The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II (Common Core)

Wednesday, June 1, 2016 — 9:15 a.m. to 12:15 p.m., only

Student Name:_____

School Name:_____

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 When $b > 0$ and d is a positive integer, the expression $(3b)^{\frac{2}{d}}$ is equivalent to

(1) $\dfrac{1}{\left(\sqrt[d]{3b}\right)^2}$

(2) $\left(\sqrt{3b}\right)^d$

(3) $\dfrac{1}{\sqrt{3b^d}}$

(4) $\left(\sqrt[d]{3b}\right)^2$

2 Julie averaged 85 on the first three tests of the semester in her mathematics class. If she scores 93 on each of the remaining tests, her average will be 90. Which equation could be used to determine how many tests, T, are left in the semester?

(1) $\dfrac{255 + 93T}{3T} = 90$

(2) $\dfrac{255 + 90T}{3T} = 93$

(3) $\dfrac{255 + 93T}{T + 3} = 90$

(4) $\dfrac{255 + 90T}{T + 3} = 93$

3 Given i is the imaginary unit, $(2 - yi)^2$ in simplest form is

(1) $y^2 - 4yi + 4$

(2) $-y^2 - 4yi + 4$

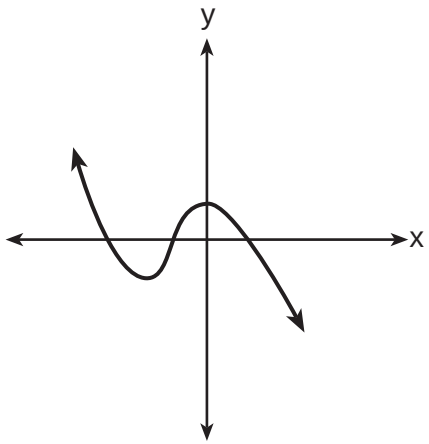
(3) $-y^2 + 4$

(4) $y^2 + 4$

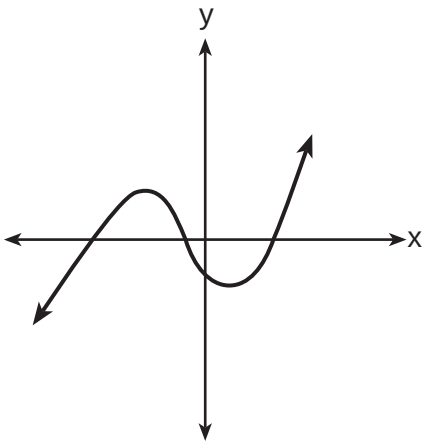
Use this space for computations.

4 Which graph has the following characteristics?

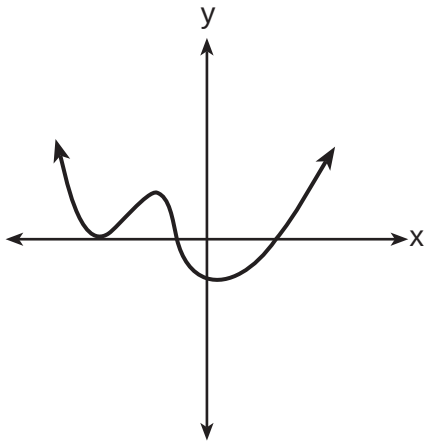
- three real zeros
- as $x \to -\infty$, $f(x) \to -\infty$
- as $x \to \infty$, $f(x) \to \infty$



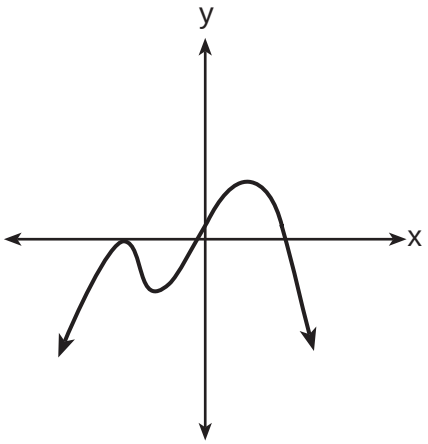
(1)



(3)



(2)



(4)

5 The solution set for the equation $\sqrt{56 - x} = x$ is

(1) $\{-8,7\}$

(2) $\{-7,8\}$

(3) $\{7\}$

(4) $\{\ \}$

Use this space for computations.

6 The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are

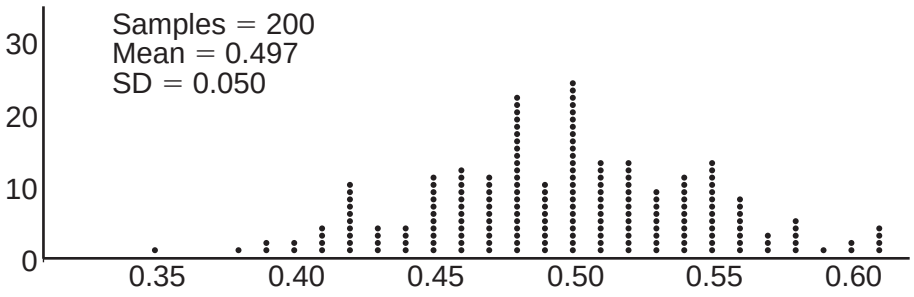
(1) $\{0,\pm 3,4\}$

(2) $\{0,3,4\}$

(3) $\{0,\pm 3,-4\}$

(4) $\{0,3,-4\}$

7 Anne has a coin. She does not know if it is a fair coin. She flipped the coin 100 times and obtained 73 heads and 27 tails. She ran a computer simulation of 200 samples of 100 fair coin flips. The output of the proportion of heads is shown below.



Given the results of her coin flips and of her computer simulation, which statement is most accurate?

(1) 73 of the computer's next 100 coin flips will be heads.

(2) 50 of her next 100 coin flips will be heads.

(3) Her coin is not fair.

(4) Her coin is fair.

8 If $g(c) = 1 - c^2$ and $m(c) = c + 1$, then which statement is *not* true?

(1) $g(c) \bullet m(c) = 1 + c - c^2 - c^3$

(2) $g(c) + m(c) = 2 + c - c^2$

(3) $m(c) - g(c) = c + c^2$

(4) $\frac{m(c)}{g(c)} = \frac{-1}{1-c}$

Use this space for computations.

9 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the *nearest whole percent,* is

(1) 6
(2) 48
(3) 68
(4) 95

10 The formula below can be used to model which scenario?

$$a_1 = 3000$$
$$a_n = 0.80a_{n-1}$$

(1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.

(2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.

(3) A bank account starts with a deposit of $3000, and each year it grows by 80%.

(4) The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

11 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

(1) independent
(2) dependent
(3) mutually exclusive
(4) complements

Use this space for computations.

12 A solution of the equation $2x^2 + 3x + 2 = 0$ is

(1) $-\frac{3}{4} + \frac{1}{4}i\sqrt{7}$

(2) $-\frac{3}{4} + \frac{7}{4}i$

(3) $-\frac{3}{4} + \frac{1}{4}\sqrt{7}$

(4) $\frac{1}{2}$

13 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, H, in feet, above the ground of one of the six-person cars can be modeled by $H(t) = 70 \sin\left(\frac{2\pi}{7}(t - 1.75)\right) + 80$, where t is time, in minutes. Using $H(t)$ for one full rotation, this car's minimum height, in feet, is

(1) 150

(2) 70

(3) 10

(4) 0

14 The expression $\frac{4x^3 + 5x + 10}{2x + 3}$ is equivalent to

(1) $2x^2 + 3x - 7 + \frac{31}{2x + 3}$

(2) $2x^2 - 3x + 7 - \frac{11}{2x + 3}$

(3) $2x^2 + 2.5x + 5 + \frac{15}{2x + 3}$

(4) $2x^2 - 2.5x - 5 - \frac{20}{2x + 3}$

15 Which function represents exponential decay?

(1) $y = 2^{0.3t}$

(2) $y = 1.2^{3t}$

(3) $y = \left(\frac{1}{2}\right)^{-t}$

(4) $y = 5^{-t}$

Use this space for computations.

16 Given $f^{-1}(x) = -\frac{3}{4}x + 2$, which equation represents $f(x)$?

(1) $f(x) = \frac{4}{3}x - \frac{8}{3}$

(2) $f(x) = -\frac{4}{3}x + \frac{8}{3}$

(3) $f(x) = \frac{3}{4}x - 2$

(4) $f(x) = -\frac{3}{4}x + 2$

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of $\cos\theta$?

(1) $-\frac{3}{5}$

(2) $-\frac{3}{4}$

(3) $\frac{3}{5}$

(4) $\frac{4}{5}$

18 Which statement about the graph of $c(x) = \log_6 x$ is *false*?

(1) The asymptote has equation $y = 0$.

(2) The graph has no y-intercept.

(3) The domain is the set of positive reals.

(4) The range is the set of all real numbers.

19 The equation $4x^2 - 24x + 4y^2 + 72y = 76$ is equivalent to

(1) $4(x - 3)^2 + 4(y + 9)^2 = 76$

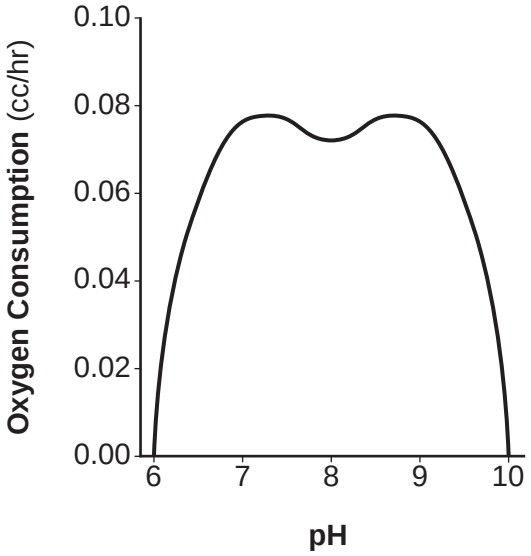
(2) $4(x - 3)^2 + 4(y + 9)^2 = 121$

(3) $4(x - 3)^2 + 4(y + 9)^2 = 166$

(4) $4(x - 3)^2 + 4(y + 9)^2 = 436$

Use this space for computations.

20 There was a study done on oxygen consumption of snails as a function of pH, and the result was a degree 4 polynomial function whose graph is shown below.



Which statement about this function is *incorrect*?

(1) The degree of the polynomial is even.

(2) There is a positive leading coefficient.

(3) At two pH values, there is a relative maximum value.

(4) There are two intervals where the function is decreasing.

21 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let m represent months.]

(1) $(1.0525)^m$

(2) $(1.0525)^{\frac{12}{m}}$

(3) $(1.00427)^m$

(4) $(1.00427)^{\frac{m}{12}}$

Use this space for computations.

22 Which value, to the *nearest tenth*, is *not* a solution of $p(x) = q(x)$ if $p(x) = x^3 + 3x^2 - 3x - 1$ and $q(x) = 3x + 8$?

(1) -3.9

(2) -1.1

(3) 2.1

(4) 4.7

23 The population of Jamesburg for the years 2010 – 2013, respectively, was reported as follows:

250,000 250,937 251,878 252,822

How can this sequence be recursively modeled?

(1) $j_n = 250{,}000(1.00375)^{n-1}$

(2) $j_n = 250{,}000 + 937^{(n-1)}$

(3) $j_1 = 250{,}000$
$j_n = 1.00375\, j_{n-1}$

(4) $j_1 = 250{,}000$
$j_n = j_{n-1} + 937$

24 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles *every second*. Which equation best represents the value of the voltage as it flows through the electric wires, where t is time in seconds?

(1) $V = 120 \sin(t)$

(2) $V = 120 \sin(60t)$

(3) $V = 120 \sin(60\pi t)$

(4) $V = 120 \sin(120\pi t)$

Part II

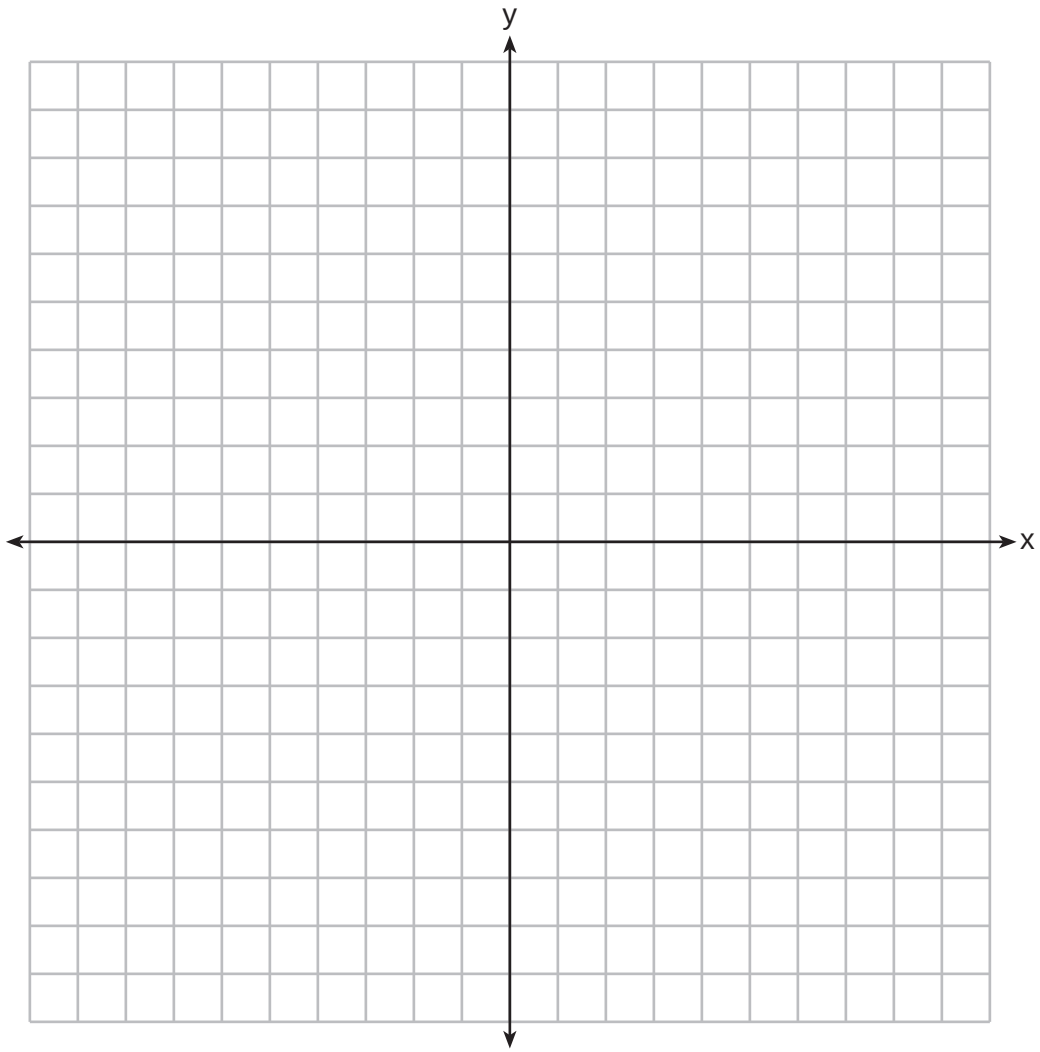
Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

31 Algebraically prove that $\frac{x^3 + 9}{x^3 + 8} = 1 + \frac{1}{x^3 + 8}$, where $x \neq -2$.

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

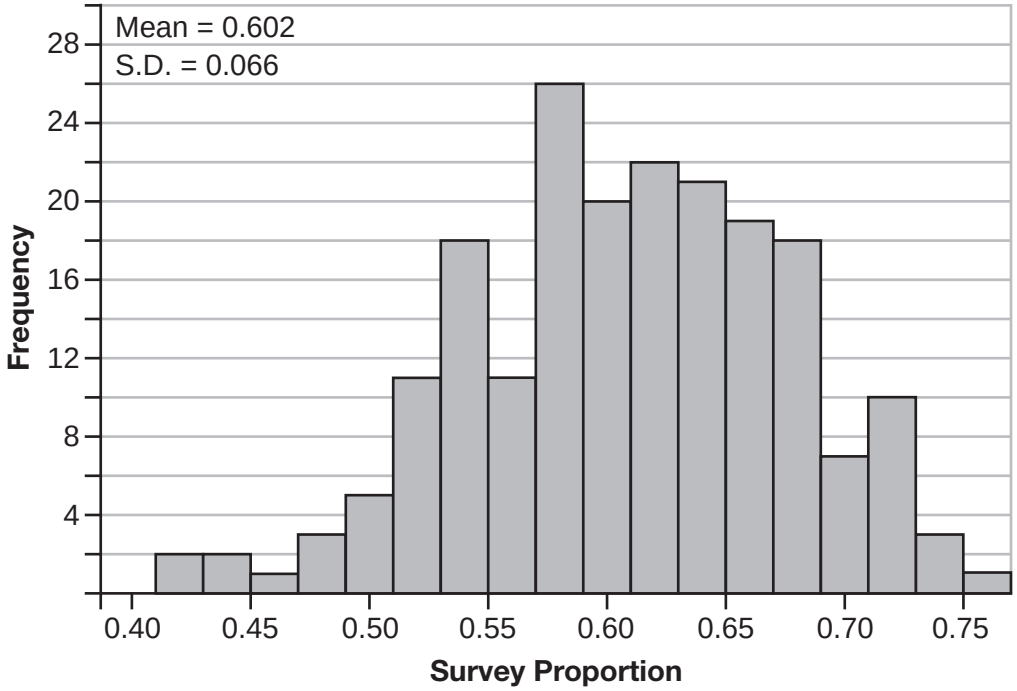
34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

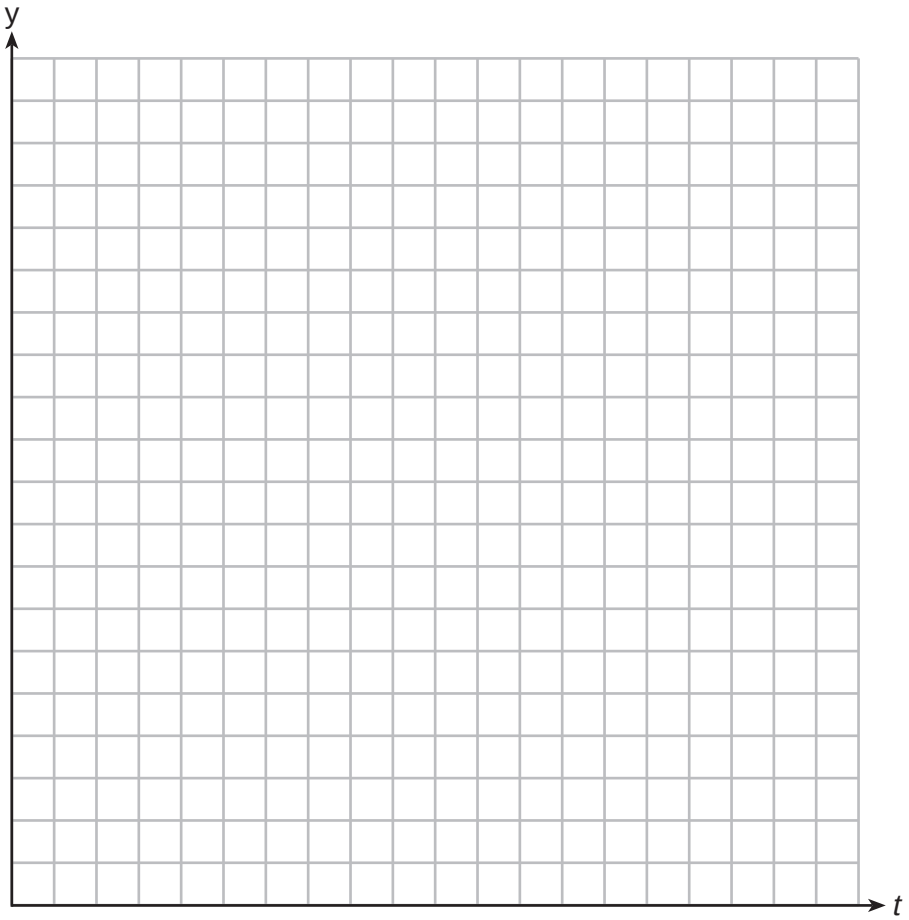
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

Graph each function on the set of axes below.



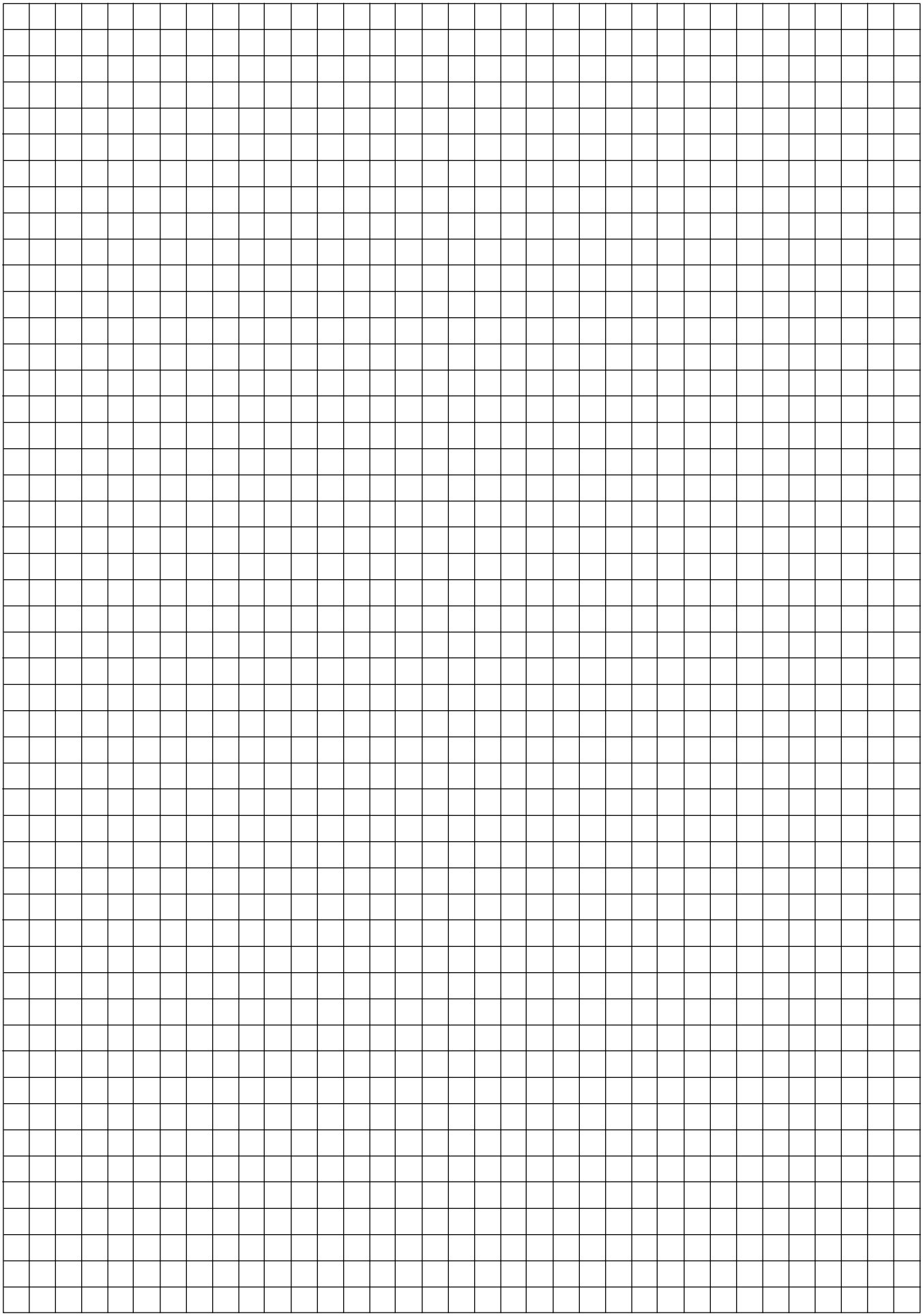
To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

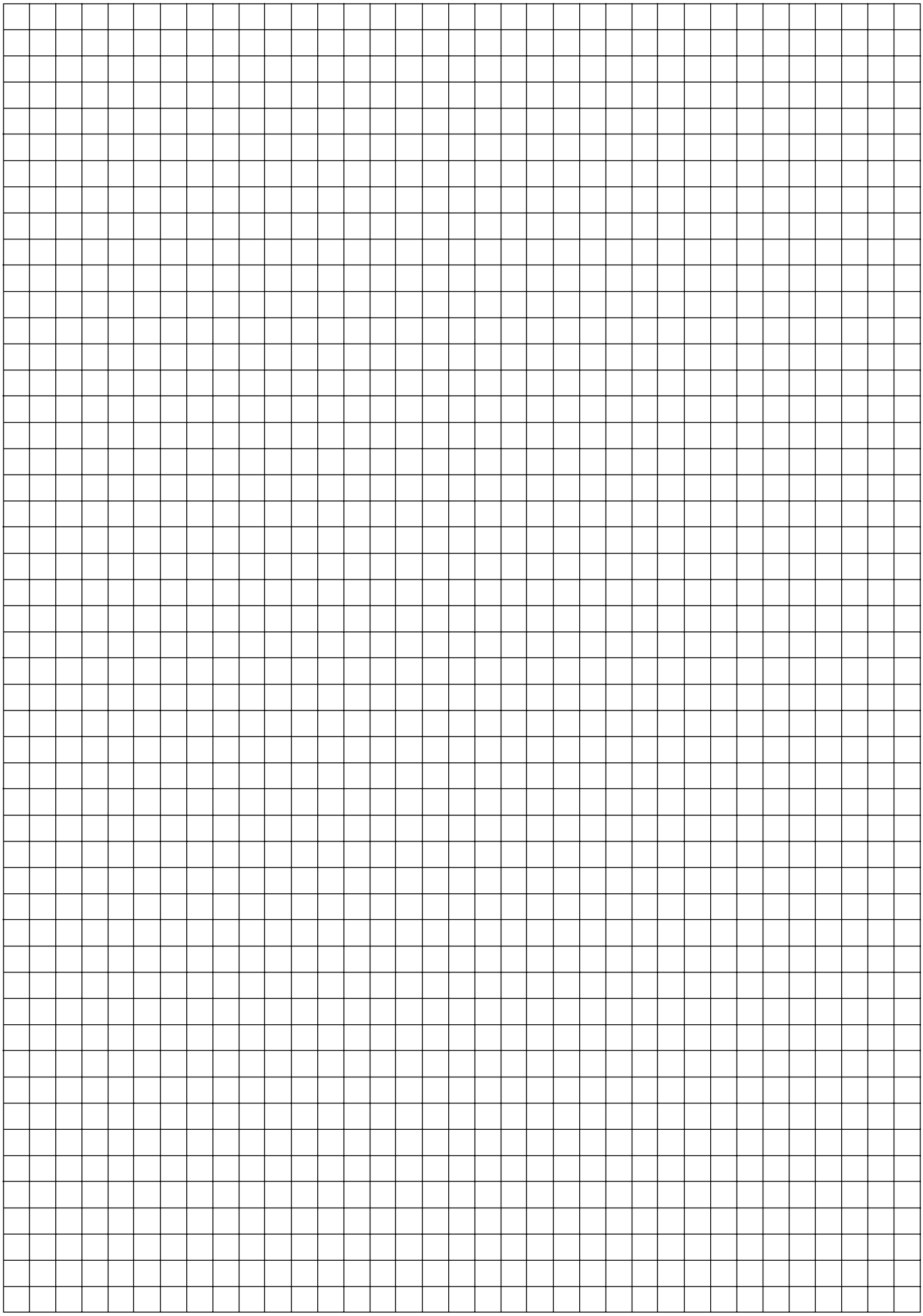
Tear Here

Tear Here

Scrap Graph Paper — This sheet will *not* be scored.



