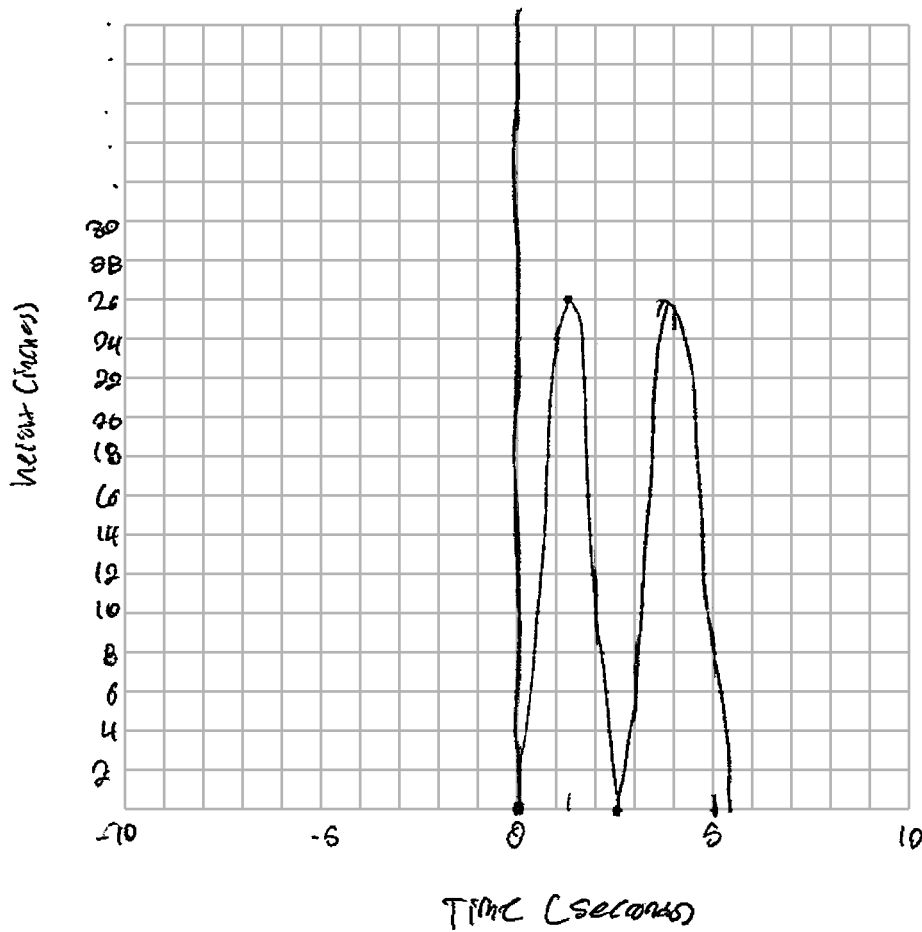
$f(t)$  represents the cycle of the wheel  
like how high the nail is from the ground.

**Question 37 is continued on the next page.**

**Score 1:** The student received one credit for the graph.

Question 37 continued.

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

---

**Question 37**

---

**37** Griffin is riding his bike down the street in Churchville, N.Y. at a constant speed, when a nail gets caught in one of his tires. The height of the nail above the ground, in inches, can be represented by the trigonometric function  $f(t) = -13\cos(0.8\pi t) + 13$ , where  $t$  represents the time (in seconds) since the nail first became caught in the tire.

Determine the period of  $f(t)$ .

Interpret what the period represents in this context.