Scrap Graph Paper — This sheet will *not* be scored.



Tear Here

Tear Here

Tear Here

High School Math Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilogram
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallon
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n - 1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

Tear Here

ALGEBRA II (COMMON CORE)

FOR TEACHERS ONLY

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II (Common Core)

Wednesday, June 1, 2016 — 9:15 a.m. to 12:15 p.m., only

SCORING KEY AND RATING GUIDE

Mechanics of Rating

The following procedures are to be followed for scoring student answer papers for the Regents Examination in Algebra II (Common Core). More detailed information about scoring is provided in the publication *Information Booklet for Scoring the Regents Examination in Algebra II (Common Core)*.

Do *not* attempt to correct the student's work by making insertions or changes of any kind. In scoring the constructed-response questions, use check marks to indicate student errors. Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Each student's answer paper is to be scored by a minimum of three mathematics teachers. No one teacher is to score more than approximately one-third of the constructed-response questions on a student's paper. Teachers may not score their own students' answer papers. On the student's separate answer sheet, for each question, record the number of credits earned and the teacher's assigned rater/scorer letter.

Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Raters should record the student's scores for all questions and the total raw score on the student's separate answer sheet. Then the student's total raw score should be converted to a scale score by using the conversion chart that will be posted on the Department's web site at: http://www.p12.nysed.gov/assessment/ by Thursday, June 23, 2016. Because scale scores corresponding to raw scores in the conversion chart may change from one administration to another, it is crucial that, for each administration, the conversion chart provided for that administration be used to determine the student's final score. The student's scale score should be entered in the box provided on the student's separate answer sheet. The scale score is the student's final examination score.

If the student's responses for the multiple-choice questions are being hand scored prior to being scanned, the scorer must be careful not to make any marks on the answer sheet except to record the scores in the designated score boxes. Marks elsewhere on the answer sheet will interfere with the accuracy of the scanning.

Part I

Allow a total of 48 credits, 2 credits for each of the following.

(1) 4	(9) 2	(17) . . . . . 1 . . . . .
(2) 3	(10) 4	(18) . . . . . 1 . . . . .
(3) 2	(11) 1	(19) . . . . . 4 . . . . .
(4) 3	(12) 1	(20) . . . . . 2 . . . . .
(5) 3	(13) 3	(21) . . . . . 3 . . . . .
(6) 1	(14) 2	(22) . . . . . 4 . . . . .
(7) 3	(15) 4	(23) . . . . . 3 . . . . .
(8) 4	(16) 2	(24) . . . . . 4 . . . . .

Updated information regarding the rating of this examination may be posted on the New York State Education Department's web site during the rating period. Check this web site at: http://www.p12.nysed.gov/assessment/ and select the link "Scoring Information" for any recently posted information regarding this examination. This site should be checked before the rating process for this examination begins and several times throughout the Regents Examination period.

The Department is providing supplemental scoring guidance, the "Model Response Set," for the Regents Examination in Algebra II (Common Core). This guidance is recommended to be part of the scorer training. Schools are encouraged to incorporate the Model Response Sets into the scorer training or to use them as additional information during scoring. While not reflective of all scenarios, the model responses selected for the Model Response Set illustrate how less common student responses to constructed-response questions may be scored. The Model Response Set will be available on the Department's web site at http://www.nysedregents.org/algebratwo/.

General Rules for Applying Mathematics Rubrics

I. General Principles for Rating

The rubrics for the constructed-response questions on the Regents Examination in Algebra II (Common Core) are designed to provide a systematic, consistent method for awarding credit. The rubrics are not to be considered all-inclusive; it is impossible to anticipate all the different methods that students might use to solve a given problem. Each response must be rated carefully using the teacher's professional judgment and knowledge of mathematics; all calculations must be checked. The specific rubrics for each question must be applied consistently to all responses. In cases that are not specifically addressed in the rubrics, raters must follow the general rating guidelines in the publication *Information Booklet for Scoring the Regents Examination in Algebra II (Common Core)*, use their own professional judgment, confer with other mathematics teachers, and/or contact the State Education Department for guidance. During each Regents Examination administration period, rating questions may be referred directly to the Education Department. The contact numbers are sent to all schools before each administration period.

II. Full-Credit Responses

A full-credit response provides a complete and correct answer to all parts of the question. Sufficient work is shown to enable the rater to determine how the student arrived at the correct answer.

When the rubric for the full-credit response includes one or more examples of an acceptable method for solving the question (usually introduced by the phrase "such as"), it does not mean that there are no additional acceptable methods of arriving at the correct answer. Unless otherwise specified, mathematically correct alternative solutions should be awarded credit. The only exceptions are those questions that specify the type of solution that must be used; e.g., an algebraic solution or a graphic solution. A correct solution using a method other than the one specified is awarded half the credit of a correct solution using the specified method.

III. Appropriate Work

Full-Credit Responses: The directions in the examination booklet for all the constructed-response questions state: "Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc." The student has the responsibility of providing the correct answer ***and*** showing how that answer was obtained. The student must "construct" the response; the teacher should not have to search through a group of seemingly random calculations scribbled on the student paper to ascertain what method the student may have used.

Responses With Errors: Rubrics that state "Appropriate work is shown, but…" are intended to be used with solutions that show an essentially complete response to the question but contain certain types of errors, whether computational, rounding, graphing, or conceptual. If the response is incomplete; i.e., an equation is written but not solved or an equation is solved but not all of the parts of the question are answered, appropriate work has ***not*** been shown. Other rubrics address incomplete responses.

IV. Multiple Errors

Computational Errors, Graphing Errors, and Rounding Errors: Each of these types of errors results in a 1-credit deduction. Any combination of two of these types of errors results in a 2-credit deduction. No more than 2 credits should be deducted for such mechanical errors in a 4-credit question and no more than 3 credits should be deducted in a 6-credit question. The teacher must carefully review the student's work to determine what errors were made and what type of errors they were.

Conceptual Errors: A conceptual error involves a more serious lack of knowledge or procedure. Examples of conceptual errors include using the incorrect formula for the area of a figure, choosing the incorrect trigonometric function, or multiplying the exponents instead of adding them when multiplying terms with exponents.

If a response shows repeated occurrences of the same conceptual error, the student should not be penalized twice. If the same conceptual error is repeated in responses to other questions, credit should be deducted in each response.

For 4- and 6-credit questions, if a response shows one conceptual error and one computational, graphing, or rounding error, the teacher must award credit that takes into account both errors. Refer to the rubric for specific scoring guidelines.

Part II

For each question, use the specific criteria to award a maximum of 2 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(25) **[2]** 4, and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 4, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(26) **[2]** A correct description of a controlled experiment is written, such as indicating two randomly assigned groups, one with ingredient X and one without ingredient X.

[1] One conceptual error is made.

or

[1] An incomplete description of a controlled experiment is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(27) **[2]** No, and correct work is shown, and a correct explanation is written.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] Correct work is shown, but no explanation or an incorrect explanation is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(28) **[2]** A correct graph is drawn.

[1] One graphing error is made.

or

[1] One conceptual error is made, such as graphing more than one cycle.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(29) **[2]** $\frac{108}{1376}$ or an equivalent fraction, and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] $\frac{108}{1376}$, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(30) **[2]** (4,0), and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] (4,0), but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(31) **[2]** A correct algebraic proof is shown.

[1] Appropriate work is shown, but one computational or simplification error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(32) **[2]** 6, and correct work is shown.

[1] Appropriate work is shown, but one computational or rounding error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 6, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Part III

For each question, use the specific criteria to award a maximum of 4 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(33) **[4]** (7,−2) and (3,2) or equivalent solutions, and correct algebraic work is shown.

[3] Appropriate work is shown, but one computational, factoring, or substitution error is made.

or

[3] Appropriate work is shown, but only one correct solution is found or only the x-values or y-values are found.

[2] Appropriate work is shown, but two or more computational, factoring, or substitution errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] A correct substitution into the quadratic formula is made, but no further correct work is shown.

or

[2] (7,−2) and (3,2), but a method other than algebraic is used.

[1] Appropriate work is shown, but one conceptual error and one computational, factoring, or substitution error are made.

or

[1] A correct quadratic equation in one variable is written, but no further correct work is shown.

or

[1] (7,−2) and (3,2), but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(34) **[4]** $S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1-1.04}$ or an equivalent equation is written and 660,778.39, and correct work is shown.

[3] Appropriate work is shown, but one computational or simplification error is made.

or

[3] Appropriate work is shown, but one notation error is made, such as writing the expression $\frac{33{,}000 - 33{,}000(1.04)^n}{1-1.04}$, or not using n.

[2] Appropriate work is shown, but two or more computational, notation, or simplification errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] $S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1-1.04}$, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual error and one computational, notation, or simplification error are made.

or

[1] 660,778.39, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(35) **[4]** (0.47, 0.73) and correct work is shown, and a correct statistical explanation is written.

[3] Appropriate work is shown, but one computational error is made.

or

[3] Correct work is shown to find (0.47, 0.73), but the explanation is incomplete or nonstatistical.

[2] Appropriate work is shown, but two or more computational errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find (0.47, 0.73), but no further correct work is shown.

or

[2] A correct statistical explanation is written, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual error and one computational error are made.

or

[1] (0.47, 0.73), but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

(36) **[4]** *g*, and a correct justification is given.

[3] Appropriate work is shown, but one computational error is made.

or

[3] Correct rates of change are computed, but no function is indicated.

[2] Appropriate work is shown, but two or more computational errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find the rate of change for f is 13.125 or the rate of change for g is 38, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual error and one computational error are made.

or

[1] 13.125 and 38, but no work is shown.

[0] 13.125 or 38, but the rates of change are not labeled, and no further correct work is shown.

or

[0] *g*, but no work is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Part IV

For this question, use the specific criteria to award a maximum of 6 credits. Unless otherwise specified, mathematically correct alternative solutions should be awarded appropriate credit.

(37) **[6]** $A(t) = 800e^{-0.347t}$ and $B(t) = 400e^{-0.231t}$, correct graphs are drawn and at least one is labeled, 6, 5.5, and correct work is shown.

[5] Appropriate work is shown, but one computational, graphing, labeling, or rounding error is made.

[4] Appropriate work is shown, but two computational, graphing, labeling, or rounding errors are made.

or

[4] Appropriate work is shown, but one conceptual error is made.

[3] Appropriate work is shown, but three or more computational, graphing, labeling, or rounding errors are made.

or

[3] Appropriate work is shown, but one conceptual error and one computational, graphing, labeling, or rounding error are made.

[2] Appropriate work is shown, but two conceptual errors are made.

or

[2] Correct graphs are drawn and at least one is labeled, but no further correct work is shown.

or

[2] 5.5 and correct work is shown, but no further correct work is shown.

or

[2] 6 and 5.5, but no work is shown.

[1] Appropriate work is shown, but two conceptual errors and one computational, graphing, labeling, or rounding errors are made.

or

[1] $A(t) = 800e^{-0.347t}$ and $B(t) = 400e^{-0.231t}$, but no further correct work is shown.

or

[1] $A(t)$ or $B(t)$ is graphed correctly, but no further correct work is shown.

or

[1] 6 or 5.5, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Map to the Common Core Learning Standards
Algebra II (Common Core)
June 2016

Question	Type	Credits	Cluster
1	Multiple Choice	2	N-RN.A
2	Multiple Choice	2	A-CED.A
3	Multiple Choice	2	N-CN.A
4	Multiple Choice	2	F-IF.C
5	Multiple Choice	2	A-REI.A
6	Multiple Choice	2	A-APR.B
7	Multiple Choice	2	S-IC.A
8	Multiple Choice	2	F-BF.A
9	Multiple Choice	2	S-ID.A
10	Multiple Choice	2	F-BF.A
11	Multiple Choice	2	S-CP.A
12	Multiple Choice	2	N-CN.C
13	Multiple Choice	2	F-IF.B
14	Multiple Choice	2	A-APR.D
15	Multiple Choice	2	F-IF.C
16	Multiple Choice	2	F-BF.B
17	Multiple Choice	2	F-TF.A
18	Multiple Choice	2	F-IF.C
19	Multiple Choice	2	A-SSE.A
20	Multiple Choice	2	F-IF.B

21	Multiple Choice	2	A-SSE.B
22	Multiple Choice	2	A-REI.D
23	Multiple Choice	2	F-BF.A
24	Multiple Choice	2	F-TF.B
25	Constructed Response	2	A-REI.A
26	Constructed Response	2	S-IC.B
27	Constructed Response	2	A-APR.B
28	Constructed Response	2	F-IF.C
29	Constructed Response	2	S-CP.B
30	Constructed Response	2	G-GPE.A
31	Constructed Response	2	A-SSE.A
32	Constructed Response	2	F-LE.A
33	Constructed Response	4	A-REI.C
34	Constructed Response	4	A-SSE.B
35	Constructed Response	4	S-IC.B
36	Constructed Response	4	F-IF.C
37	Constructed Response	6	A-REI.D

Regents Examination in Algebra II (Common Core)

June 2016

Chart for Converting Total Test Raw Scores to Final Examination Scores (Scale Scores)

The *Chart for Determining the Final Examination Score for the June 2016 Regents Examination in Algebra II (Common Core)* will be posted on the Department's web site at: http://www.p12.nysed.gov/assessment/ by Thursday, June 23, 2016.

Online Submission of Teacher Evaluations of the Test to the Department

Suggestions and feedback from teachers provide an important contribution to the test development process. The Department provides an online evaluation form for State assessments. It contains spaces for teachers to respond to several specific questions and to make suggestions. Instructions for completing the evaluation form are as follows:

1. Go to http://www.forms2.nysed.gov/emsc/osa/exameval/reexameval.cfm.

2. Select the test title.

3. Complete the required demographic fields.

4. Complete each evaluation question and provide comments in the space provided.

5. Click the SUBMIT button at the bottom of the page to submit the completed form.

New York State Testing Program

Regents Examination in Algebra II (Common Core)

Selected Questions with Annotations

June 2016

engage ny

Our Students. Their Moment.

THE STATE EDUCATION DEPARTMENT /
THE UNIVERSITY OF THE STATE OF NEW YORK / ALBANY, NY 12234

New York State Testing Program
Regents Examination in Algebra II (Common Core)
Selected Questions with Annotations

With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. In Spring 2014, New York State administered the first set of Regents Exams designed to assess student performance in accordance with the instructional shifts and the rigor demanded by the Common Core State Standards (CCSS). To aid in the transition to new tests, New York State released a number of resources including sample questions, test blueprints and specifications, and criteria for writing test questions. These resources can be found at http://www.engageny.org/resource/regents-exams.

New York State administered the first Algebra II (Common Core) Regents Exam in June 2016 and is now annotating a portion of the questions from this test available for review and use. These annotated questions will help students, families, educators, and the public better understand how the test has changed to assess the instructional shifts demanded by the Common Core and to assess the rigor required to ensure that all students are on track to college and career readiness.

Annotated Questions Are Teaching Tools

The annotated questions are intended to help students, families, educators, and the public understand how the Common Core is different. The annotated questions demonstrate the way the Common Core should drive instruction and how tests have changed to better assess student performance in accordance with the instructional shifts demanded by the Common Core. They are also intended to help educators identify how the rigor of the Regents Examinations can inform classroom instruction and local assessment. The annotations will indicate common student misunderstandings related to content clusters; educators should use these to help inform unit and lesson planning. In some cases, the annotations may offer insight into particular instructional elements (conceptual thinking, mathematical modeling) that align to the Common Core that may be used in curricular design. It should not be assumed, however, that a particular cluster will be measured with identical items in future assessments.

The annotated questions include both multiple-choice and constructed-response questions. With each multiple-choice question annotated, a commentary is available to demonstrate why the question measures the intended cluster. The rationales describe why the correct answer is correct and why the wrong answer choices are plausible but incorrect, based on common misconceptions or common procedural errors. While these rationales speak to a possible and likely reason for the selection of the incorrect option by the student, these rationales do not contain definitive statements as to why the student chose the incorrect option, or what we can infer about the knowledge and skills of the student based on the student's selection of an incorrect response. These multiple-choice questions are designed to assess student proficiency, not to diagnose specific misconceptions/errors with each and every incorrect option.

For each constructed-response question, there is a commentary describing how the question measures the intended cluster, plus sample student responses representing possible student errors or misconceptions at each possible score point.

The annotated questions do not represent the full spectrum of standards assessed on the State test, nor do they represent the full spectrum of how the Common Core should be taught and assessed in the classroom. Specific criteria for writing test questions as well as test information are available at http://www.engageny.org/resource/regents-exams.

Understanding Math Annotated Questions

All questions on the Regents Exam in Algebra II (Common Core) are designed to measure the Common Core Learning Standards identified by the PARCC Model Content Framework for Algebra II. More information about the relationship between the New York State Testing Program and PARCC can be found here: http://www.p12.nysed.gov/assessment/math/ccmath/parccmcf.pdf.

Multiple Choice

Multiple-choice questions will primarily be used to assess procedural fluency and conceptual understanding. Multiple-choice questions measure the Standards for Mathematical Content and may incorporate Standards for Mathematical Practices and real-world applications. Some multiple-choice questions require students to complete multiple steps. Likewise, questions may measure more than one cluster, drawing on the simultaneous application of multiple skills and concepts. Within answer choices, distractors will all be based on plausible missteps.

Constructed Response

Constructed-response questions will require students to show a deep understanding of mathematical procedures, concepts, and applications. The Regents Examination in Algebra II (Common Core) contains 2-, 4-, and 6-credit constructed-response questions.

2-credit constructed-response questions require students to complete a task and show their work. Like multiple-choice questions, 2-credit constructed-response questions may involve multiple steps, the application of multiple mathematics skills, and real-world applications. These questions may ask students to explain or justify their solutions and/or show their process of problem solving.

4-credit and 6-credit constructed-response questions require students to show their work in completing more extensive problems that may involve multiple tasks and concepts. Students will be asked to make sense of mathematical and real-world problems in order to demonstrate procedural and conceptual understanding. For 6-credit constructed-response questions, students will analyze, interpret, and/or create mathematical models of real-world situations to solve multi-step problems that connect multiple major clusters or a major cluster to supporting or additional content.

#1

1 When $b > 0$ and d is a positive integer, the expression $(3b)^{\frac{2}{d}}$ is equivalent to

(1) $\dfrac{1}{\left(\sqrt[d]{3b}\right)^2}$

(2) $\left(\sqrt{3b}\right)^d$

(3) $\dfrac{1}{\sqrt{3b^d}}$

(4) $\left(\sqrt[d]{3b}\right)^2$

Measured CCLS Cluster: N-RN.A

Key: 4

Commentary: The question measures the knowledge and skills described by the standards within N-RN.A because it requires the student to extend the properties of exponents to rational exponents. The student must rewrite an expression involving rational exponents into an equivalent radical form. This question requires the student to employ Mathematical Practice 7(Look for and make use of structure), because the student must shift perspective from one form to another.

Rationale: Choices (1), (2), (3) are plausible but incorrect. They represent common student errors made when a student has to rewrite rational exponents in radical form. Choosing the correct solution requires students to know how to manipulate rational exponents.

Answer Choice: (1) This response is incorrect and represents an incorrect equivalent expression. The student may have understood the fractional exponent, but did not understand that the reciprocal should only be written when the exponent is negative.

Answer Choice: (2) This response is incorrect. The expression represents the fractional exponent applied with the numerator as the root and the denominator as the power instead of the numerator as the power and the denominator as the root.

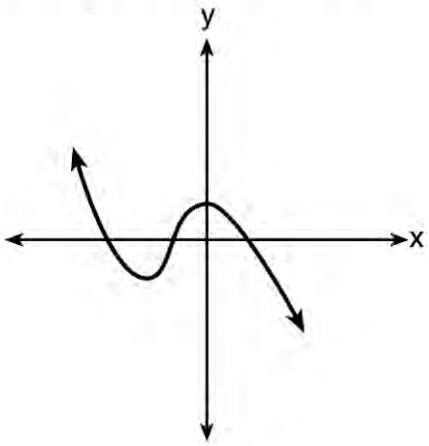
Answer Choice: (3) This response is incorrect. The student response is a combination of choice (1) and choice (2). The student used a reciprocal and made the denominator of the exponent the power instead of the root.

Answer Choice: (4) This response is correct and represents an equivalent expression. A student who selects this choice understands how to apply a fractional exponent by using the numerator as the power and the denominator as the root.

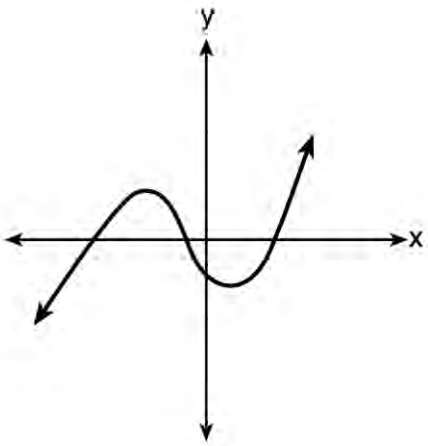
#4

4 Which graph has the following characteristics?

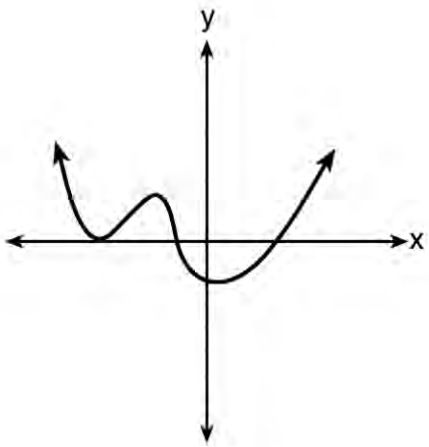
- three real zeros
- as $x \to -\infty$, $f(x) \to -\infty$
- as $x \to \infty$, $f(x) \to \infty$



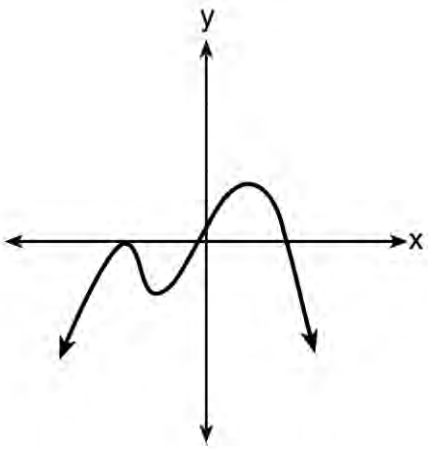
(1)



(3)



(2)



(4)

Measured CCLS Cluster: F-IF.C

Key: 3

Commentary: The question measures the knowledge and skills described by the standards within F-IF.C because it requires the student to analyze functions using different representations. The student must understand how to identify the graph of a polynomial function given zeros and end behavior.

Rationale: Choices (1), (2), (4) are plausible but incorrect. They represent common errors made when a student has to identify zeros and end behavior. Choosing the correct solution requires students to know how to determine the characteristics of a function from a graph. The graph's characteristics include three real zeros, where the left-end behavior is decreasing and the right-end behavior is increasing. Compare with question 36, which also assesses F-IF.C.

Answer Choice: (1) This response is incorrect. Although the graph shows three real zeros, the end behavior is opposite to the given characteristics. For this function, the left-end behavior is increasing and the right-end behavior is decreasing.

Answer Choice: (2) This response is incorrect. Although the graph shows three real zeros and correct right-end behavior, the left-end behavior is incorrect because it is increasing.

Answer Choice: (3) This response is correct, and a student who selects this response understands zeros and end behavior. The graph shows three real zeros, and has end behavior matching the given end behavior.

Answer Choice: (4) This response is incorrect. Although the graph shows three real zeros and correct left-end behavior, the right-end behavior is incorrect because it is decreasing.

#6

6 The zeros for $f(x) = x^4 - 4x^3 - 9x^2 + 36x$ are

(1) $\{0,\pm 3,4\}$

(2) $\{0,3,4\}$

(3) $\{0,\pm 3,-4\}$

(4) $\{0,3,-4\}$

Measured CCLS Cluster: A-APR.B

Key: 1

Commentary: This question measures the knowledge and skills described by the standards within A-APR.B because it requires the student to understand the relationship between zeros and factors of polynomials. The student must identify zeros of a quartic polynomial either by factoring or by constructing a graph of the function. This question requires the student to employ Mathematical Practice 5 (Use appropriate tools strategically) if the student chooses to analyze the graph of the functions using a graphing calculator, or Mathematical Practice 7 (Look for and make use of structure) if the student chooses to factor the polynomial.

Rationale: Choices (2), (3), (4) are plausible but incorrect. They represent common student errors made when finding the zeros of a factorable polynomial function. Choosing the correct solution requires students to know how to rewrite a polynomial expression using factor by grouping, and solving for the zeros or analyzing key features of the graph of the function.

Answer Choice: (1) This response is correct. The student understands how to find the zeros for a higher degree factorable polynomial function.

$$0 = x^2(x^2 - 9) - 4x(x^2 - 9)$$
$$0 = (x^2 - 4x)(x^2 - 9)$$
$$0 = x(x - 4)(x + 3)(x - 3)$$
$$x = 0, 4, -3, 3$$

Answer Choice: (2) This response is incorrect. The student found the positive square root of 9, but failed to find the negative square root of 9.

Answer Choice: (3) This response is incorrect. The student may have factored incorrectly to get $x + 4$ and $x = -4$ or factored correctly to get $x - 4$, but concluded $x = -4$. Using a graphical approach, the student may have found an x-intercept at 4, but interpreted the zero as -4.

Answer Choice: (4) This response is incorrect. The student failed to find the negative square root of 9 and incorrectly found $x = -4$ possibly due to a factoring or graphing error.

#9

9 The heights of women in the United States are normally distributed with a mean of 64 inches and a standard deviation of 2.75 inches. The percent of women whose heights are between 64 and 69.5 inches, to the *nearest whole percent*, is

(1) 6
(2) 48
(3) 68
(4) 95

Measured CCLS Cluster: S-ID.A

Key: 2

Commentary: The question measures the knowledge and skills described by the standards within S-ID.A because it requires the student to use the mean and standard deviation to estimate population percentages. The student must use a statistical function of a calculator or use knowledge of common normal curve percentages and z-scores to obtain a correct percent. This question requires students to employ Mathematical Practice 4 (Model with mathematics) as students apply mathematics to solve a problem arising in everyday life.

Rationale: Choices (1), (3), (4) are plausible but incorrect. They represent common student errors made when a student has to determine population percentages, with or without using technology. Choosing the correct solution requires students to know how to compute a population percentage using the normal cumulative density function on a graphing calculator or by finding the area associated with z-scores on a normal curve.

Answer Choice: (1) This response is incorrect. The student subtracted the mean of 64 from the value 69.5 and rounded. No connection to population percentages was made.

Answer Choice: (2) This response is correct and represents the percentage of women whose heights are between 64 and 69.5 inches to the nearest whole percent. The student understands how to find the population percentage using a graphing calculator or using knowledge of common normal curve percentages.

Answer Choice: (3) This response is incorrect. By the empirical rule, 68% of data lie within one standard deviation of the mean. The student found the percentage of women whose heights are between 61.25 and 66.75 inches rather than between 64 and 69.5 inches.

Answer Choice: (4) This response is incorrect. By the empirical rule, 95% of data lie within two standard deviations of the mean. The student found the percentage of women whose heights are between 58.5 and 69.5 inches rather than between 64 and 69.5 inches.

#10

10 The formula below can be used to model which scenario?

$$a_1 = 3000$$
$$a_n = 0.80a_{n-1}$$

(1) The first row of a stadium has 3000 seats, and each row thereafter has 80 more seats than the row in front of it.

(2) The last row of a stadium has 3000 seats, and each row before it has 80 fewer seats than the row behind it.

(3) A bank account starts with a deposit of $3000, and each year it grows by 80%.

(4) The initial value of a specialty toy is $3000, and its value each of the following years is 20% less.

Measured CCLS Cluster: F-BF.A

Key: 4

Commentary: The question measures the knowledge and skills described by the standards within F-BF.A because it requires the student to determine a scenario modeled by a recursive process. This question requires the student to employ Mathematical Practice 4 (Model with mathematics) as the student will interpret mathematical results within a given context and reflect on whether the results make sense.

Rationale: Choices (1), (2), (3) are plausible but incorrect. They represent common errors made when a student has to know which scenario models a geometric sequence expressed recursively. Choosing the correct solution requires students to recognize a geometric sequence with an initial term of 3000 and subsequent terms decreasing by 20%.

Answer Choice: (1) This response is incorrect. The scenario represents a model whose seats are increasing as an arithmetic sequence with a common difference of 80.

Answer Choice: (2) This response is incorrect. Although the scenario represents a decreasing number of seats, it is an arithmetic sequence with a common difference of -80.

Answer Choice: (3) This response is incorrect. The scenario represents a model where an account is growing as an increasing geometric sequence with a common ratio of 1.80.

Answer Choice: (4) This response is correct. The student understands that the scenario represents a decreasing geometric sequence with a common ratio of 0.80.

#11

11 Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

(1) independent
(2) dependent
(3) mutually exclusive
(4) complements

Measured CCLS Cluster: S-CP.A

Key: 1

Commentary: The question measures the knowledge and skills described by the standards within S-CP.A because it requires the student to understand independence and conditional probability. This question also requires the student to employ Mathematical Practice 3 (Construct viable arguments and critique the reasoning of others) because the student can make conjectures, build a logical progression of statements, and reason inductively about data.

Rationale: Choices (2), (3), (4) are plausible but incorrect. They represent common errors made when a student has to determine that if $P(A|B) = P(A)$, then events A and B are independent.

Answer Choice: (1) This response is correct. The student understands the relationship between conditional probability and independence.

$$P(Rain|Sean\ pitches) = \frac{P(Rain \cap Sean\ pitches)}{P(Sean\ pitches)}$$

$$0.4 = \frac{P(Rain \cap Sean\ pitches)}{0.5}$$

$$P(Rain \cap Sean\ pitches) = 0.2$$

Since $P(Rain) \cdot P(Sean\ pitches) = 0.2$, the two events are independent.

Another method is to show: $P(Rain|Sean\ pitches) = P(Rain)$

$$0.4 = 0.4$$

Answer Choice: (2) This response is incorrect. There is no evidence that the probability of rain impacts the probability that Sean pitches.

Answer Choice: (3) This response is incorrect. If the events were mutually exclusive, then the 2 events could not occur at the same time. Since Sean could pitch in the rain, $P(Rain \cap Sean\ pitches) = 0.2$, the events are not mutually exclusive.

Answer Choice: (4) This response is incorrect. If the events were complements, the probabilities would add to 1. Additionally, complements are mutually exclusive.

#13

13 The Ferris wheel at the landmark Navy Pier in Chicago takes 7 minutes to make one full rotation. The height, H, in feet, above the ground of one of the six-person cars can be modeled by $H(t) = 70 \sin\left(\frac{2\pi}{7}(t-1.75)\right) + 80$, where t is time, in minutes. Using $H(t)$ for one full rotation, this car's minimum height, in feet, is

(1) 150
(2) 70
(3) 10
(4) 0

Measured CCLS Cluster: F-IF.B

Key: 3

Commentary: The question measures the knowledge and skills described by the standards within F-IF.B because it requires the student to interpret functions that arise in applications. This question requires the student to employ Mathematical Practice 7 (Look for and make use of structure), because the student recognizes the parameters of a sinusoidal function that relate to the characteristics of the model.

Rationale: Choices (1), (2), (4) are plausible but incorrect. They represent common errors made when a student has to determine the minimum height for a trigonometric function by subtracting the amplitude (70) from the vertical shift (80).

Answer Choice: (1) This response is incorrect. The student found the maximum value by adding the amplitude to the vertical shift.

Answer Choice: (2) This response is incorrect. The student only found the amplitude and did not understand how to use it to get the minimum height.

Answer Choice: (3) This response is correct. The student who selects this response has an understanding of how to determine the minimum height knowing the amplitude and the vertical shift.

Answer Choice: (4) This response is incorrect. The student did not take into account the vertical shift or amplitude.

#17

17 A circle centered at the origin has a radius of 10 units. The terminal side of an angle, θ, intercepts the circle in Quadrant II at point C. The y-coordinate of point C is 8. What is the value of cos θ?

(1) $-\frac{3}{5}$

(2) $-\frac{3}{4}$

(3) $\frac{3}{5}$

(4) $\frac{4}{5}$

Measured CCLS Cluster: F-TF.A

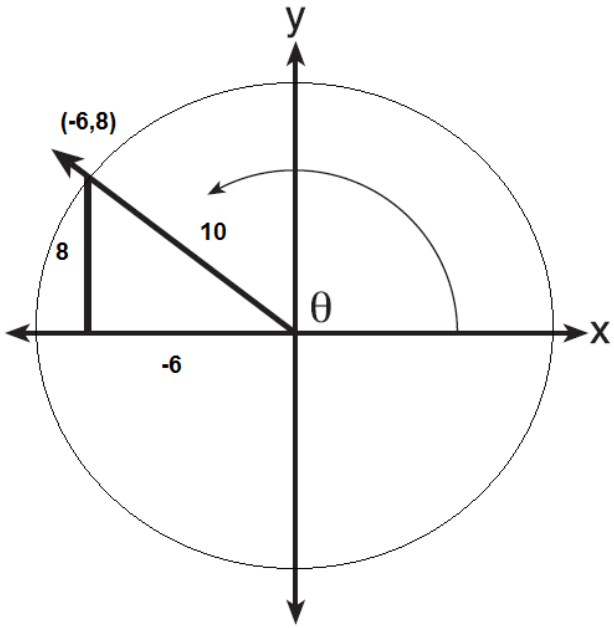
Key: 1

Commentary: The question measures the knowledge and skills described by the standards within F-TF.A because it requires the student to extend the domain of trigonometric functions by using the unit circle. This question requires the student to employ Mathematical Practice 2 (Reason abstractly and quantitatively) because the student contextualizes their knowledge of right triangle trigonometry and applies that knowledge within the unit circle.

Rationale: Choices (2), (3), (4) are plausible but incorrect. They represent common errors made when a student has to determine the value of a trigonometric function when extending the domain to any quadrant.

Answer Choice: (1) This response is correct. The student who selects this response understands how to find the value of the cosine of an angle in Quadrant II.

$$cos\ \theta = -\frac{6}{10} = -\frac{3}{5}$$



Answer Choice: (2) This response is incorrect. Although the student understood the value should be negative, the cotangent of the angle was found.

Answer Choice: (3) This response is incorrect. The student does not understand that the cosine of an angle in Quadrant II is a negative value.

Answer Choice: (4) This response is incorrect. The student may have found the sine of the angle or may have used the given *y*-coordinate as the *x*-coordinate in Quadrant I.

#19

19 The equation $4x^2 - 24x + 4y^2 + 72y = 76$ is equivalent to

(1) $4(x - 3)^2 + 4(y + 9)^2 = 76$

(2) $4(x - 3)^2 + 4(y + 9)^2 = 121$

(3) $4(x - 3)^2 + 4(y + 9)^2 = 166$

(4) $4(x - 3)^2 + 4(y + 9)^2 = 436$

Measured CCLS Cluster: A-SSE.A

Key: 4

Commentary: The question measures the knowledge and skills described by the standards within A-SSE.A because it requires the student to use the structure of an equation to identify ways to rewrite it. This question requires the student to employ Mathematical Practice 7 (Look for and make use of structure) because the student must recognize an opportunity to write the original equation as a circle in center-radius form.

Rationale: Choices (1), (2), (3) are plausible but incorrect. They represent common errors made when a student has to determine how to write an equivalent equation using the completing the square process. Compare with item 31, which also assesses A-SSE.A.

Answer Choice: (1) This response is incorrect. When completing the square, the student did not add the distributed values of 36 and 324 to the right side.

$$4x^2 - 24x + 4y^2 + 72y = 76$$
$$4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76$$

Answer Choice: (2) This response is incorrect. The student added 36 and 9 to the right side rather than 36 and 324.

Answer Choice: (3) This response is incorrect. The student added 9 and 81 to the right side by failing to distribute the 4 to each term.

Answer Choice: (4) This response is correct. The student who selects this response has an understanding of structure and the process of completing the square.

$$4x^2 - 24x + 4y^2 + 72y = 76$$
$$4(x^2 - 6x + 9) + 4(y^2 + 18y + 81) = 76 + 36 + 324$$
$$4(x - 3)^2 + 4(y + 9)^2 = 436$$

#21

21 Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let m represent months.]

(1) $(1.0525)^m$

(2) $(1.0525)^{\frac{12}{m}}$

(3) $(1.00427)^m$

(4) $(1.00427)^{\frac{m}{12}}$

Measured CCLS Cluster: A-SSE.B

Key: 3

Commentary: The question measures the knowledge and skills described by the standards within A-SSE.B because it requires the student to model an exponential growth rate as an expression. The student must write an equivalent monthly exponential expression from a yearly exponential growth rate. This item requires skills associated with both Mathematical Practice 4 (Model with mathematics) and Mathematical Practice 7 (Look for and make use of structure).

Rationale: Choices (1), (2), (4) are plausible but incorrect. They represent common errors made when a student has to determine how to use the properties of exponents to transform an exponential expression. Compare with item 34, which also assesses A-SSE.B.

Answer Choice: (1) This response is incorrect. The student applied the annual rate to the commonly used form of $(1 + r)^n$.

Answer Choice: (2) This response is incorrect. The student applied the annual rate to the commonly used form of $(1 + r)^n$ and stated the reciprocal of the correct exponent.

Answer Choice: (3) This response is correct. The student who selected this response understands how to convert from an annual rate to a monthly rate, and correctly expressed and applied the new exponent.

$$(1 + 0.0525)^{\frac{m}{12}}$$
$$\left((1.0525)^{\frac{1}{12}}\right)^m$$
$$\approx (1.00427)^m$$

Answer Choice: (4) This response is incorrect. The student converted to a monthly rate but left the exponent expressed in years instead of months.

#23

23 The population of Jamesburg for the years 2010 – 2013, respectively, was reported as follows:

250,000 250,937 251,878 252,822

How can this sequence be recursively modeled?

(1) $j_n = 250{,}000(1.00375)^{n-1}$

(2) $j_n = 250{,}000 + 937^{(n-1)}$

(3) $j_1 = 250{,}000$
$j_n = 1.00375\, j_{n-1}$

(4) $j_1 = 250{,}000$
$j_n = j_{n-1} + 937$

Measured CCLS Cluster: F-BF.A

Key: 3

Commentary: The question measures the knowledge and skills described by the standards within F-BF.A because it requires the student to write a geometric sequence recursively. The student must recognize the difference between recursive and an explicit formulas, and arithmetic and geometric sequences.

Rationale: Choices (1), (2), (4) are plausible but incorrect. They represent common errors made when a student has to determine a recursive model for a geometric sequence. Choosing the correct solution requires the student to develop a recursive formula generating the four given values. Compare with item 10, which also assesses F-BF.A.

Answer Choice: (1) This response is incorrect. An explicit geometric sequence is expressed rather than a recursive geometric sequence.

Answer Choice: (2) This response is incorrect. The student found a sequence in which the difference grows exponentially.

Answer Choice: (3) This response is correct. The student who selects this response understands how to model a geometric sequence recursively by stating an initial term and a formula for all subsequent terms.

Answer Choice: (4) This response is incorrect. Although the initial term is correctly represented, subsequent terms are incorrectly represented as an arithmetic sequence.

#24

24 The voltage used by most households can be modeled by a sine function. The maximum voltage is 120 volts, and there are 60 cycles *every second*. Which equation best represents the value of the voltage as it flows through the electric wires, where t is time in seconds?

(1) $V = 120 \sin(t)$

(2) $V = 120 \sin(60t)$

(3) $V = 120 \sin(60\pi t)$

(4) $V = 120 \sin(120\pi t)$

Measured CCLS Cluster: F-TF.B

Key: 4

Commentary: The question measures the knowledge and skills described by the standards within F-TF.B because it requires the student to model periodic phenomena with trigonometric functions. This question requires the students to employ Mathematical Practice 4 (Model with Mathematics) because the student must choose a sine function that models voltage given the maximum voltage and the number of cycles per time period.

Rationale: Choices (1), (2), (3) are plausible but incorrect. They represent common errors made when a student has to model periodic phenomena with specified amplitude and period.

Answer Choice: (1) This response is incorrect. The student lacks a coefficient of *t*, and the function therefore does not complete the required cycles per second.

Answer Choice: (2) This response is incorrect. The student simply used 60 as the coefficient of x. As a result, this function completes 60 cycles in 2π seconds, rather than in one second.

Answer Choice: (3) This response is incorrect. The student likely solved $Period = \frac{\pi}{B}$, omitting the 2 in the numerator.

Answer Choice: (4) This response is correct. The student who selects this response has an understanding of how to determine the period and amplitude of a trigonometric model.

$$Period = \frac{1}{60}$$
$$\frac{1}{60} = \frac{2\pi}{B}$$
$$B = 120\pi$$
$$V = 120sin(120\pi t)$$

#25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

Measured CCLS Cluster: A-REI.A

Commentary: The question measures the knowledge and skills described by the standards within A-REI.A because it requires the student to solve a simple rational equation. Additionally, the item requires the student to employ Mathematical Practice 1 because the student must make sense of a problem and persevere in solving it.

Rationale: This question requires students to solve a simple rational equation by eliminating the denominator by either multiplying by the LCD or finding a common denominator, and setting numerators equal to each other.

$$\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$$

$$\left(\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}\right) 3x$$

$$3 - x = -1$$

$$x = 4$$

Sample student responses and scores appear on the following pages.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$3\left(\frac{1}{x}\right) - \left(\frac{1}{3}\right)x = \frac{-1}{3x}$$

$$\frac{3}{3x} - \frac{1x}{3x} = \frac{-1}{3x}$$

$$3 - 1x = -1$$
$$-3 \qquad -3$$

$$\frac{-1x}{-1} = \frac{-4}{-1}$$

$$x = 4$$

Score 2: The student gave a complete and correct response.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$\frac{(3)1}{(3)x} - \frac{(x)1}{(x)3} = \frac{-1}{3x}$

$\frac{3}{3x} - \frac{x}{3x} = \frac{-1}{3x}$

$\frac{3-x}{3x} = \frac{-1}{3x}$

Score 1: The student only found a common denominator and combined like terms.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$\frac{1}{x} - \frac{1}{3} = \frac{1}{3x}$$

$$\frac{0}{x-3} \times \frac{1}{3x}$$

$$0 = x - 3$$

$$x = 3$$

Score 0: The student made an error combining the fractions, and also made a transcription error by omitting the negative.

#26:

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

Measured CCLS Cluster: S-IC.B

Commentary: The question measures the knowledge and skills described by the standards within S-IC.B because the requirements of a controlled experiment by describing how a control group can be created to examine the effect of including an ingredient in toothpaste. Additionally, the item requires the student to employ Mathematical Practice 6 because the student must examine claims and make explicit use of definitions.

Rationale: This question requires students to describe a controlled experiment. Two critical aspects of this controlled experiment involve random assignment and the use of a control group. Experiments generally do not require a randomly selected group of individuals, but rather volunteers who are available at the time of study. However, random assignment of the volunteers to groups is critical to reduce bias. Additionally, a control group is required so that ingredient X is the sole differentiator in the outcomes of the experiment, rather than potential confounding variables that could influence results.

Compare with item 35, which also assesses S-IC.B.

Sample student responses and scores appear on the following pages.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.

I would collect two groups of individuals that are of equal age and sex to ensure accuracy and eliminate any other variables that can have an effect. I would use a large group of people say 40 in each. Then. I would give one random group an equal amount of toothpaste with the ingredient, where as the other random group will receive toothpaste with no ingredient. It will be given in the morning at the same time. By the end of the day at the same time for a week, I will record the results to determine the impact of the ingredient

Score 2: The student gave a complete and correct response.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient X in a toothpaste.

One group of people will use the version with ingredient X and another will use the tooth paste without. Compare the results.

Score 1: The student wrote an incomplete description by omitting the random assignment of two groups.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.

A controlled experiment can be used by distributing products with the ingredients to a group, while giving the control group to ~~dotted~~ a different group of people.

Score 0: The student's response lacked random assignment and had an insufficient explanation of a control group.

#27:

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

Measured CCLS Cluster: A-APR.B

Commentary: The question measures the knowledge and skills described by the standards within A-APR.B because the student is required to apply understanding of the relationship between zeros and factors of polynomials. The student might directly apply the remainder theorem or use long or synthetic division with an appropriate explanation.

Rationale: This question instructs the student to determine whether $(x - 5)$ is a factor of the given polynomial. One possible method is to substitute 5 into the polynomial and observe the value is not zero, along with an explanation using the remainder theorem. Another possible method is to apply synthetic or long division and calculate the remainder, along with noting the nonzero remainder implies that $x - 5$ is not a factor.

Sample student responses and scores appear on the following pages.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$x - 5 = 0$

$x = 5$

$2(5)^3 - 4(5)^2 - 7(5) - 10 \stackrel{?}{=} 0$

$250 - 100 - 35 - 10 \stackrel{?}{=} 0$

$105 \neq 0$

$x-5$ is not a factor of $2x^3-4x^2-7x-10$.

If $x-5$ is a factor of $2x^3-4x^2-7x-10$, then when $2x^3-4x^2-7x-10$ and 5 is substituted for x, the value of $2x^3-4x^2-7x-10$ should be 0.

Score 2: The student gave a complete and correct response.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$2(-5)^3 - 4(-5)^2 - 7(-5) - 10 = 0$

$-325 \neq 0$

x-5 is not a factor because when you use the remainder theorem the remainder is -325 not 0.

Score 1: The student made one error by substituting -5 instead of 5.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$$x-5 \overline{)2x^3 - 4x^2 - 7x - 10}$$

$2x^2 - 6x + 23$

$2x^3 - 10x^2$

$6x^2 - 7x - 10$

$6x^2 - 30x$

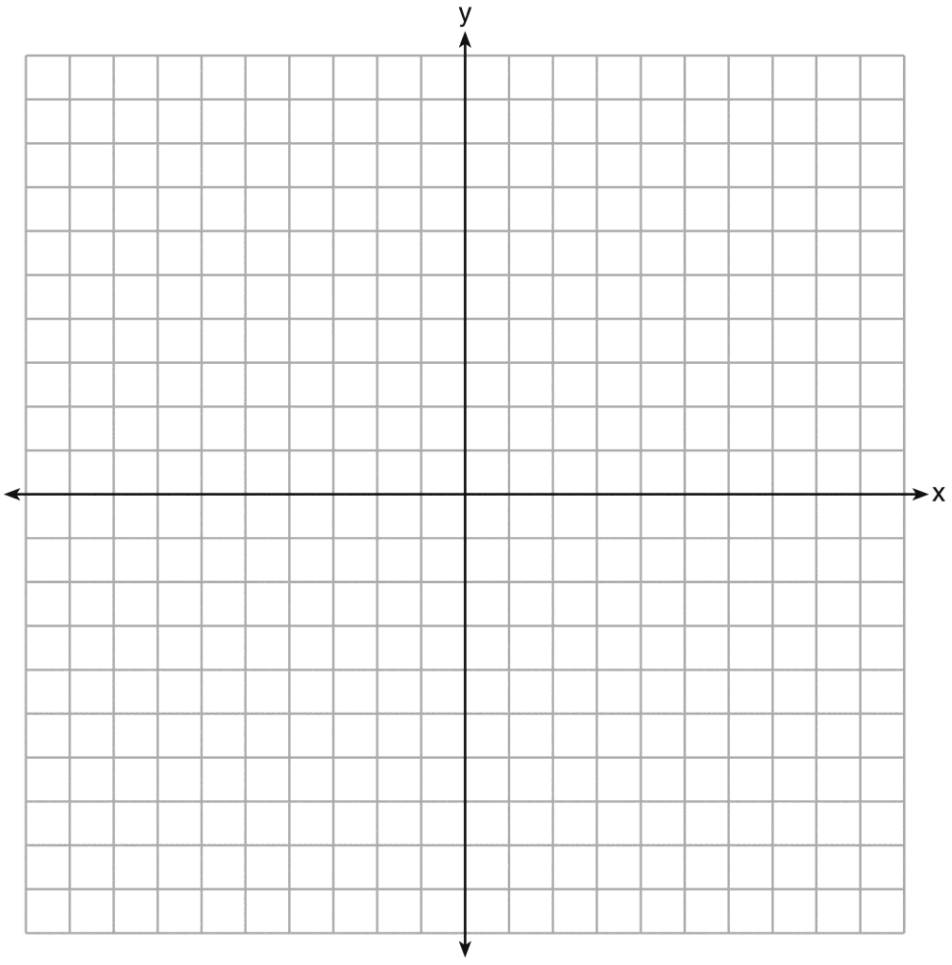
$23x - 10$

NO

Score 0: The student made multiple errors dividing and did not provide the explanation.