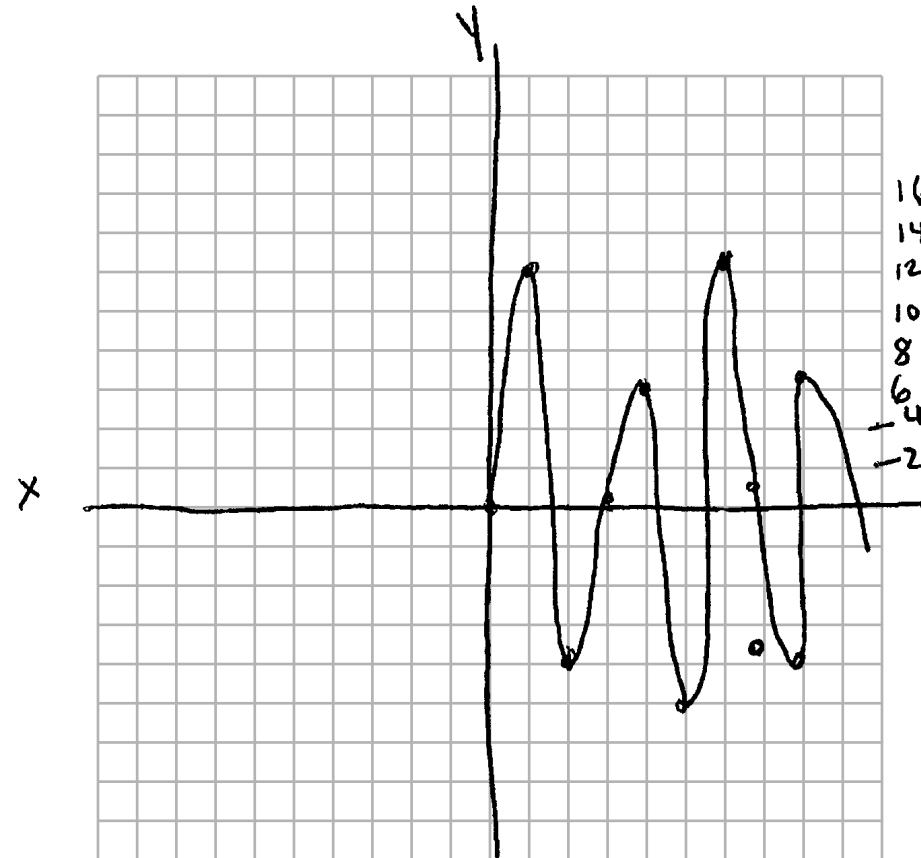
$f(t)$  represents one cycle of the wheel.  
and how high the nail is from the ground.

**Question 37 is continued on the next page.**

**Score 0:** The student did not show enough correct work to receive any credit.

Question 37 continued.

On the grid below, graph *at least one* cycle of  $f(t)$  that includes the  $y$ -intercept of the function.



Does the height of the nail ever reach 30 inches above the ground? Justify your answer.

No, the maximum height  
is 23.5 inches.



## Regents Examination in Algebra II – June 2019

Chart for Converting Total Test Raw Scores to Final Exam Scores (Scale Scores)

(Use for the June 2019 exam only.)

Raw Score	Scale Score	Performance Level	Raw Score	Scale Score	Performance Level	Raw Score	Scale Score	Performance Level
86	100	5	57	82	4	28	67	3
85	99	5	56	82	4	27	66	3
84	98	5	55	81	4	26	65	3
83	97	5	54	81	4	25	63	2
82	96	5	53	81	4	24	62	2
81	95	5	52	80	4	23	61	2
80	94	5	51	80	4	22	59	2
79	93	5	50	80	4	21	57	2
78	92	5	49	79	4	20	55	2
77	92	5	48	79	4	19	54	1
76	91	5	47	79	4	18	52	1
75	90	5	46	78	4	17	51	1
74	90	5	45	78	4	16	49	1
73	89	5	44	77	3	15	46	1
72	89	5	43	77	3	14	44	1
71	88	5	42	77	3	13	42	1
70	88	5	41	76	3	12	39	1
69	87	5	40	76	3	11	37	1
68	87	5	39	75	3	10	34	1
67	86	5	38	75	3	9	31	1
66	86	5	37	74	3	8	28	1
65	86	5	36	74	3	7	25	1
64	85	5	35	73	3	6	22	1
63	84	4	34	72	3	5	19	1
62	84	4	33	71	3	4	15	1
61	84	4	32	71	3	3	12	1
60	83	4	31	70	3	2	8	1
59	83	4	30	69	3	1	4	1
58	82	4	29	68	3	0	0	1

To determine the student’s final examination score (scale score), find the student’s total test raw score in the column labeled “Raw Score” and then locate the scale score that corresponds to that raw score. The scale score is the student’s final examination score. Enter this score in the space labeled “Scale Score” on the student’s answer sheet.

**Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.**

Because scale scores corresponding to raw scores in the conversion chart change from one administration to another, it is crucial that for each administration the conversion chart provided for that administration be used to determine the student’s final score. The chart above is usable only for this administration of the Regents Examination in Algebra II.