#28:

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Measured CCLS Cluster: F-IF.C

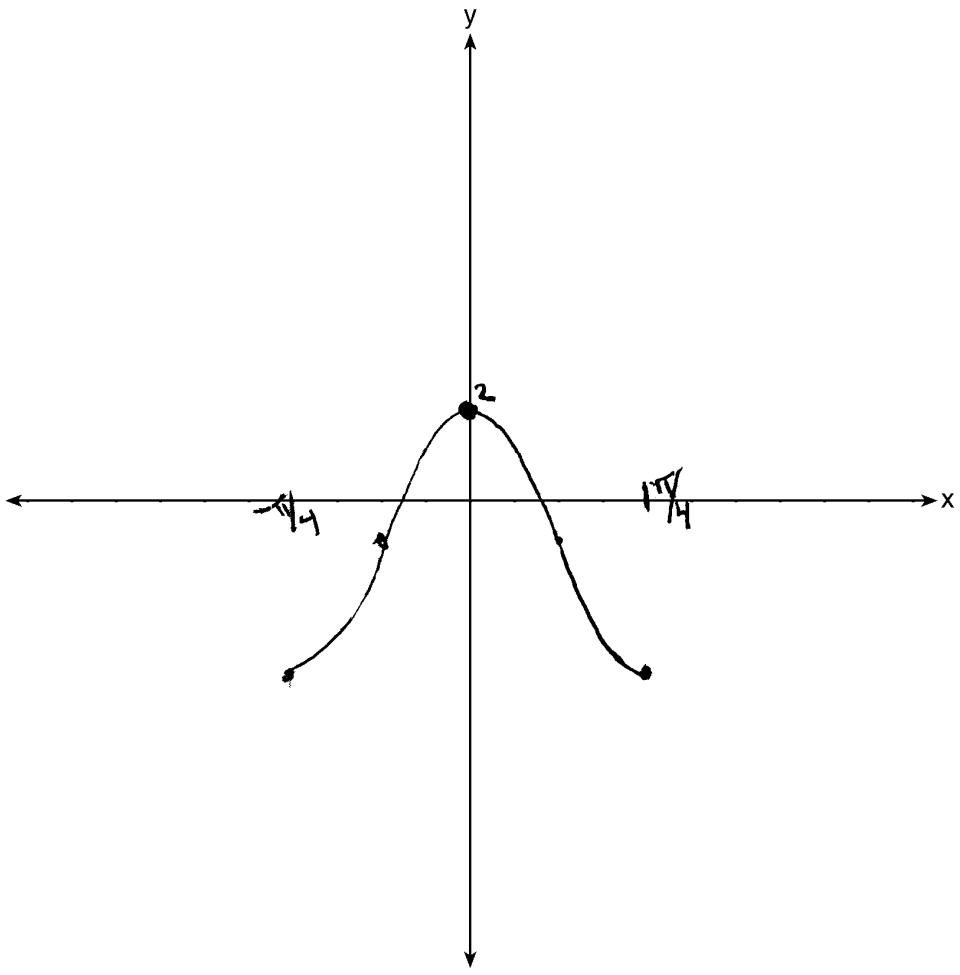
Commentary: The question measures the knowledge and skills described by the standards within F-IF.C because the student is required to graph a trigonometric function given a description of its properties. Additionally, the item requires the student to employ Mathematical Practice 6 because the student must label the axes and sketch multiple features of the graph.

Rationale: This question requires the student to graph a cosine function given its period, amplitude, midline and the point $(0,2)$. The student may create the cosine function $y = 3\cos(4x) - 1$ from the given characteristics and graph one cycle with assistance from a graphing calculator. The amplitude 3 represents the distance between the highest point on the graph and the midline, the period $\frac{\pi}{2}$ represents the length of one complete cycle of the function, and the midline represents the equation of the horizontal line halfway between the maximum and minimum values. One cycle of the function can be completed on either side of the y-axis as long as it passes through the point $(0,2)$.

Sample student responses and scores appear on the following pages.

Question 28

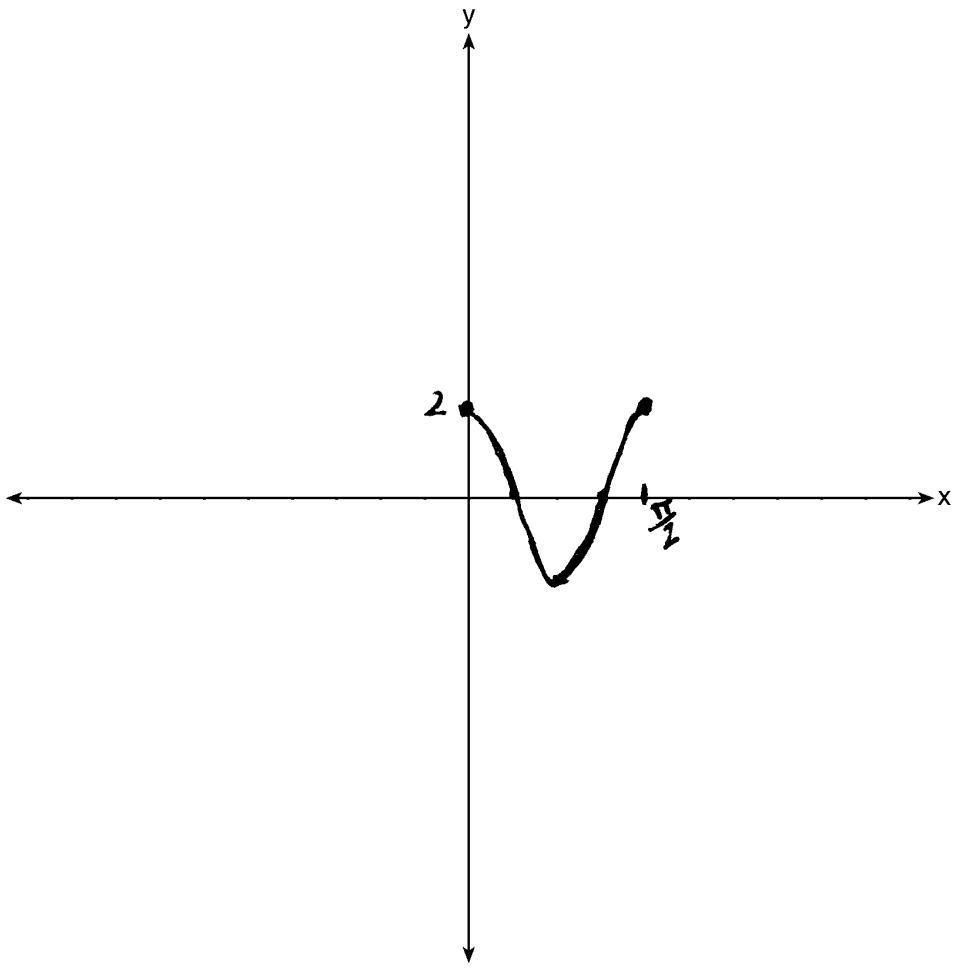
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 2: The student gave a complete and correct response.

Question 28

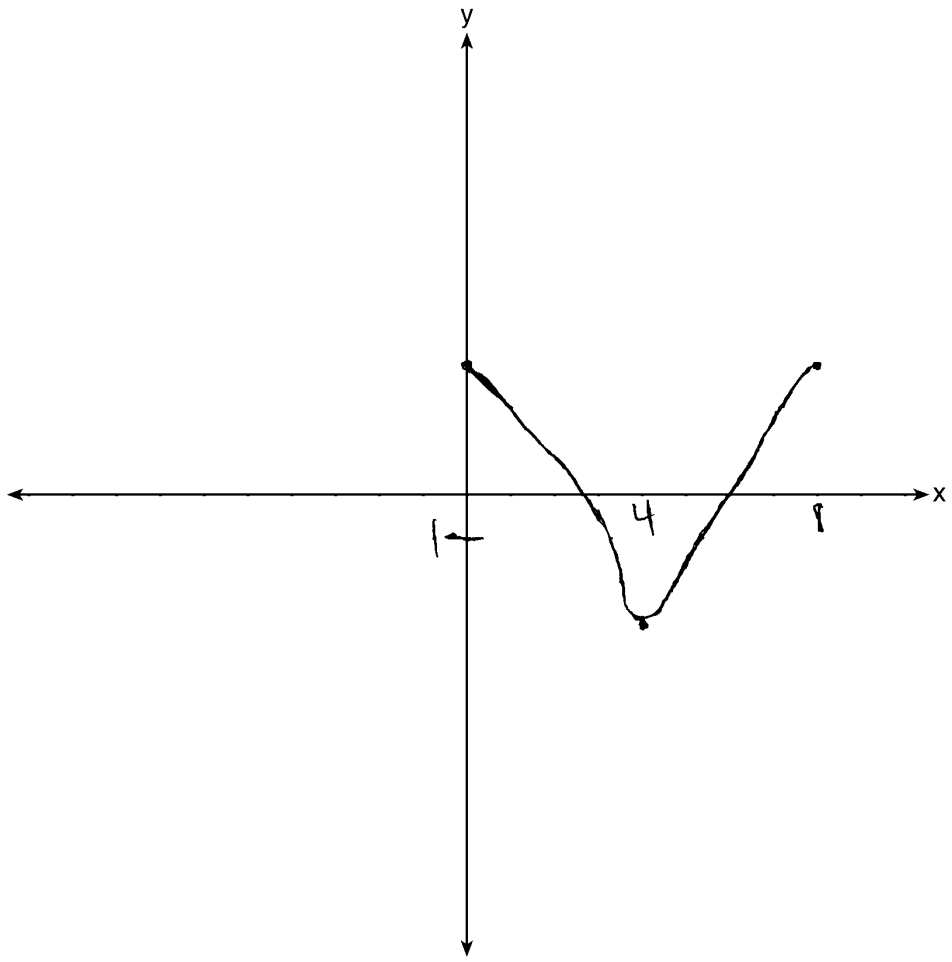
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 1: The student correctly graphed one cycle of a cosine function passing through (0,2) with period $\frac{\pi}{2}$, but used an incorrect amplitude that affected the midline.

Question 28

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 0: The student made multiple errors.

#29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

Measured CCLS Cluster: S-CP.B

Commentary: The question measures the knowledge and skills described by the standards within S-CP.B because the student is required to apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ and interpret the answer in terms of the model. Additionally, the item requires the student to employ Mathematical Practice 4 because the student must be able to identify important quantities in a practical situation and map those relationships using such tools as diagrams.

Rationale: The question requires students to apply the Addition Rule as follows:

$$P(S \text{ or } M) = P(S) + P(M) - P(S \text{ and } M)$$

$$\frac{974}{1376} = \frac{649}{1376} + \frac{433}{1376} - P(S \text{ and } M)$$

$$\frac{974}{1376} = \frac{1082}{1376} - P(S \text{ and } M)$$

$$\frac{108}{1376} = P(S \text{ and } M)$$

Sample student responses and scores appear on the following pages.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

649 + 433

1082 - 974

108

1376

Score 2: The student gave a complete and correct response.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

$\frac{649}{1376} + \frac{433}{1376}$

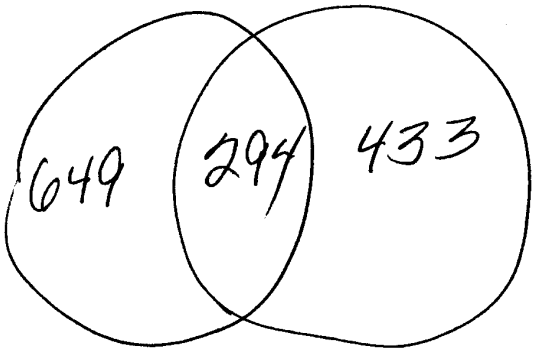
$\frac{1082}{1376}$

$\begin{array}{r} 649 \\ +433 \\ \hline 1082 \end{array}$

Score 1: The student made an error by not subtracting from $\frac{974}{1376}$.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?



1376
-1082
294

294
974

Score 0: The student made multiple errors.

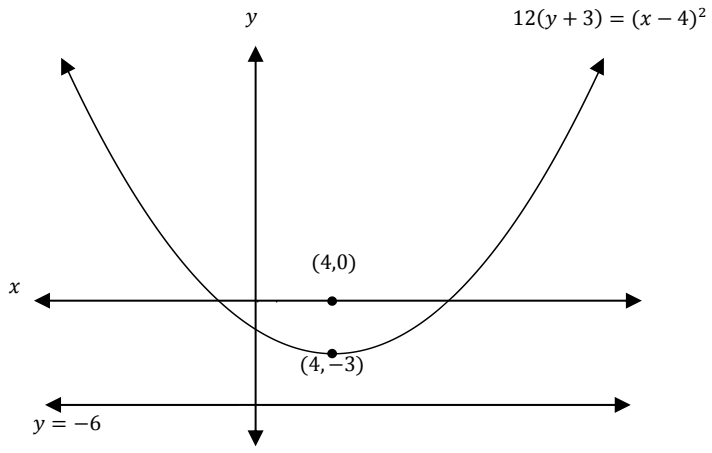
#30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

Measured CCLS Cluster: G-GPE.A

Commentary: The question measures the knowledge and skills described by the standards within G-GPE.A because the student is required to determine the focus of a parabola from its equation and its directrix. Additionally, the item requires the student to employ Mathematical Practice 7 because of the form of the equation provided.

Rationale: This question requires the student to find the vertex and use it to determine the focus. A parabola is the locus of points equidistant from a point (focus) and a line (directrix). The vertex can be determined simply from the original form of the equation as $(4, -3)$. Since the vertex is the minimum point on the given parabola, and any point on the parabola must be equidistant from the focus and directrix, the point $(4,0)$ can be determined as the focus.



Sample student responses and scores appear on the following pages.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$12y + 36 = x^2 - 8x + 16$

$-36 \quad\quad -36$

$12y = x^2 - 8x - 20$

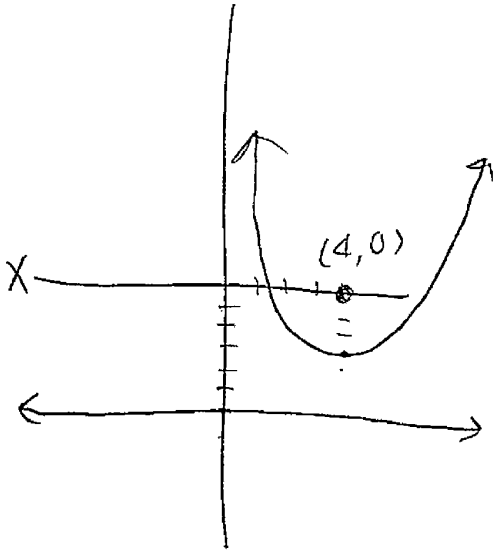
$y = \frac{x^2}{12} - \frac{2x}{3} - \frac{5}{3}$

$x = -\frac{b}{2a} = -\frac{-\frac{2}{3}}{2} = \frac{2}{3} = \frac{1}{4}$

$(x-4)^2 = 12(y+3)$

$4(3)$

(4, 0)



Score 2: The student gave a complete and correct response.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

Vertex = (4, -3)

since directrix is $y = -6$

need to add 6 to y in vertex

Focus = (4, 3)

Score 1: The student misused the directrix.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$12y + 36 = x^2 - 8x + 16$

$\frac{12y}{12} = \frac{x^2 - 8x - 20}{12}$

$y = \frac{x^2 - 8x - 20}{12}$

Focus: $(4, 0)$

$(-8, 0)$

Score 0: The student stated a partially correct answer that was obtained by an incorrect procedure.

#31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

Measured CCLS Cluster: A-SSE.A

Commentary: The question measures the knowledge and skills described by the standards within A-SSE.A because the student is asked to use the structure of an expression to identify ways to rewrite it, while proving that two expressions equal each other independently. Additionally, the item requires the student to employ Mathematical Practice 7 by asking the student to look closely to discern a pattern or structure. Students can see complicated things, such as algebraic expressions as single objects or as being composed of several objects.

Rationale: This question requires a student to prove two expressions are equal. By noticing the structure of the first expression, a student could obtain the following proof:

$$\frac{x^3+9}{x^3+8} = \frac{x^3+8+1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+8}{x^3+8} + \frac{1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

Compare with item 19, which also assesses A-SSE.A.

Sample student responses and scores appear on the following pages.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$$\frac{x^3+9}{x^3+8} \stackrel{?}{=} 1 + \frac{1}{x^3+8}$$

$$= (1)\left(\frac{x^3+8}{x^3+8}\right) + \frac{1}{x^3+8}$$

$$= \frac{x^3+8+1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+9}{x^3+8}$$

Score 2: The student gave a complete and correct response.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{(x^3+8)1}{(x^3+8)1} + \frac{1}{x^3+8}$$

$$x^3+9 = x^3+8+1$$
$$-x^3 \qquad -x^3$$

$$9 = 8+1$$

$$\boxed{9=9}$$

Score 1: The student made an error by not manipulating expressions independently in an algebraic proof.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

let $x=2$

$$\frac{2^3+9}{2^3+8} = \frac{8+9}{8+8} = \frac{17}{16}$$

$$1 + \frac{1}{2^3+8} = 1 + \frac{1}{8+8} = \frac{16}{16} + \frac{1}{16}$$

$$= \frac{17}{16}$$

$$\frac{17}{16} = \frac{17}{16}$$

Score 0: The student used an incorrect procedure by substituting a single value in for x.

#32

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

Measured CCLS Cluster: F-LE.A

Commentary: The question measures the knowledge and skills described by the standards within F-LE.A because the student is required to construct an exponential model to approximate the annual growth rate. The item requires the student to employ Mathematical Practice 4 because the student has to identify important qualities and create an equation.

Rationale: This question asks students to approximate an annual growth rate, which can be accomplished with an exponential model as shown below:

$$135000 = 100000x^5$$
$$1.35 = x^5$$
$$1.0618 = x$$

$$1.0618 - 1 = .0618$$
$$6.18 \rightarrow 6\%$$

Sample student responses and scores appear on the following pages.

Question 32

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

$\frac{135,000}{100,000} = \frac{100,000(1+x)^5}{100,000}$

$1.35 = (1+x)^5$

$\sqrt[5]{1.35} = \sqrt[5]{(1+x)^5}$

$1.06185879 = 1+x$

$-1 \quad -1$

$.06185879 = x$

$x = 6\%$

Score 2: The student gave a complete and correct response.

Question 32

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

$A = A_0 e^{k(t-t_0)} + B_0$

$A_t = A_0 (1+r)^t$

$135{,}000 = 100{,}000 (1+r)^5$

$\frac{27}{20} = (1+r)^5$

$\sqrt[5]{\frac{27}{20}} = 1+r$

Score 1: The student wrote an incomplete solution.

Question 32

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent.*

$135000 = 100000(1+r)^5$

$35000 = (1+r)^5$

$1.54 = 5\log(1+r)$

$.31 = \log(1+r)$

$10^{.31} = 10^{\log(1+r)}$

$2.042 = 1+r$

$1.042 = r$

Score 0: The student made an error by subtracting 100,000 and did not state a percentage.

#33

33 Solve the system of equations shown below algebraically.

$$(x-3)^2+(y+2)^2=16$$
$$2x+2y=10$$

Measured CCLS Cluster: A-REI.C

Commentary: The question measures the knowledge and skills described by the standards within A-REI.C because the student is required to solve a system consisting of a linear equation and a quadratic equation in two variables algebraically.

Rationale: The question requires students to solve a system consisting of a linear equation and a quadratic equation. Below, the linear equation is solved for y and substituted into the quadratic equation.

$$2x+2y=10$$
$$y=-x+5$$

$$(x-3)^2+(-x+5+2)^2=16$$
$$x^2-6x+9+x^2-14x+49=16$$
$$2x^2-20x+42=0$$
$$x^2-10x+21=0$$
$$(x-7)(x-3)=0$$
$$x=7 \text{ or } x=3$$

Substitute the x values into the linear equation to solve for the y-values of each solutions.

$$(7,-2),(3,2)$$

Sample student responses and scores appear on the following pages.

Question 33

33 Solve the system of equations shown below algebraically.

$$\begin{cases} (x-3)^2 + (y+2)^2 = 16 & (1) \\ 2x + 2y = 10 & (2) \end{cases}$$

(2) $2x+2y=10$

$2y = 10-2x$

$y = 5-x$ (3)

put (3) to (1) : $(x-3)^2 + (5-x+2)^2 = 16$

$x^2-6x+9+49-14x+x^2=16$

$2x^2-20x+58=16$

$2x^2-20x+42=0$

$x^2-10x+21=0$

$-3 \quad -7$

$1 \quad 1$

$(x-3)(x-7)=0$

$x_1=3 \quad x_2=7$

$y_1=2 \quad y_2=-2$

$2\times3+2y=16$

$2y=4$

$y=2$

$2\times7+2y=10$

$2y=-4$

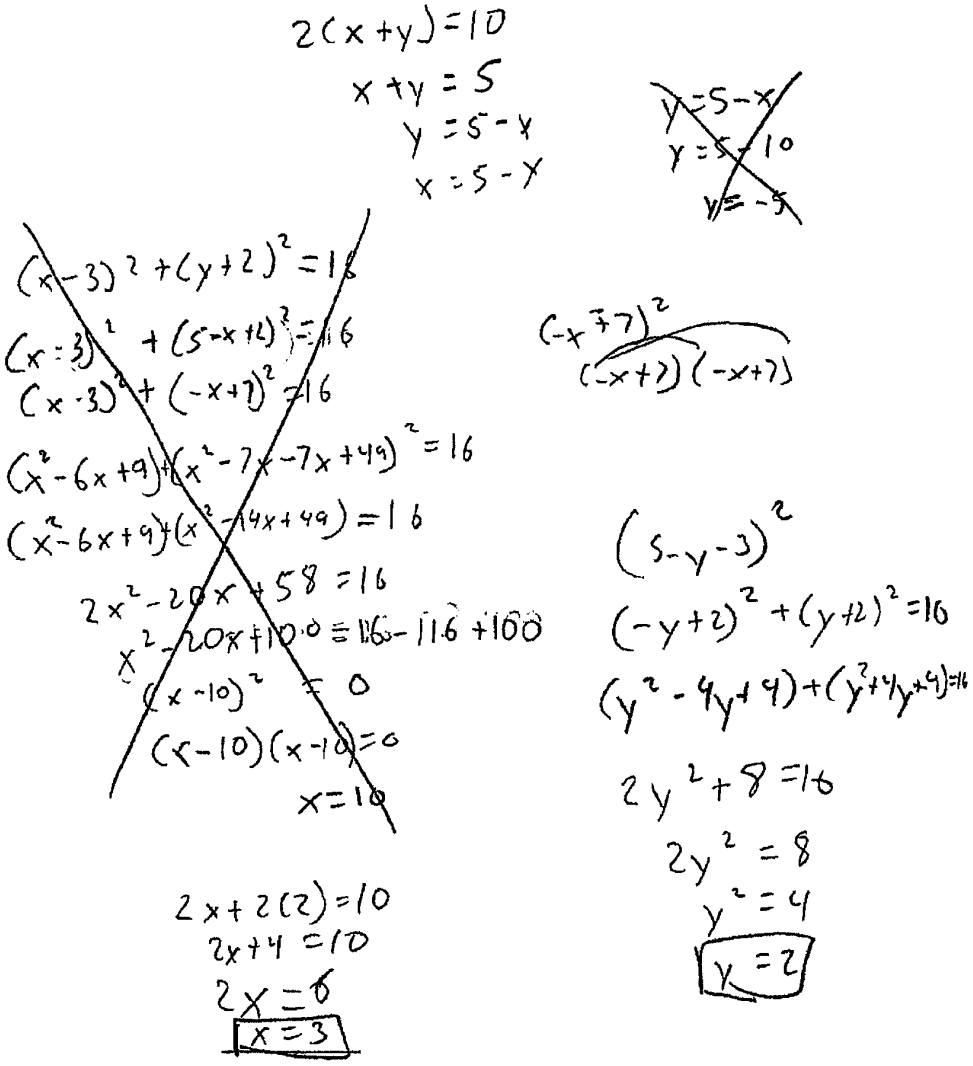
$y_2=-2$

Score 4: The student gave a complete and correct response.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$



Score 3: The student found only one correct solution of the system.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x+2y=10$

$2y = 10-2x$

$y = 5-x$

$(x-3)^2+(5-x+2)^2=16$

$(x-3)^2+(y+2)^2=16$

$x^2-3x+3x+9+y^2+2y+2y+4=16$

$x^2-6x+9+y^2+4y+4=16$

$x^2-6x+y^2+4y+13=16$

$y^2=-x^2+6x-4y+3$

$(5-x)^2=-x^2+6x-4(5-x)+3$

$5-x \quad 5-x$

$x^2-5x+5x+25$

$x^2-10x+25=-x^2+6x-20+4x+3$

$2x^2+45=10x+3$

$2x^2+10x+42=0$

$x^2-5x+21=0$

$x=\frac{5\pm\sqrt{25-4(1)(21)}}{2}$ $\quad x=\frac{5\pm i\sqrt{59}}{2}$

Score 2: The student made a transcription error by losing a $-10x$, and did not find y-values.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x+2y=10$
$2x=10-2y$
$x=\frac{10-2y}{2}$
$x=5-y$

$((5-y)-3)^2+(y+2)^2=16$
$(5-y)(5-y)-9+(y+2)^2=16$
$25-5y-5y+y^2-9+y^2+4y+4=16$
$2y^2-6y+20=16$
$2y^2-6y+4=0$
$2(y^2-3y+2)=0$
$2(y-2)(y-1)=0$

$y-2=0$
$y=2$
$y-1=0$
$y=1$

$2x+2(2)=10$
$2x+4=10$
$2x=6$
$x=3$

$2x+2=10$
$2x=8$
$x=4$

answer=
$y=2, x=3$

check:
$(3-3)^2+(2+2)^2=16$
$(0^2)+(4)^2=16$
$16=16$ ✓

$(3-3)^2+(1+2)^2=16$
$(0^2)+(3^2)=16$
$9\neq16$

$(4-3)^2+(2+2)^2=16$
$(1)^2+(4^2)=16$
$17\neq16$

$(4-3)^2+(1+2)^2=16$
$(1)^2+(3)^2=16$
$1+9=16$
$10\neq16$

Score 1: The student made a conceptual error squaring the first term and did not express both ordered pairs.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x+2y=5$
$y=5+x$

$(x-3)^2+(5+x+2)^2=16$

$x^2-6x+9+49+14x+x^2=16$

$2x^2+8x+42=0$

$x^2+4x+21=0$

$(x+7)(x+3)=0$

$x=7 \quad | \quad x=3$

Score 0: The student made several errors and did not find the y-values.

#34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

Measured CCLS Cluster: A-SSE.B

Commentary: The question measures the knowledge and skills described by the standards within A-SSE.B because the student must be able derive the formula for the sum of a finite geometric series and use the formula to solve a problem. This item requires students to employ Mathematical Practice 4 because the student must be able to identify important quantities in a practical situation and map those relationships using a formula.

Rationale: This question asks the student to write a geometric series formula and evaluate at $n = 15$ years.

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

$$S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04}$$

$$S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04}$$

$$S_{15} = 660{,}778.39$$

Compare with item 21, which also assesses A-SSE.B.

Sample student responses and scores appear on the following pages.

Question 34

34 Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33{,}000 - 33{,}000(1.04)^{15}}{1 - 1.04}$$

$$S_{15} = \frac{33{,}000 - 33{,}000(1.80)}{-.04}$$

$$S_{15} = \frac{-26{,}431.14}{-.04}$$

$$S_{15} = 660{,}778.39$$

Score 4: The student gave a complete and correct response.

Question 34

34 Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_n = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04}$$

$S_n = -26432.18$

Score 3: The student failed to use parentheses when entering the expression into the calculator.

Question 34

34 Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000(0.04)^n}{1 - 0.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33{,}000 - 33{,}000(0.04)^{15}}{1 - 0.04}$$

$$\frac{33000}{0.96}$$

$$34375$$

Score 2: The student made a conceptual error interpreting the 4% increase.

Question 34

34 Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33000 - (1.04)^N}{1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33000 - (1.04)^{15}}{1.04}$$

$$= \frac{33000 + 1.80094}{1.04}$$

$$= \$31732.50$$

Score 1: The student made a computational error in the second part, having received no credit for the first part.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ $\quad S_n = \frac{a_1 - a_1 (0.04)(t)}{1 - 0.04}$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$S_n = \frac{33{,}000 (0.04)(15)}{.96} =$

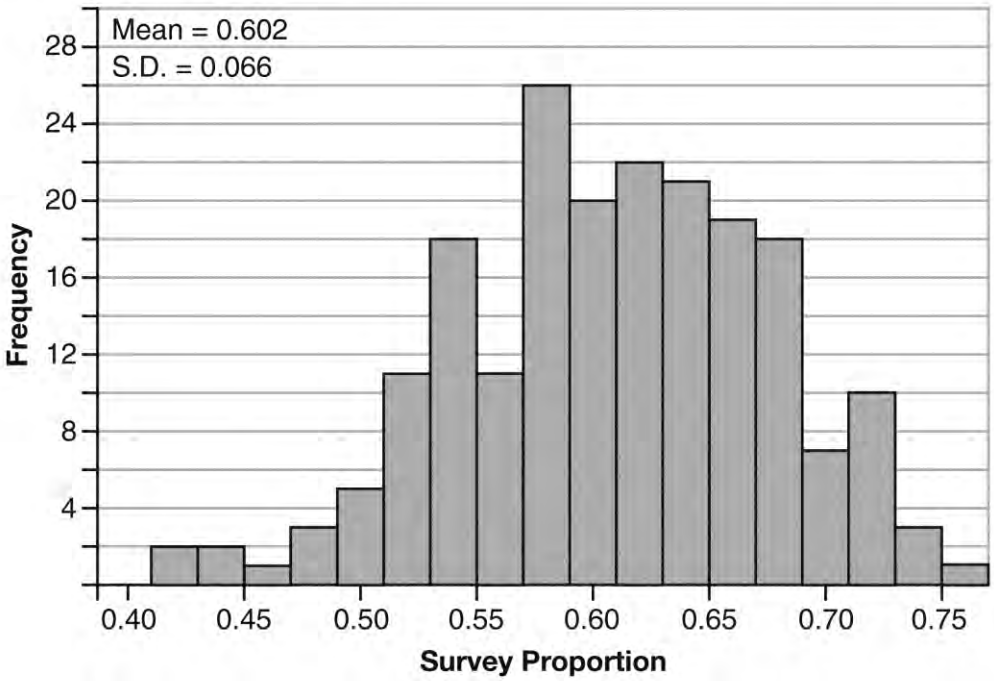
$S_n \approx \$535$

Score 0: The student made multiple errors.

#35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

Measured CCLS Cluster: S-IC.B

Commentary: The question measures the knowledge and skills described by the standards within S-IC.B because students use data from a sample survey to estimate a population mean or proportion. Students are given a graph of data, with the mean and standard deviation already computed, and asked to determine a plausible interval containing a middle 95% of the data. Additionally, the item requires the student to employ Mathematical Practice 6 because the student examines claims and makes explicit use of definitions.

Rationale: This question requires students to determine a 95% interval given the results of a normal simulation in a graph and related mean and standard deviations. Students are also asked to explain the statistical evidence of a 50%-50% split by referencing the interval that was calculated. About 95% of the simulated proportions lie within two standard deviations of the mean. This produces the interval 0.60 ± 0.13 or (0.47, 0.73). The reason the prom committee may be concerned is because 0.50 falls within the interval, making a 50%-50% split vote a possibility.

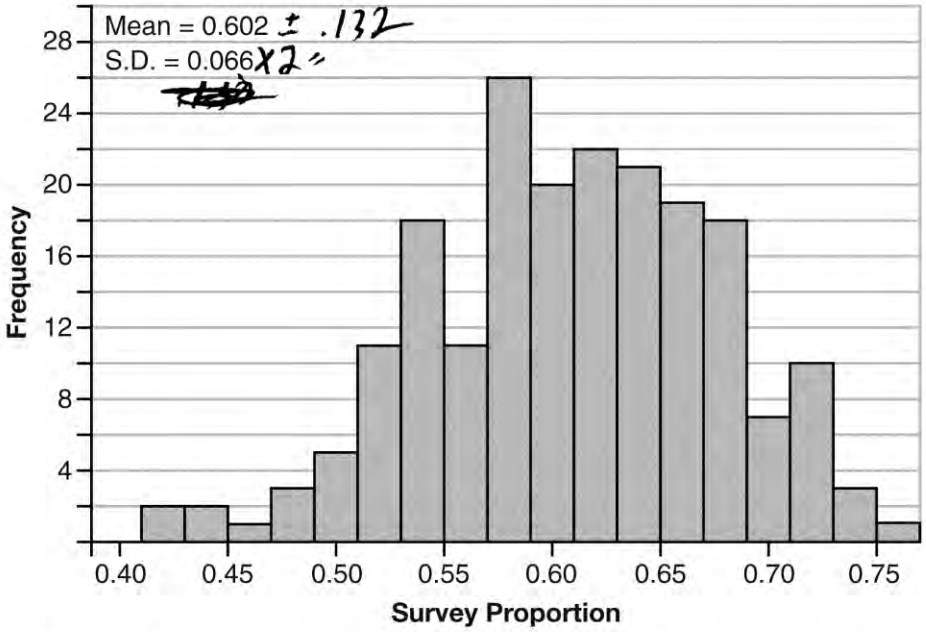
Compare with item 26, which also assesses S-IC.B.

Sample student responses and scores appear on the following pages.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

0.47 - 0.73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

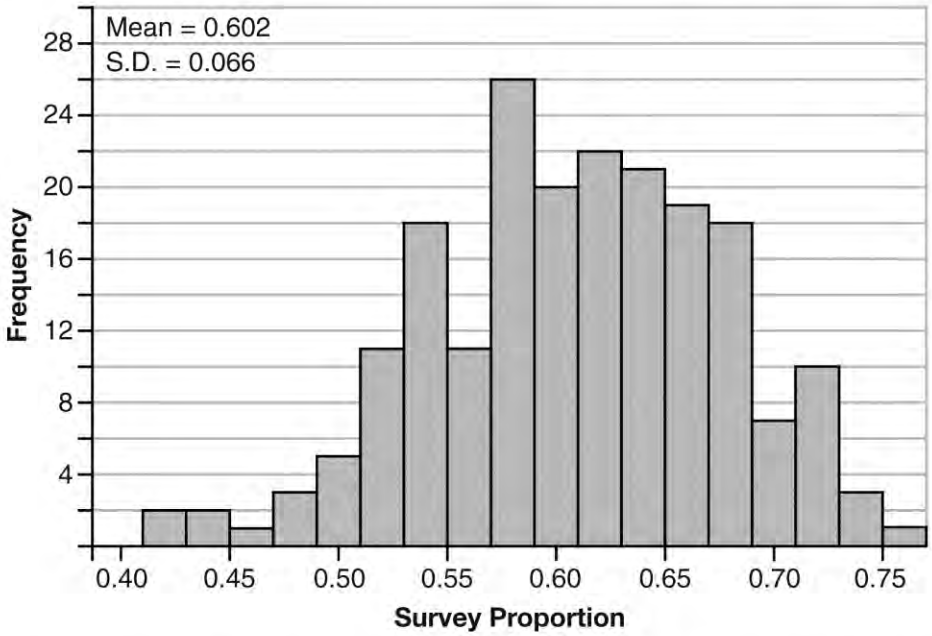
.50 is within this interval so its possible to get a split vote.

Score 4: The student gave a complete and correct response.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

$0.602 + 2 \cdot 0.066 = 0.734$
$0.602 - 2 \cdot 0.066 = 0.47$

0.47 to 0.73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

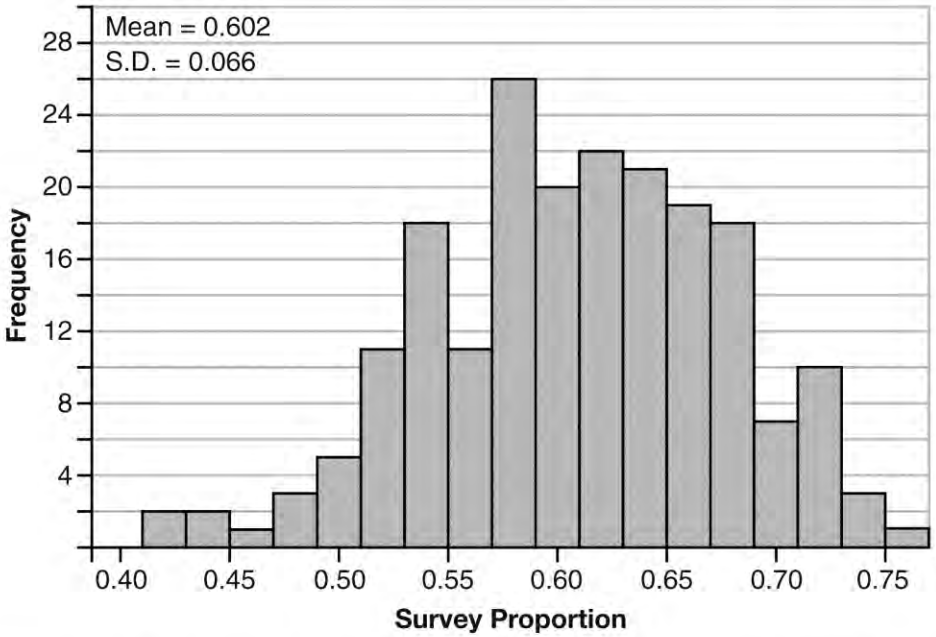
It is not a concern since .50 falls within the interval.

Score 3: The student determined a correct interval, but provided contradictory statistical evidence.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

$0.602 \pm 2 \cdot 0.066$ → .47, .734

.47 to .73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

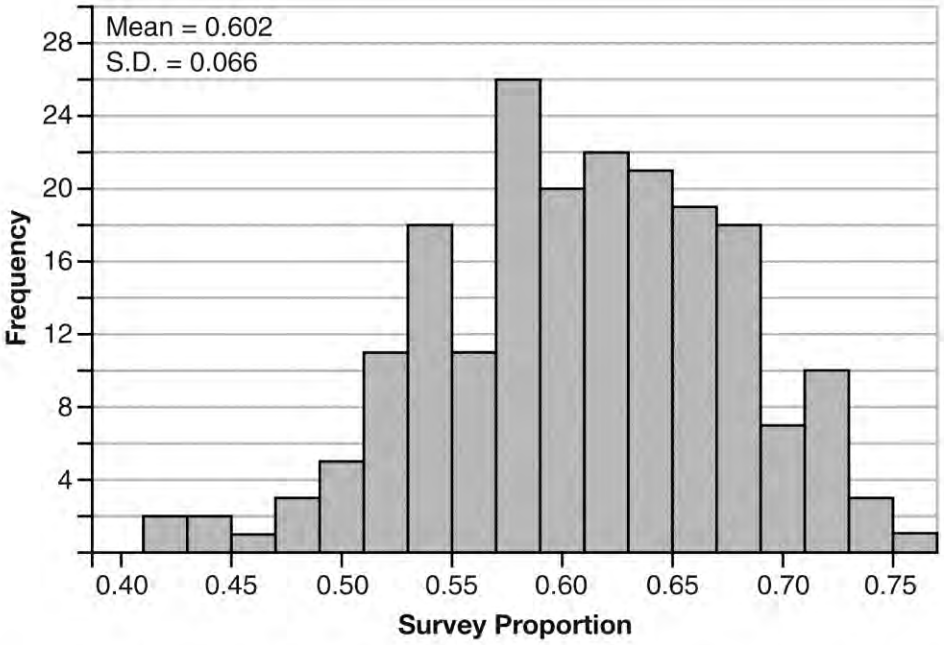
The graph shows 0.50

Score 2: The student gave no statistical explanation.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

.066 × 2 = .132 → .13
.066 × 2 = .132 → .13
.602 + .13 = .732 → .73
.602 − .13 = .472 → .47

.47 to .73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

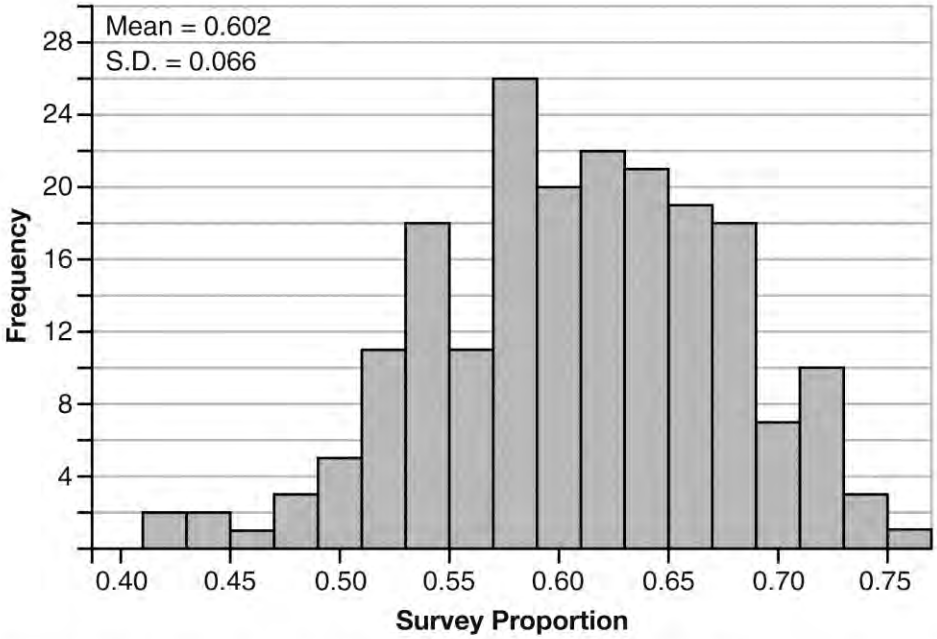
Since 60% of the students voted and 50% is close to 60%, they could be concerned that 50% is possible because voting can vary.

Score 1: The student used the standard deviation as the center and rounded incorrectly. The student gave an incomplete explanation.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth*.

.602 + .132 = .734 (.602, .734)

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

.5 is in the interval.

Score 0: The student made multiple conceptual and computational errors.

#36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

Measured CCLS Cluster: F-IF.C

Commentary: The question measures the knowledge and skills described by the standards within F-IF.C because the student must analyze two functions, one of which is represented numerically in a table and the other is represented algebraically. The item requires the student to employ Mathematical Practice 2 and Mathematical Practice 3 because the student has to compare average rates of change and draw a correct conclusion.

Rationale: This question asks students to determine which function has a greater average rate of change over the interval $[-2,4]$. The average rate of change for f is 13.125 and the average rate of change for g is 38. A student could also argue that the change in y values over the same interval $[-2,4]$ is greater for g, which implies a greater average rate of change.

Compare with item 4, which also assesses F-IF.C.

Sample student responses and scores appear on the following pages.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

$\frac{80 - 1.25}{4 - (-2)} = \frac{78.75}{6}$

$= 13.125$

$= \frac{105}{8}$.

$g(-2) = -32 - 20 + 3$

$= -49$

$g(4) = 179$

$\frac{179 - (-49)}{4 - (-2)}$

$= \frac{228}{6}$

$= 38$

g(x) b/c its rate of change over [-2, 4] is 38, and f(x)'s rate of change is 13.125.

Score 4: The student gave a complete and correct response.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
x_1 −2	y_1 1.25
−1	2.5
0	5
1	10
2	20
3	40
x_2 4	y_2 80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

x	y
−2	−49
−1	−26
0	3
1	2
2	15
3	60
4	179

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{80 - 1.25}{4 - (-2)}$

$\frac{78.75}{7} = 11.25$ rate

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{179 - (-49)}{4 - (-2)}$

$\frac{228}{7} = 32.57142857$ rate

$g(x) = 4x^3 - 5x^2 + 3$ has the greater average rate of change on the interval $[-2, 4]$

Score 3: The student made a computational error when calculating the denominators.

Question 36

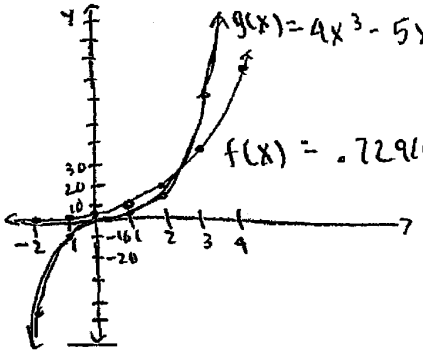
36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

x	g(x)
−2	−49
−1	−6
0	3
1	2
2	15
3	66
4	179

Function $g(x) = 4x^3 - 5x^2 + 3$ has a greater average rate of change in the interval $[-2, 4]$. This is because when both functions are graphed, g(x)'s slope is steeper than f(x)'s.



$f(x) = .7291667x^3 + 1.27976x^2 + 1.80059x + 5$

Score 2: The student made a conceptual error by creating an appropriate model for $f(x)$, but wrote an appropriate explanation for that model.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

78.75

$g(x) = 4x^3 - 5x^2 + 3$

$g(-2) = -8(4) - 5(4) + 3$

$= -32 - 20 + 3$

$= -52 + 3$

$= -49$

$g(4) = 4(64) - 5(16) + 3$

$= 252 - 80 + 3$

$= 169$

$169 > 78.75$

$g(x) = 4x^3 - 5x^2$ has a greater change.

Score 1: The student made an error finding the average rates of change by not dividing by Δx, and made one computational error.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

↑ this function had a greater ~~start~~ average change on the interval $[-2, 4]$ because this function is a geometric sequence which doubles its x-values.

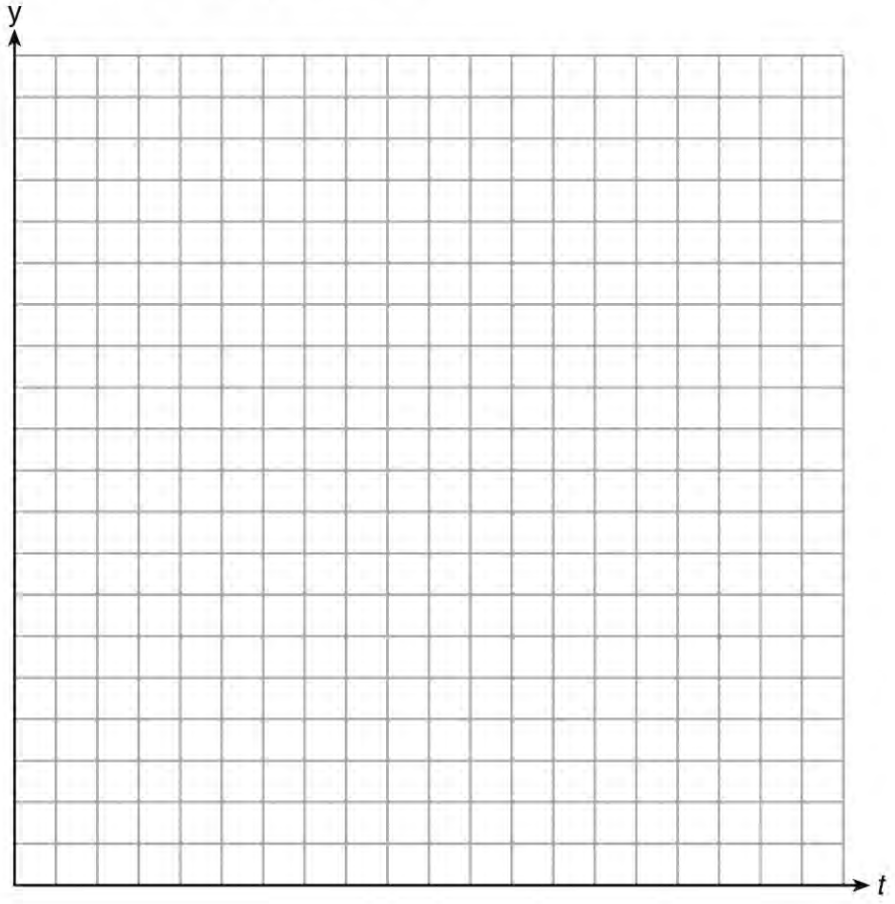
Score 0: The student did not calculate an average rate of change and wrote an irrelevant explanation.

#37

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

Graph each function on the set of axes below.



To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

Measured CCLS Cluster: A-REI.D

Commentary: The question measures the knowledge and skills described by the standards within A-REI.D because the student must represent, graph, and solve exponential equations. Knowledge and skills within clusters A-CED.A and F-LE.A are also assessed as students must create exponential equations and possibly solve an exponential equation algebraically. The item requires the student to employ Mathematical Practice 4 and 5 because the student uses a graphing calculator to graph exponential functions and solve a problem arising in everyday life, society, or the workplace.

Rationale: In this question, the student creates functions $A(t) = 800e^{-0.347t}$ and $B(t) = 400e^{-0.231t}$ by substituting given values into the general form of a given exponential equation. The student then graphs these functions using an appropriate scale. Using the graph, the student determines where the functions intersect since the t-value of this intersection point represents the nearest hour when the amount of the drug remaining in patient *B* begins to exceed that of patient *A*. Finally, the student solves an exponential equation graphically or using logarithms to determine when it is safe for patient *A* to take another dosage.

Sample student responses and scores appear on the following pages.

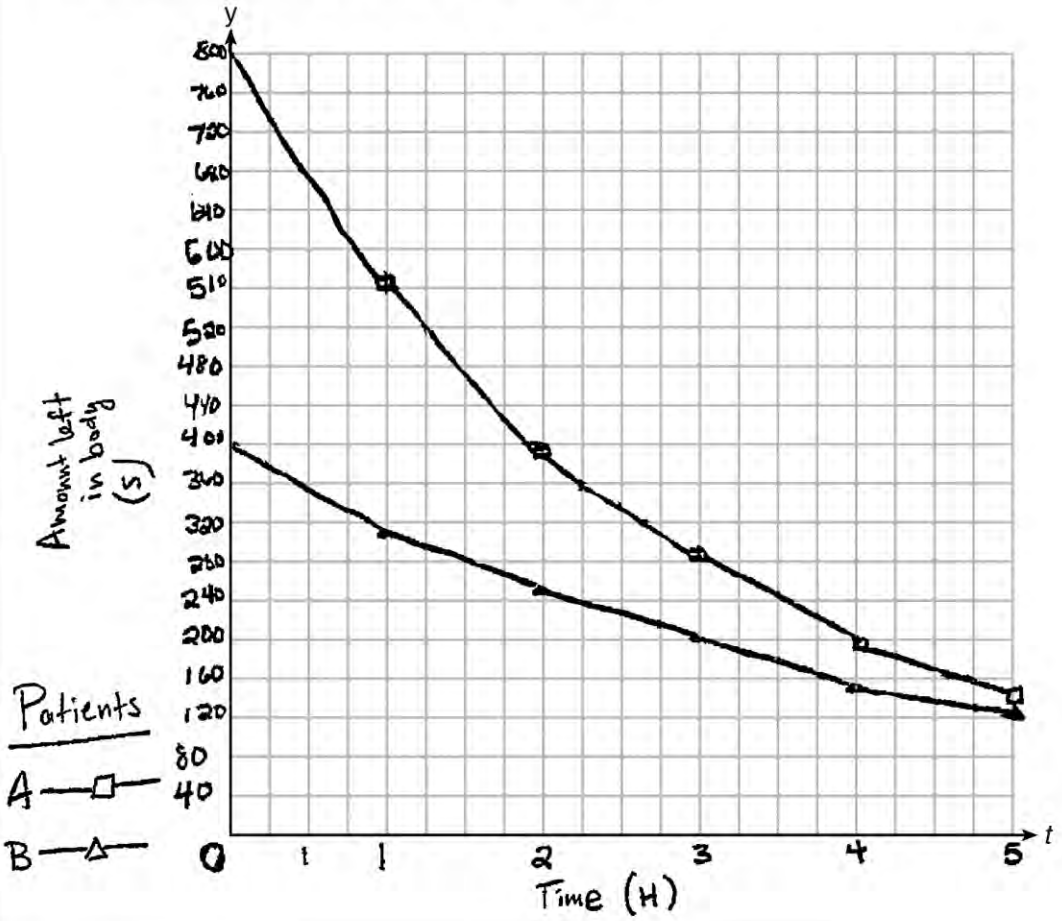
Question 37

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

P_A $A(t) = 800\text{mg}(e)^{-.347t}$ P_B $B(t) = 400\text{mg}(e)^{-.231t}$

Graph each function on the set of axes below.



Score 6: The student gave a complete and correct response.

Question 37

To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

Hour 6 because when the equation is solved the amount left in B is 100 while A has about 99.7.

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

$800(.16) = 120mg$

$H_5 = 141$

$\frac{120}{800} = \frac{800mg(e)^{-.347(t)}}{800}$

$.15 = (e)^{-.347(t)}$

$\log_e .15 = \log_e e^{-.347(t)}$

$\frac{-1.897}{-.347} = \frac{-.347(t)}{-.347}$

$5.5 hrs = t$

Question 37

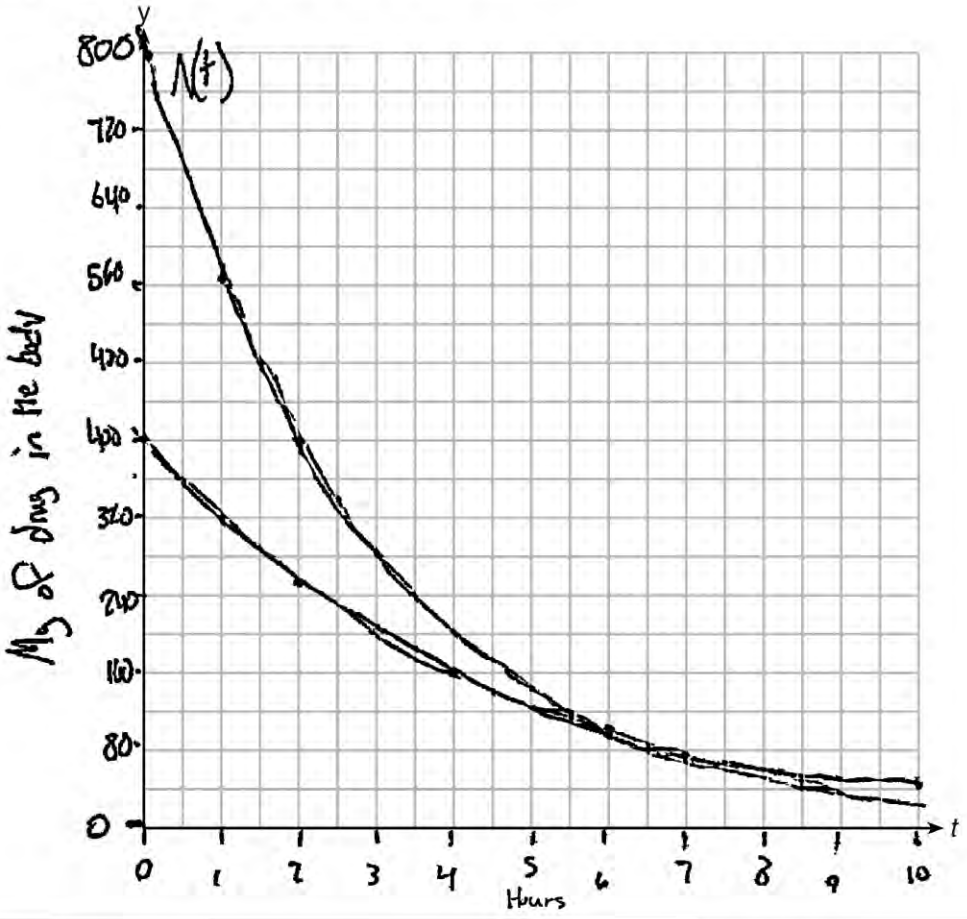
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$N(t) = 800(e)^{-.347t}$

$400(e)^{-.231t}$

Graph each function on the set of axes below.



Score 5: The student did not indicate which function models which patient.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

Hour 6

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$\frac{120}{800} = \frac{800(e)^{-.347t}}{800}$

$.15 = e^{-.347t}$

$\log_e .15 = -.347t$

$\frac{\ln .15}{-.347} = \frac{-.347t}{-.347}$

$5.5 = t$

5.5 hours

Question 37

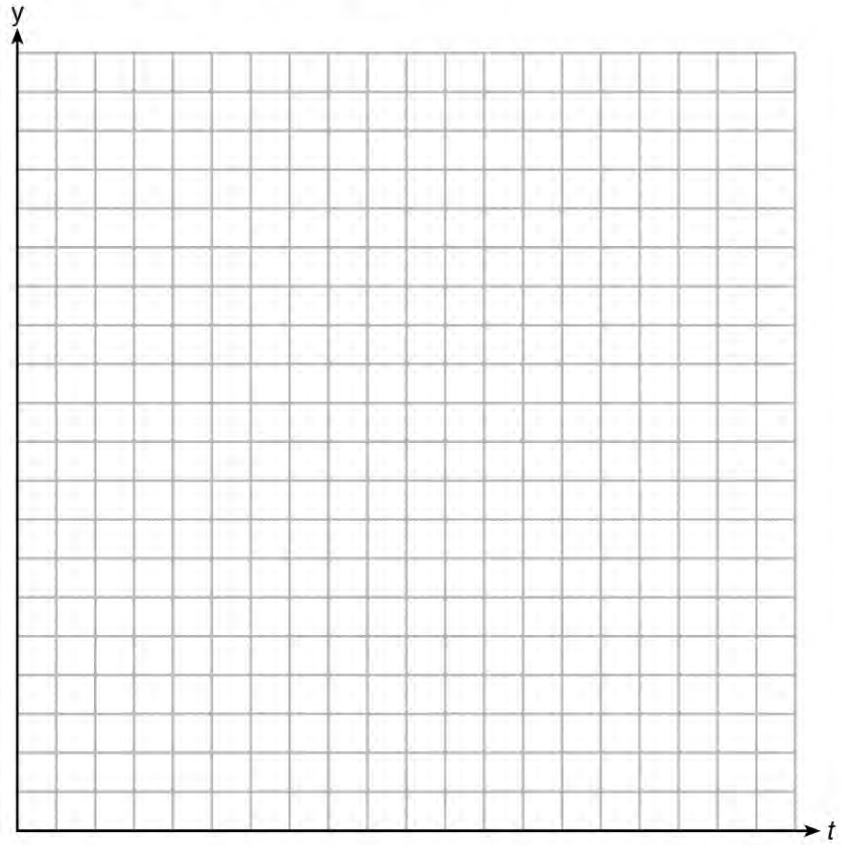
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$A(t) = 800mg(e)^{-.347t}$

$B(t) = 400mg(e)^{-.231t}$

Graph each function on the set of axes below.



Score 4: The student did not graph either function.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

After 6 hours. I see this after graphing both functions on my calculator and looking at the table. I could then see that at 6 hours patient a would have 99.74 mg of drug, while patient b would have 100.03, This is probably because, despite starting with more drug, Patient A's decay rate is also greater.

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$$\frac{15}{100} = \frac{x}{800}$$

$$\frac{15 \cdot 800}{100} = \frac{100x}{100}$$

$120 = x$

120 mg left

Using my graphing calculator...

I know that Patient A would have to wait approximately 5.5 hours or 5 hours and 30 minutes.

Question 37

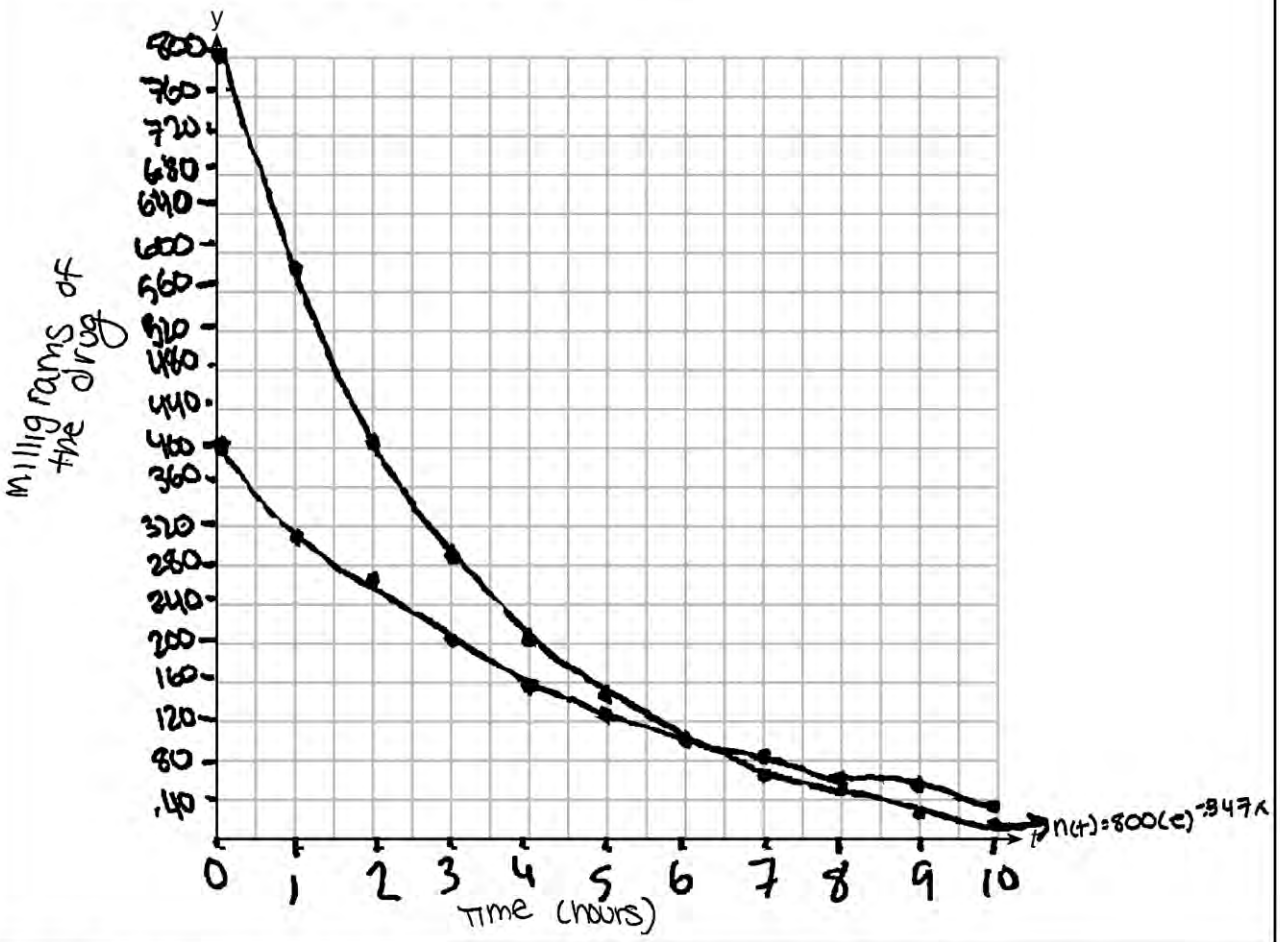
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-.347x}$

$400(e)^{-.231x}$

Graph each function on the set of axes below.



Score 3: The student drew a correct graph and gave a correct answer of 5.5 hours.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

Hour 7

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

5.5 hours

Question 37

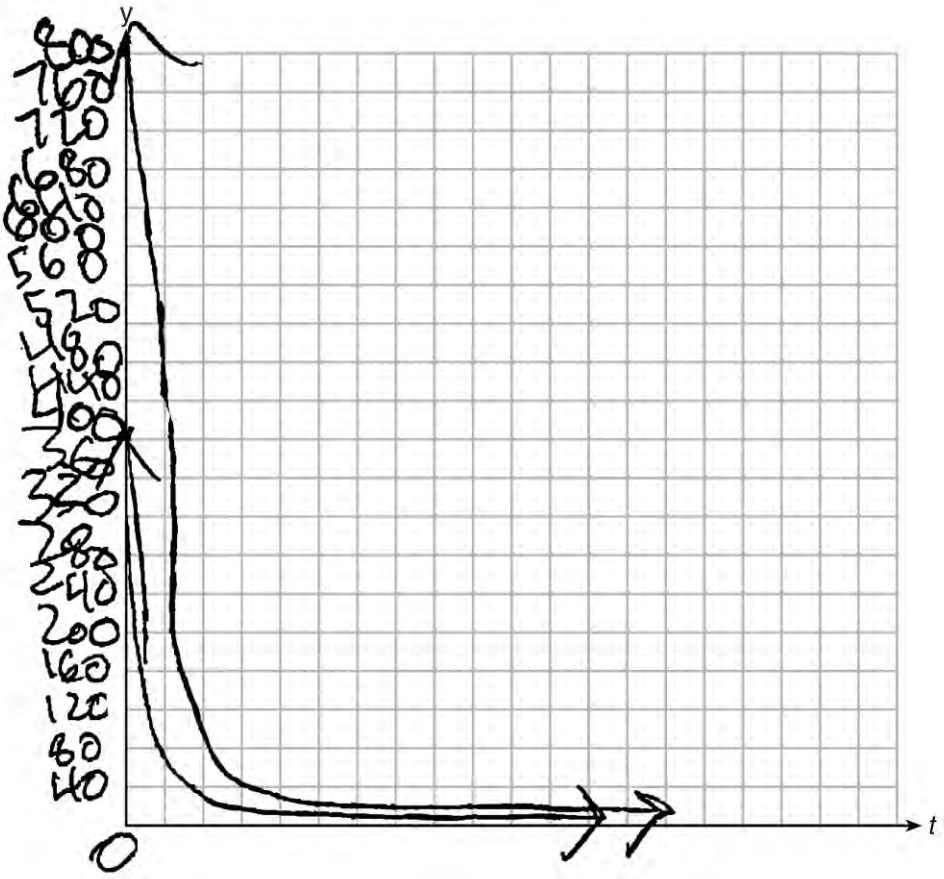
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-0.347t}$

$400(e)^{-0.231t}$

Graph each function on the set of axes below.



Score 2: The student correctly identified 6 hours and 5.5 hours.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

6 hours because that is what N(t) = when you plug 6 in for t

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

5.5 hours

Question 37

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

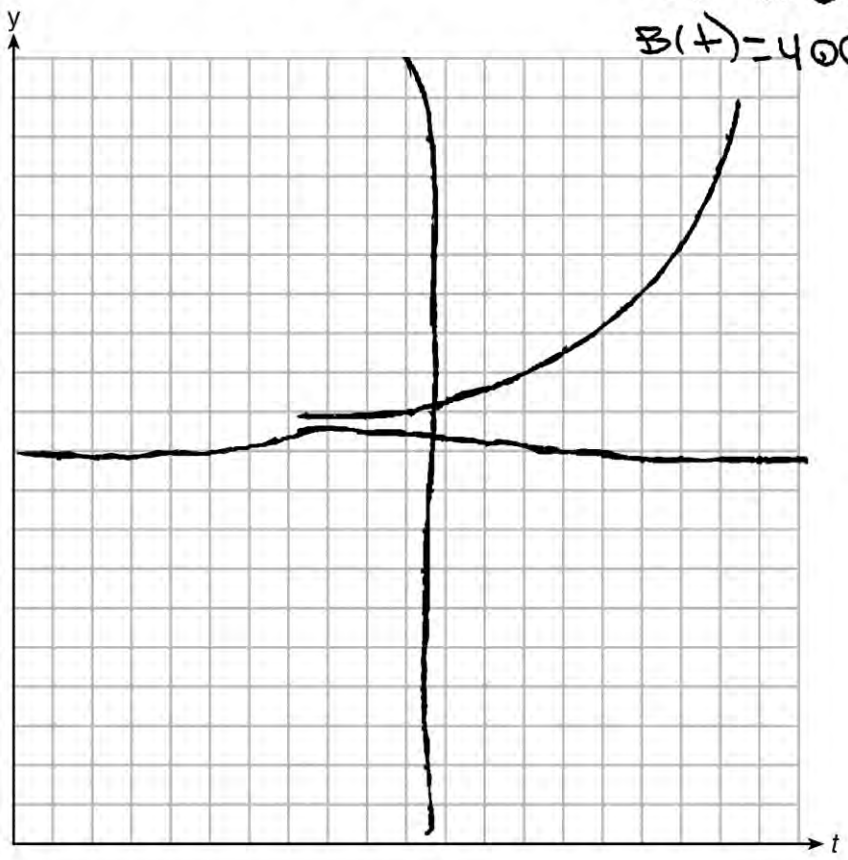
Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

800A

$A(t) = 800(e)^{-.347t}$

$B(t) = 400(e)^{-.231t}$

Graph each function on the set of axes below.



Score 1: The student created and labeled correct functions.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$.15 = 800(e)^{-.347t}$

$.15 = 800(e)^{-.347\ln t}$

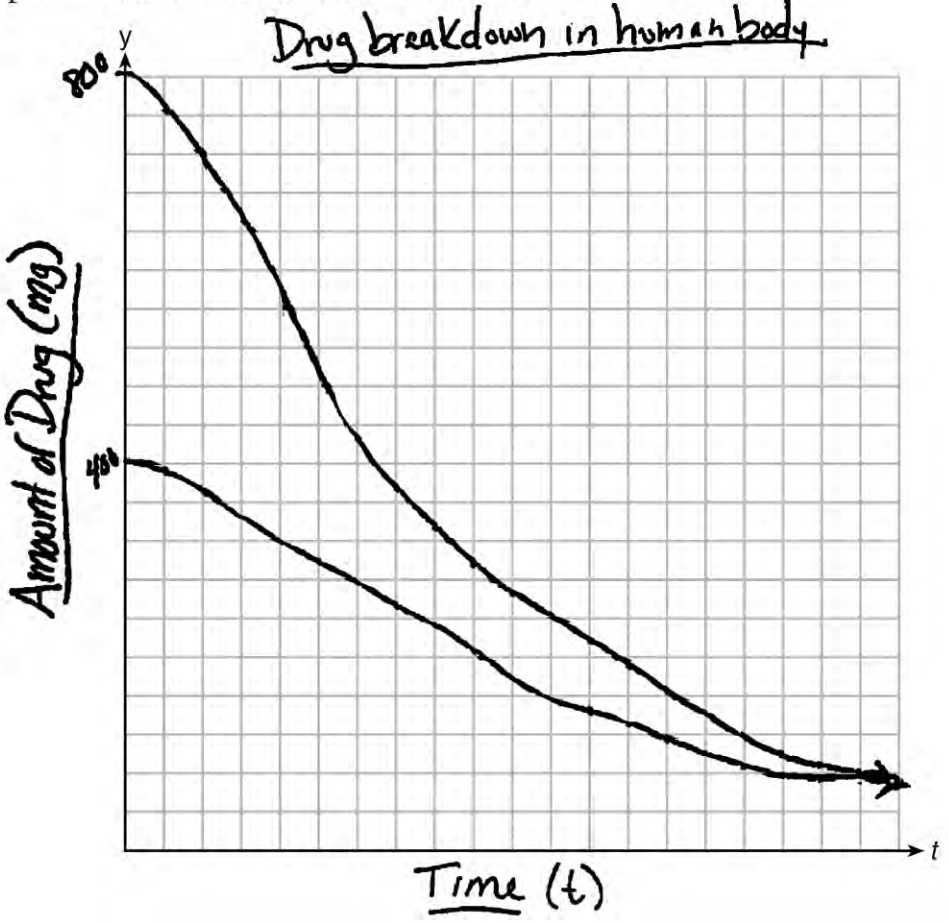
Question 37

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-.347t}$
400

Graph each function on the set of axes below.



Score 0: The student did not complete enough correct work in any part to receive credit.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

Hour 5

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour*, how long patient A will have to wait to take another 800 milligram dose of the drug.

$120 = 800(e)^{-.347t}$

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II (Common Core)

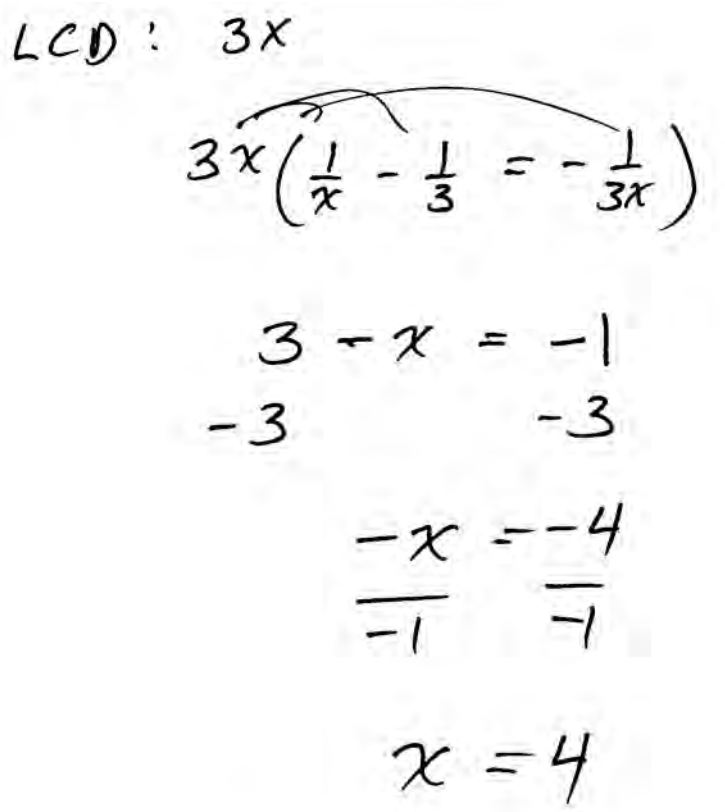
Wednesday, June 1, 2016 — 9:15 a.m. to 12:15 p.m.

MODEL RESPONSE SET

Table of Contents

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$



Score 2: The student gave a complete and correct response.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$3\left(\frac{1}{x}\right) - \left(\frac{1}{3}\right)x = \frac{-1}{3x}$

$\frac{3}{3x} - \frac{1x}{3x} = \frac{-1}{3x}$

$3 - 1x = -1$

$-3 \quad -3$

$\frac{-1x}{-1} = \frac{-4}{-1}$

$x = 4$

Score 2: The student gave a complete and correct response.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$\frac{(3)1}{(3)x} - \frac{(x)1}{(x)3} = \frac{-1}{3x}$$

$$\frac{3}{3x} - \frac{x}{3x} = \frac{-1}{3x}$$

$$\frac{3-x}{3x} = \frac{-1}{3x}$$

Score 1: The student only found a common denominator and combined like terms.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$3x - x = -1$

$2x = -1$

$x = -\frac{1}{2}$

Score 1: The student made an error reducing the first term.

Question 25

25 Solve for x: $\frac{1}{x} - \frac{1}{3} = -\frac{1}{3x}$

$$\frac{1}{x} - \frac{1}{3} = \frac{1}{3x}$$

$$\frac{0}{x-3} \times \frac{1}{3x}$$

$$0 = x - 3$$

$$x = 3$$

Score 0: The student made an error combining the fractions, and also made a transcription error by omitting the negative.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.

Randomly separate 10 volunteers into two groups. Have 5 people try a toothpaste with ingredient X & have 5 people try one without it.

Score 2: The student wrote a correct description of a controlled experiment, including random assignment and a control group.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.

I would collect two groups of individuals that are of equal age and sex to ensure accuracy and eliminate any other variables that can have an effect. I would use a large group of people say 40 in each. Then. I would give one random group an equal amount of toothpaste with the ingredient, where as the other random group will receive toothpaste with no ingredient. It will be given in the morning at the same time. By the end of the day at the same time for a week, I will record the results to determine the impact of the ingredient

Score 2: The student gave a complete and correct response.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.

One group of people will use the version with ingredient X and another will use the tooth paste without. Compare the results.

Score 1: The student wrote an incomplete description by omitting the random assignment of two groups.

Question 26

26 Describe how a controlled experiment can be created to examine the effect of ingredient *X* in a toothpaste.

A controlled experiment can be used by distributing products with the ingredients to a group, while giving the control group to a different group of people.

Score 0: The student's response lacked random assignment and had an insufficient explanation of a control group.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$x - 5 = 0$

$x = 5$

$2(5)^3 - 4(5)^2 - 7(5) - 10 \stackrel{?}{=} 0$

$250 - 100 - 35 - 10 \stackrel{?}{=} 0$

$105 \neq 0$

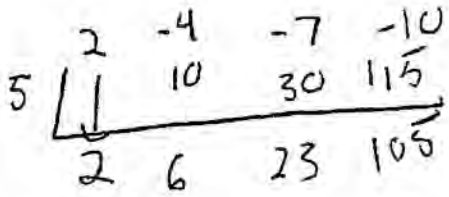
$x-5$ is not a factor of $2x^3-4x^2-7x-10$.

If $x-5$ is a factor of $2x^3-4x^2-7x-10$, then when $2x^3-4x^2-7x-10$ and 5 is substituted for x, the value of $2x^3-4x^2-7x-10$ should be 0.

Score 2: The student gave a complete and correct response.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.



$(x-5)$ is not a factor because the last value (105) does not equal 0

Score 2: The student gave a complete and correct response.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

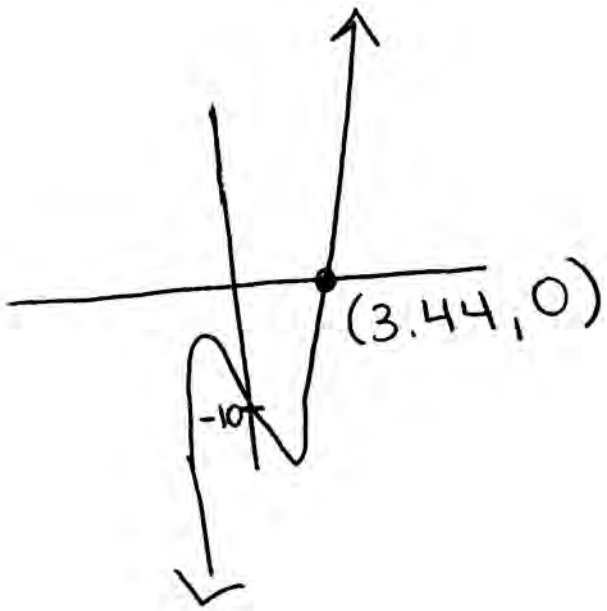
$$
\begin{array}{r|l}
 & 2x^2+6x+23 \\
\hline
x-5 & 2x^3-4x^2-7x-10 \\
 & -2x^3+10x^2 \\
\hline
 & 6x^2-7x \\
 & -6x^2+30x \\
\hline
 & 23x-10 \\
 & -23x+115 \\
\hline
 & 105 = R
\end{array}
$$

x-5 is not a factor because it did not divide evenly out of $2x^3-4x^2-7x-10$.

Score 2: The student gave a complete and correct response.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.



There is one root @ 3.44.
Since there is no root @ 5, x-5
is not a factor.

Score 2: The student gave a complete and correct response.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$x - 5 = 0$

$x = 5$

$2(5)^3 - 4(5)^2 - 7(5) - 10 = 0$

$105 \neq 0$

$x - 5$ is not a factor.

Score 1: The student wrote no explanation.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

$2(-5)^3 - 4(-5)^2 - 7(-5) - 10 = 0$

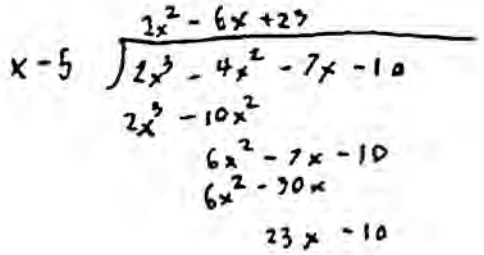
$-325 \neq 0$

x-5 is not a factor because when you use the remainder theorem the remainder is -325 not 0.

Score 1: The student made one error by substituting -5 instead of 5.

Question 27

27 Determine if $x - 5$ is a factor of $2x^3 - 4x^2 - 7x - 10$. Explain your answer.

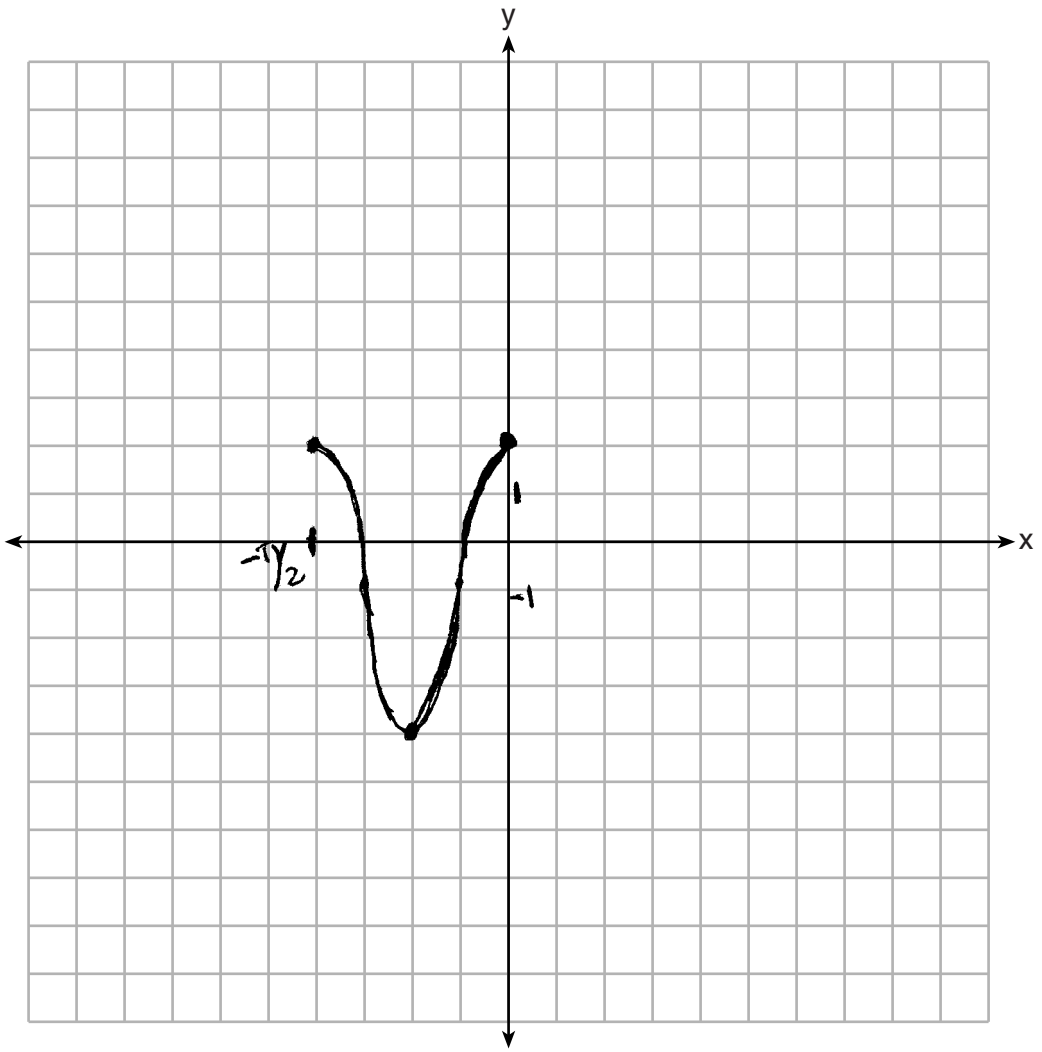


NO

Score 0: The student made multiple errors dividing and did not provide the explanation.

Question 28

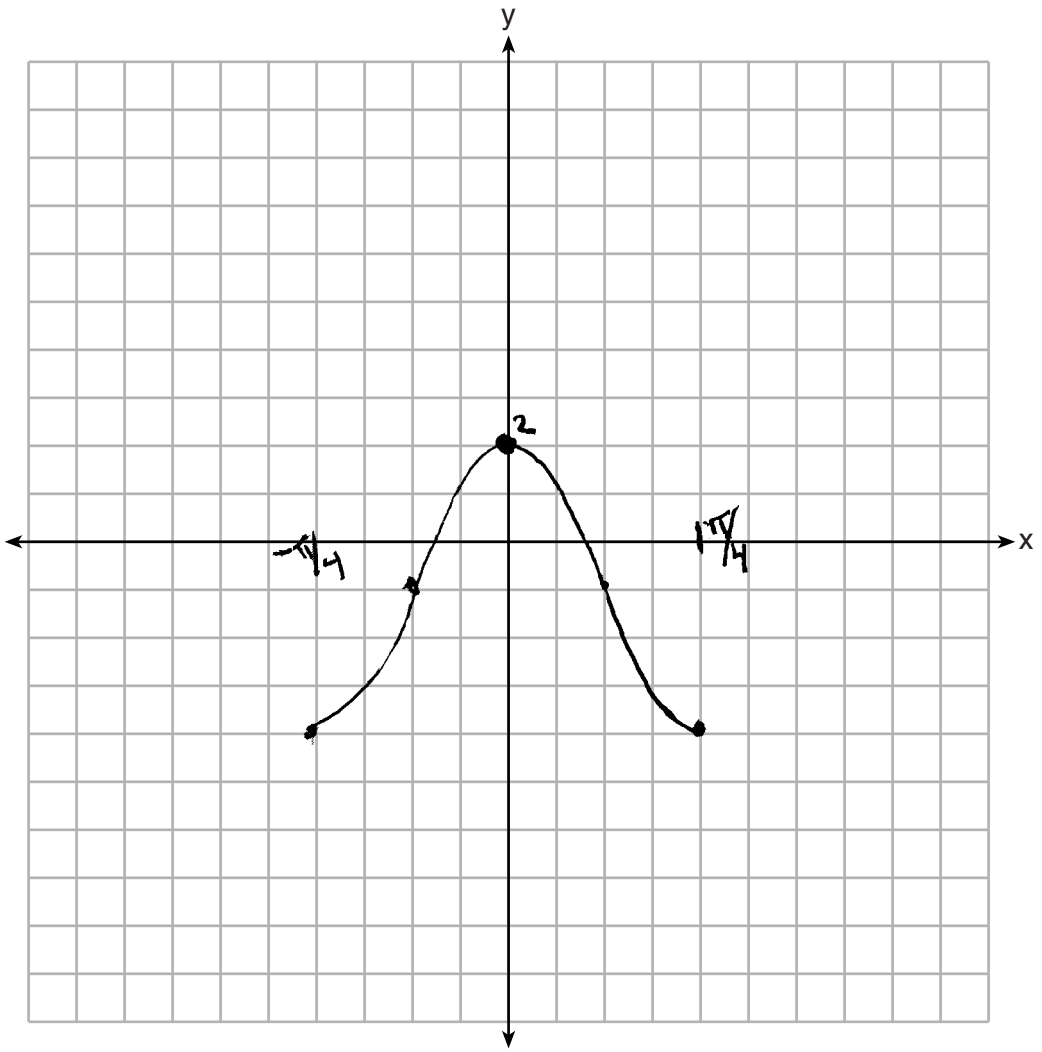
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 2: The student gave a complete and correct response.

Question 28

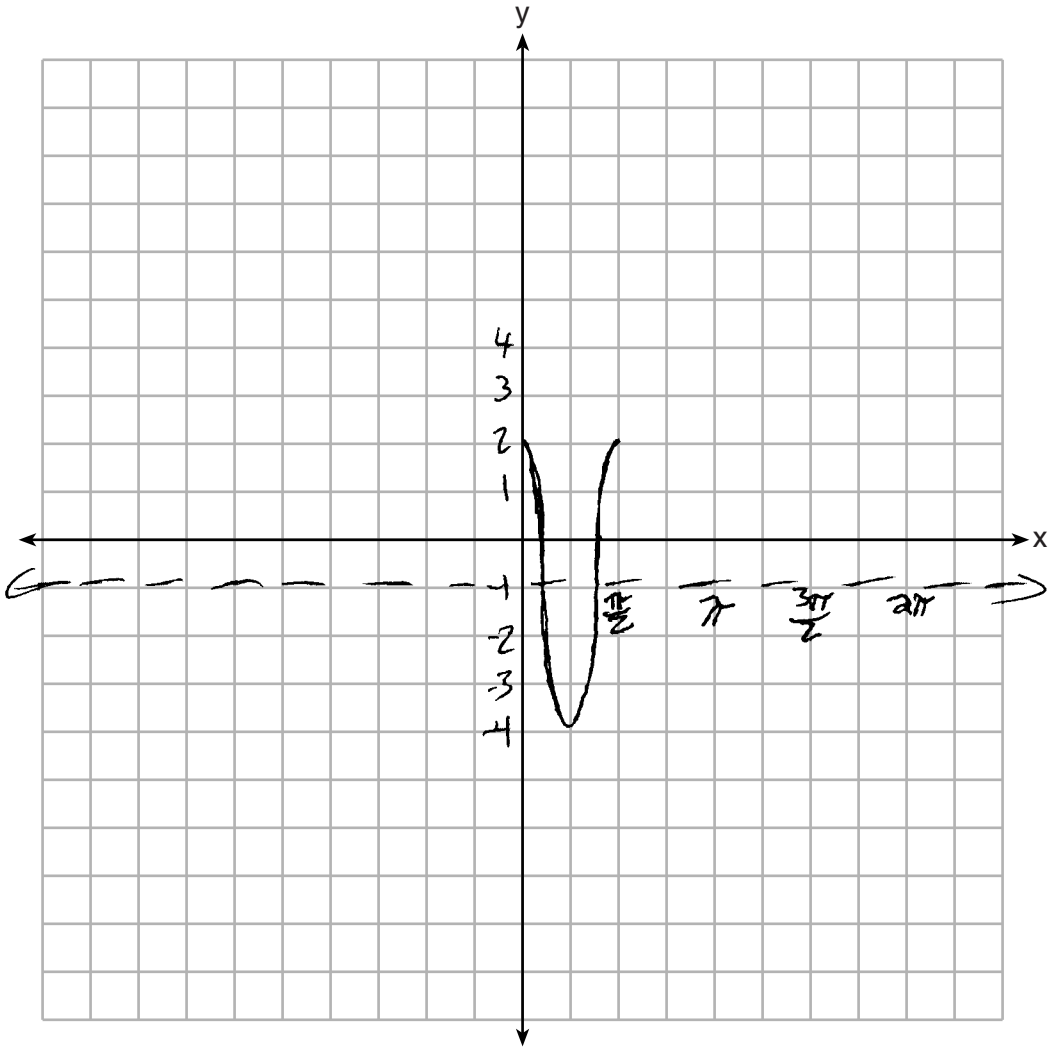
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 2: The student gave a complete and correct response.

Question 28

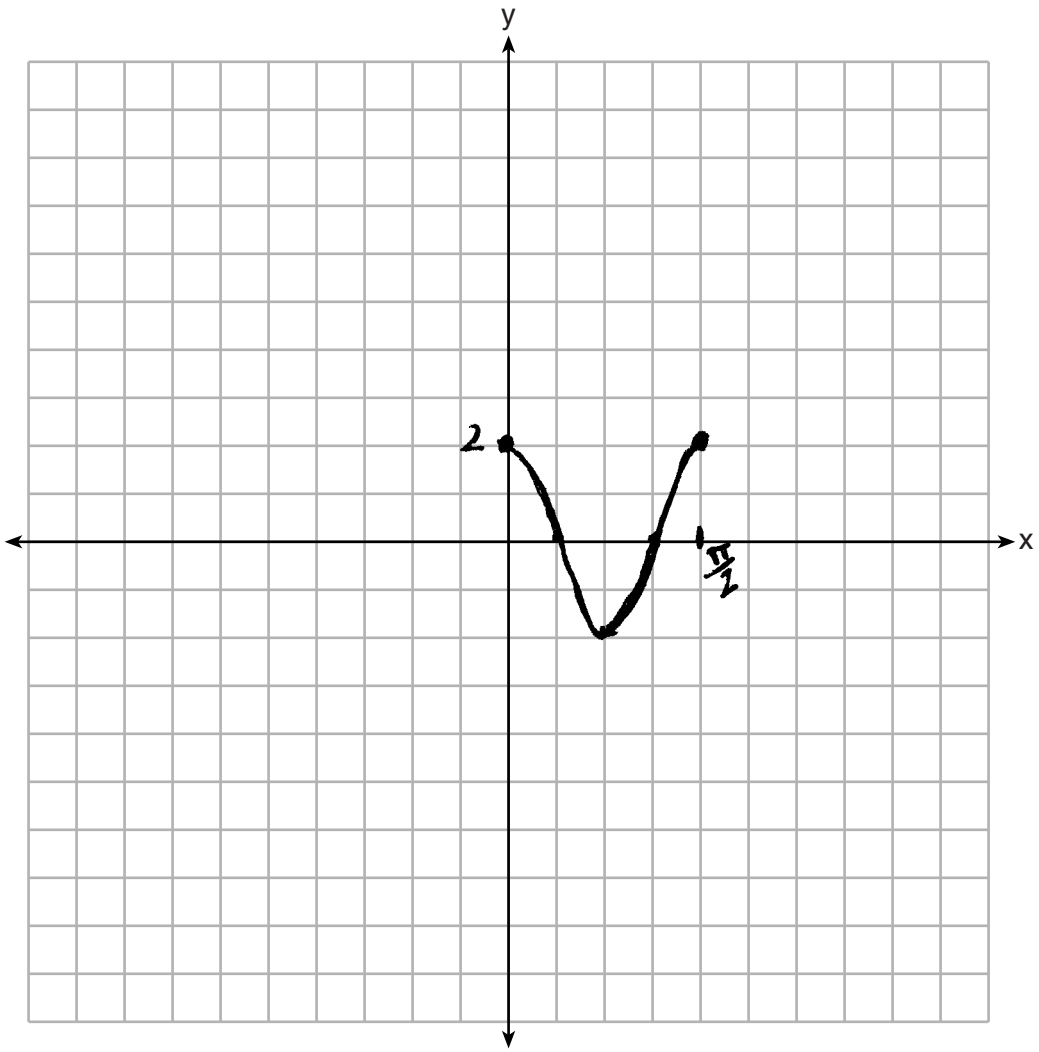
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 2: The student gave a complete and correct response.

Question 28

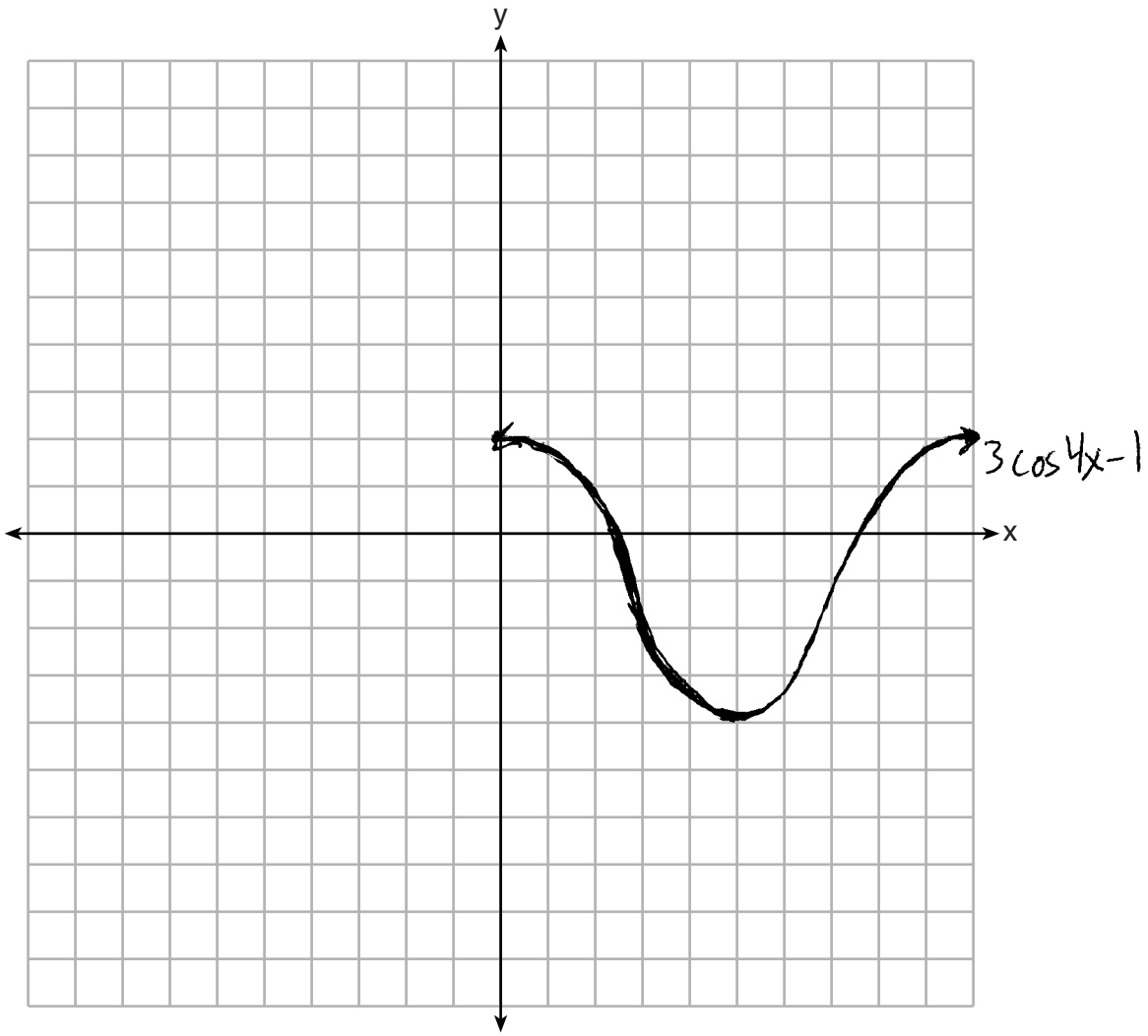
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 1: The student correctly graphed one cycle of a cosine function passing through (0,2) with period $\frac{\pi}{2}$, but used an incorrect amplitude that affected the midline.

Question 28

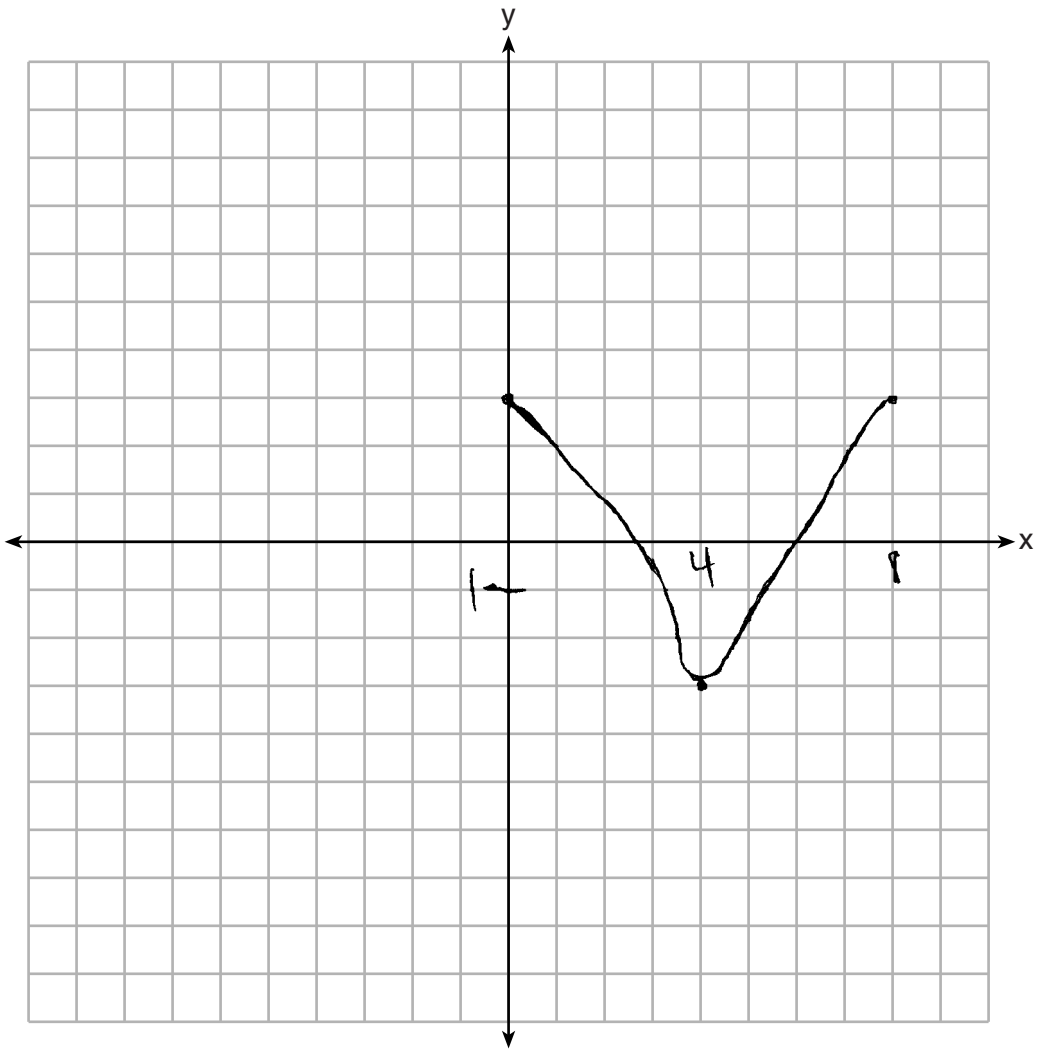
28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 1: The student did not label the axes with appropriate values.

Question 28

28 On the axes below, graph *one* cycle of a cosine function with amplitude 3, period $\frac{\pi}{2}$, midline $y = -1$, and passing through the point (0,2).



Score 0: The student made multiple errors.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

649 + 433

1082 − 974

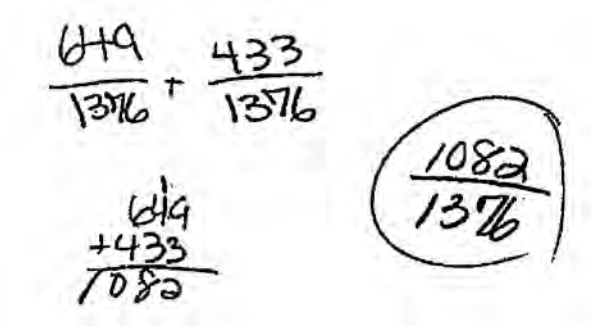
108

1376

Score 2: The student gave a complete and correct response.

Question 29

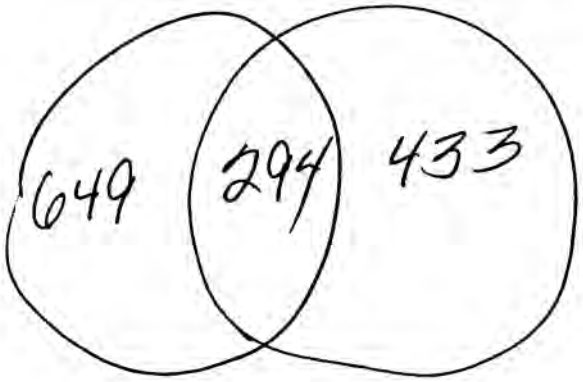
29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?



Score 1: The student made an error by not subtracting from $\frac{974}{1376}$.

Question 29

29 A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?



1376
−1082
294

294
974

Score 0: The student made multiple errors.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$12y + 36 = x^2 - 8x + 16$

$-36 \quad\quad +36$

$12y = x^2 - 8x + 20$

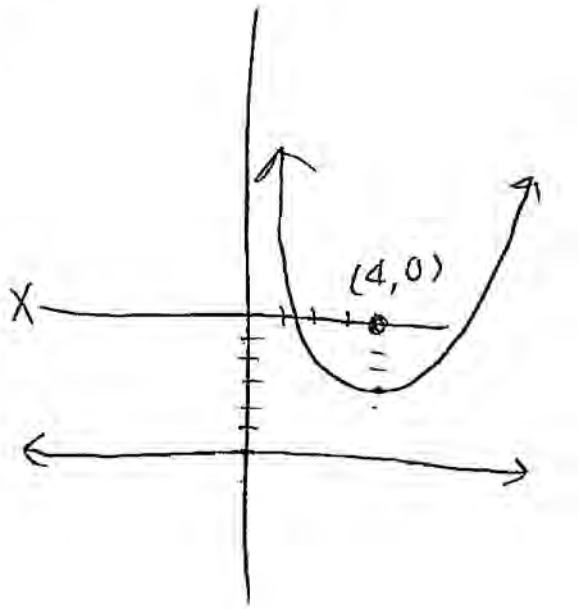
$y = \frac{x^2}{12} - \frac{2x}{3} - \frac{5}{3}$

$x = -\frac{b}{2a} = -\frac{-\frac{2}{3}}{\frac{2}{4}} = \frac{2}{\frac{2}{4}} = \frac{1}{\frac{2}{72}}$

$(x-4)^2 = 12(y+3)$

$4(3)$

$(4, 0)$



Score 2: The student gave a complete and correct response.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$y+3=\frac{1}{12}(x-4)^2$

$y-k=a(x-h)^2$

vertex $(4,-3)$

directix $y=-6$

0

-3

-6

3

$(4,0)$

$(4,0)$

(x,y) (x_2,y_2)

$(x_1,0)$

$d=\sqrt{(x-x_1)^2+(x^2+6)^2}$

$d=\sqrt{(x-x_2)^2+(x^2-y_2)^2}$

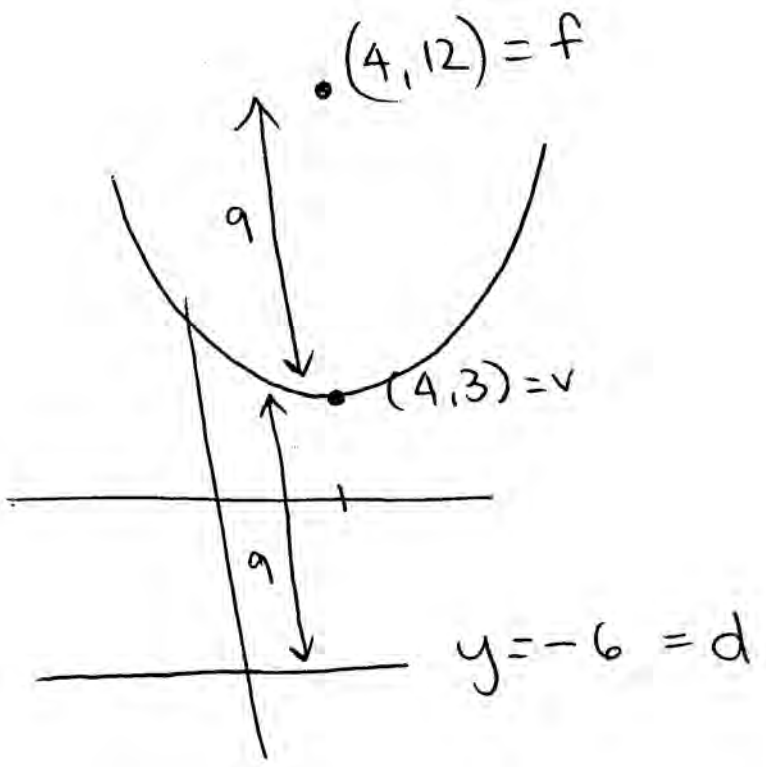
$\sqrt{\quad}=\sqrt{\quad}$

$(4,-3)$ ~~$(4-x_2)^2+(-3)^2-y_2)^2$~~

Score 2: The student gave a complete and correct response.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.



(4,12)

Score 1: The student found an incorrect vertex.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

Vertex = (4, -3)

since directrix is $y = -6$

need to add 6 to y in vertex

Focus = (4, 3)

Score 1: The student misused the directrix.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$(x-4)(x-4)$
$x^2-4x-4x+16$
$x^2-8x+16$

$12(y+3) = (x-4)^2$
$12y+36 = x^2-8x+16$
$-36 \qquad -36$
$\frac{12y}{12} = \frac{x^2-8x-20}{12}$
$y = \frac{x^2-8x-20}{12}$

Focus : $(4, -3)$

Score 0: The student stated the vertex as the focus.

Question 30

30 The directrix of the parabola $12(y + 3) = (x - 4)^2$ has the equation $y = -6$. Find the coordinates of the focus of the parabola.

$12y + 36 = x^2 - 8x + 16$

$\frac{12y}{12} = \frac{x^2 - 8x - 20}{12}$

$y = \frac{x^2 - 8x - 20}{12}$

Focus: (4, 0)
(-8, 0)

Score 0: The student stated a partially correct answer that was obtained by an incorrect procedure.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$$

$$\frac{x^3+9-1+1}{x^3+8}$$

$$\frac{x^3+8+1}{x^3+8}$$

$$\frac{x^3+8}{x^3+8} + \frac{1}{x^3+8}$$

$$1 + \frac{1}{x^3+8} \quad \checkmark$$

Score 2: The student gave a complete and correct response.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$$x^3+8 \overline{)\, x^3+0x^2+0x+9}$$

$$1 + \frac{1}{x^3+8}$$

$$x^3 \qquad +8$$

$$1$$

Score 2: The student gave a complete and correct response.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$$\frac{x^3+9}{x^3+8} \stackrel{?}{=} 1 + \frac{1}{x^3+8}$$

$$= (1)\left(\frac{x^3+8}{x^3+8}\right) + \frac{1}{x^3+8}$$

$$= \frac{x^3+8+1}{x^3+8}$$

$$\frac{x^3+9}{x^3+8} = \frac{x^3+9}{x^3+8}$$

Score 2: The student gave a complete and correct response.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

$\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$

$\frac{x^3+9}{x^3+8} = \frac{(x^3+8)}{(x^3+8)1} + \frac{1}{x^3+8}$

$x^3+9 = x^3+8+1$

$-x^3 \quad -x^3$

$9 = 8+1$

$\boxed{9=9}$

Score 1: The student made an error by not manipulating expressions independently in an algebraic proof.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

MULTIPLY BY COMMON DENOMINATOR: x^3+8

$$\left(x^3+8\right)\left(\frac{x^3+9}{x^3+8}\right) = \left(1+\frac{1}{x^3+8}\right)\left(x^3+8\right)$$

$$x^3+9 = x^3+8+1$$

$$x^3+9 = x^3+9$$

Score 1: The student made an error by not manipulating expressions independently in an algebraic proof.

Question 31

31 Algebraically prove that $\frac{x^3+9}{x^3+8} = 1 + \frac{1}{x^3+8}$, where $x \neq -2$.

let $x=2$

$$\frac{2^3+9}{2^3+8} = \frac{8+9}{8+8} = \frac{17}{16}$$

$$1 + \frac{1}{2^3+8} = 1 + \frac{1}{8+8} = \frac{16}{16} + \frac{1}{16}$$

$$= \frac{17}{16}$$

$$\frac{17}{16} = \frac{17}{16}$$

Score 0: The student used an incorrect procedure by substituting a single value in for x.

Question 32

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent.*

$\frac{135,000}{100,000} = \frac{100,000(1+x)^5}{100,000}$

$1.35 = (1+x)^5$

$\sqrt[5]{1.35} = \sqrt[5]{(1+x)^5}$

$1.06185875 = 1+x$

$-1 \quad -1$

$.06185875 = x$

$x = 6\%$

Score 2: The student gave a complete and correct response.

Question 32

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent.*

$A = A_0 e^{k(t-t_0)} + B_0$

$A_t = A_0 (1+r)^t$

$135{,}000 = 100{,}000 (1+r)^5$

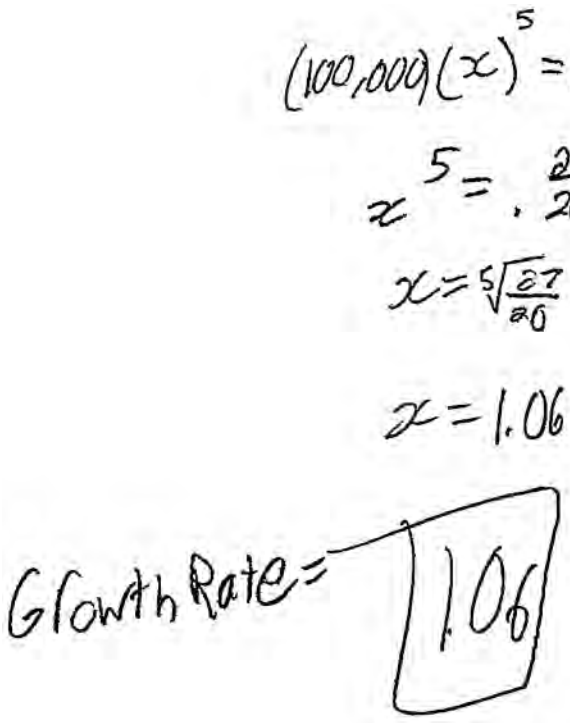
$\frac{27}{20} = (1+r)^5$

$\sqrt[5]{\frac{27}{20}} = 1+r$

Score 1: The student wrote an incomplete solution.

Question 32

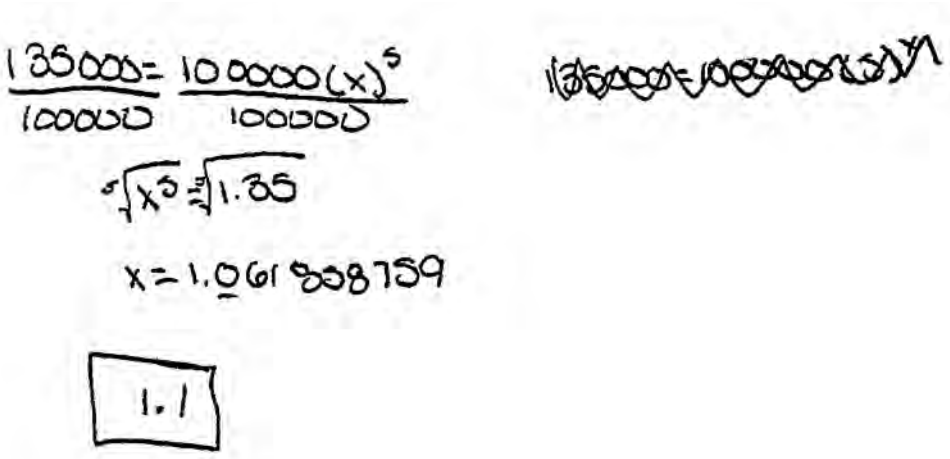
32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent.*



Score 1: The student found the growth factor correctly, but incorrectly stated the annual growth rate percentage.

Question 32

32 A house purchased 5 years ago for \$100,000 was just sold for \$135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent.*



Score 1: The student found the growth factor correctly, but stated an incorrect annual growth rate percentage.

Question 32

32 A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent.*

$135000 = 100000(1+r)^5$

$35000 = (1+r)^5$

$1.54 = 5 \log(1+r)$

$.31 = \log(1+r)$

$10^{.31} = 10^{\log(1+r)}$

$2.042 = 1+r$

$1.042 = r$

Score 0: The student made an error by subtracting 100,000 and did not state a percentage.

Question 33

33 Solve the system of equations shown below algebraically.

$$\begin{cases} (x-3)^2 + (y+2)^2 = 16 & (1) \\ 2x + 2y = 10 & (2) \end{cases}$$

(2) $2x+2y=10$

$2y = 10-2x$

$y = 5-x$ (3)

put (3) to (1): $(x-3)^2 + (5-x+2)^2 = 16$

$x^2-6x+9+49-14x+x^2=16$

$2x^2-20x+58=16$

$2x^2-20x+42=0$

$x^2-10x+21=0$

$-3 \quad -7$

1

$(x-3)(x-7)=0$

$x_1=3 \quad x_2=7$

$y_1=2 \quad y_2=-2$

$2\times3+2y=16$

$2y=4$

$y=2$

$2\times7+2y=10$

$2y=-4$

$y_2=-2$

Score 4: The student gave a complete and correct response.

Question 33

33 Solve the system of equations shown below algebraically.

$(x - 3)^2 + (y + 2)^2 = 16$

$2x + 2y = 10$ –> $2y = -2x + 10$

$y = -1x + 5$

$(x-3)^2 + (y+2)^2 = 16$

$(x-3)(x-3) + (-1x+5+2)(-1x+5+2) = 16$

$x^2 - 6x + 9 + 1x^2 - 5x - 2x - 5x + 25 + 10 - 2x + 10 + 4 = 16$

$2x^2 - 20x + 58 = 16$

$-16 \quad -16$

$2x^2 - 20x + 42 = 0$

84, -6, -14, -20

$2x^2 - 14x - 6x + 42 = 0$

$2x(x-7) - 6(x-7) = 0$

$2x - 6 = 0 \quad x - 7 = 0$

$x = 3 \quad x = 7$

$y = -1(3) + 5$

$y = -3 + 5$

$y = 2$

$(3, 2)$

$y = -1(7) + 5$

$y = -7 + 5$

$y = -2$

$(7, -2)$

Score 4: The student gave a complete and correct response.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x + 2y = 10$
$x + y = 5$
$y = 5 - x$

$(x-3)^2 + (y+2)^2 = 16$
$x^2 - 6x + 9 + y^2 + 4y + 4 = 16$
$x^2 - 6x + 9 + (5-x)^2 + 4(5-x) + 4 = 16$
$x^2 - 6x + 9 + 25 - 10x + x^2 + 20 - 4x + 4 = 16$
$2x^2 - 20x + 42 = 0$
$x^2 - 10x + 21 = 0$
$(x-7)(x-3) = 0$
$x = 7 \quad x = 3$

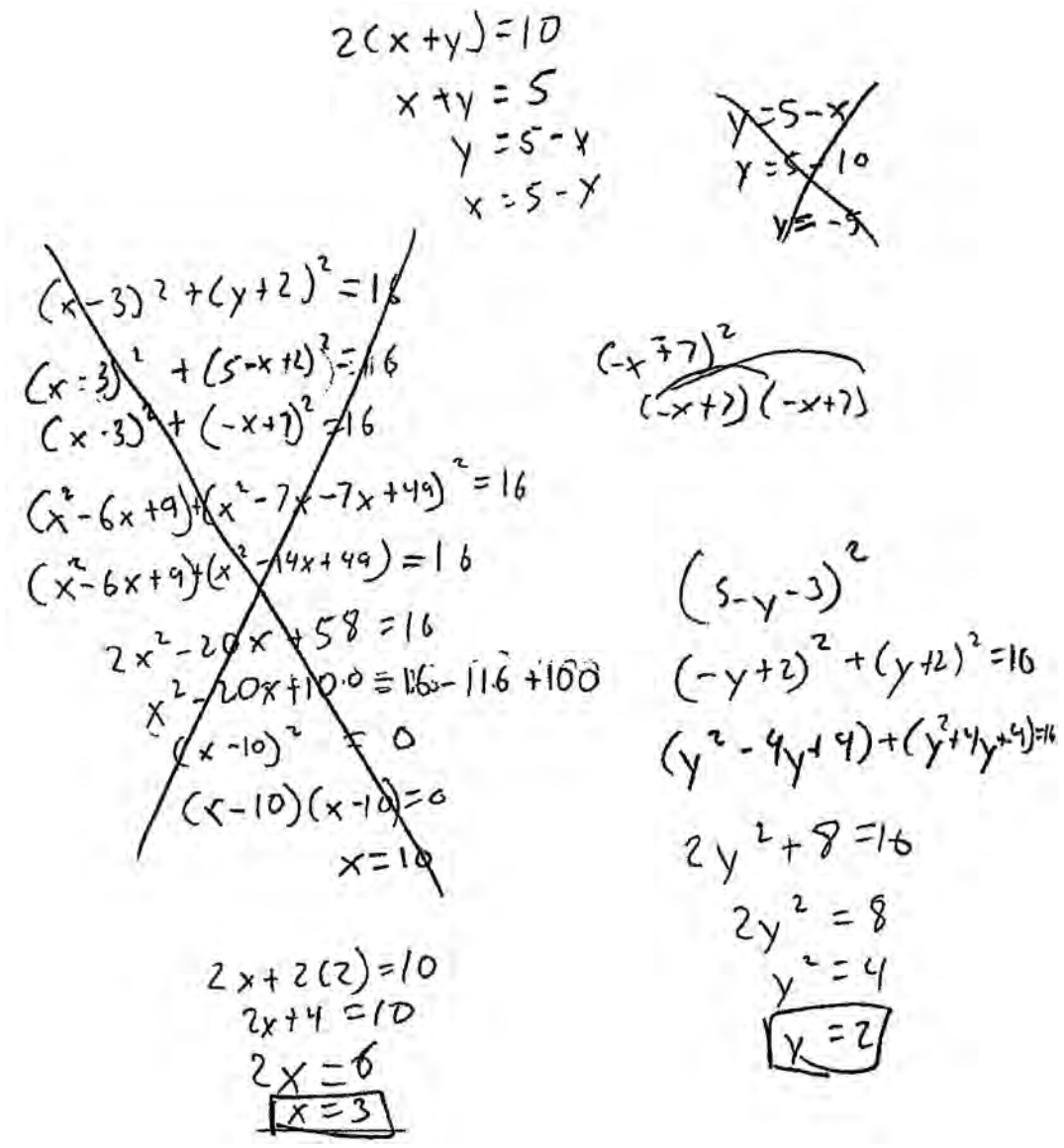
58
-16
42

Score 3: The student only found the correct x-values of the system.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

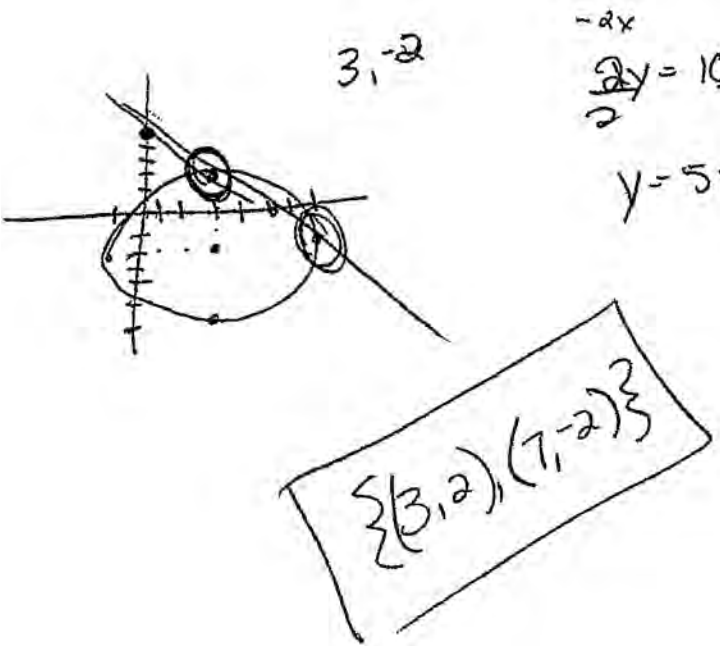


Score 3: The student found only one correct solution of the system.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$



Score 2: The student obtained the correct solution, but used a method other than algebraic.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x+2y=10$

$2y = 10-2x$

$y = 5-x$

$(x-3)^2+(y+2)^2=16$

$x^2-3x+3x+9+y^2+2y+2y+4=16$

$x^2-6x+9+y^2+4y+4=16$

$x^2-6x+y^2+4y+13=16$

$y^2 = -x^2+6x-4y+3$

$(5-x)^2 = -x^2+6x-4(5-x)+3$

$5-x \quad 5-x$

$x^2-5x+5x+25$

$x^2-10x+25 = -x^2+6x-20+4x+3$

$2x^2+45 = 10x+3$

$2x^2-10x+42x=0$

$x^2-5x+21=0$

$x = \frac{5 \pm \sqrt{25-4(1)(21)}}{2}$

$x = \frac{5 \pm i\sqrt{59}}{2}$

Score 2: The student made a transcription error by losing a $-10x$, and did not find y-values.

Question 33

33 Solve the system of equations shown below algebraically.

$(x-3)^2 + (y+2)^2 = 16$

$2x + 2y = 10$

$(x-3)(x-3) + ((5-x)+2)((5-x)+2) = 16$

$x^2 - 3x - 3x + 9 + (3-x)(3-x) = 16$

$x^2 - 6x + 9 + 9 - 3x - 3x + x^2 = 16$

$2x^2 - 12x + 18 = 16$

$x^2 - 12x + 2 = 0$

$\frac{+12 \pm \sqrt{(-12)^2 - 4(2)(1)}}{2(1)} = \frac{12 \pm \sqrt{144}}{2} = \frac{12 \pm 2\sqrt{35}}{2} = 6 \pm \sqrt{35} = x$

$\frac{2x}{2} = \frac{10-2x}{2}$

$y = 5 - x$

$2(6+\sqrt{35}) + 2y = 10$

$23.8 + 2y = 10$

$2y = -13.8$

$y = -6.92$

$2(6-\sqrt{35}) + 2y = 10$

$0167.8 + 2y = 10$

$2y = 9.8$

$y = 4.92$

$(6+\sqrt{35}, -6.92), (6-\sqrt{35}, 4.92)$

Score 2: The student made several computational errors.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x+2y=10$
$2x=10-2y$
$x=\frac{10-2y}{2}$
$x=5-y$

$((5-y)-3)^2+(y+2)^2=16$
$(5-y)(5-y)-9+(y+2)^2=16$
$25-5y-5y+y^2-9+y^2+4y+4=16$
$2y^2-6y+20=16$
$2y^2-6y+4=0$
$2(y^2-3y+2)=0$
$2(y-2)(y-1)=0$

$y-2=0$
$y=2$
$y-1=0$
$y=1$

$2x+2(2)=10$
$2x+4=10$
$2x=6$
$x=3$

$2x+2=10$
$2x=8$
$x=4$

answer=
$y=2, x=3$

check:
$(3-3)^2+(2+2)^2=16$
$(0^2)+(4)^2=16$
$16=16$ ✓

$(3-3)^2+(1+2)^2=16$
$(0^2)+(3^2)=16$
$9\neq16$

$(4-3)^2+(2+2)^2=16$
$(1)^2+(4^2)=16$
$17\neq16$

$(4-3)^2+(1+2)^2=16$
$(1)^2+(3)^2=16$
$1+9=16$
$10\neq16$

Score 1: The student made a conceptual error squaring the first term and did not express both ordered pairs.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$2x+2y=5$
$y=5+x$

$(x-3)^2+(5+x+2)^2=16$

$x^2-6x+9+49+14x+x^2=16$

$2x^2+8x+42=0$

$x^2+4x+21=0$

$(x+7)(x+3)=0$

$x=7 \quad | \quad x=3$

Score 0: The student made several errors and did not find the y-values.

Question 33

33 Solve the system of equations shown below algebraically.

$$(x - 3)^2 + (y + 2)^2 = 16$$
$$2x + 2y = 10$$

$x - 3 + y + 2 = 4$

$x + y - 1 = 4$

$x + y = 5$

$y = 5 - x$

$2x + 2(5 - x) = 10$

$2x + 10 - 2x = 10$

$10 = 10$

Score 0: The student gave a completely incorrect response.

Question 34

34 Alexa earns $33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33{,}000 - 33{,}000(1.04)^{15}}{1 - 1.04}$$

$$S_{15} = \frac{33{,}000 - 33{,}000(1.80)}{-.04}$$

$$S_{15} = \frac{-26{,}431.14}{-.04}$$

$$S_{15} = 660{,}778.39$$

Score 4: The student gave a complete and correct response.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33000 - 33000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$n = 15$$

$$S_{15} = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04}$$

$$= \frac{-26431.14}{-.04}$$

$$= 660,778.50$$

Score 3: The student rounded too early.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000(1.04)^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_n = \frac{33000 - 33000(1.04)^{15}}{1 - 1.04}$$

$$S_n = -26432.18$$

Score 3: The student failed to use parentheses when entering the expression into the calculator.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000(0.04)^n}{1 - 0.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33{,}000 - 33{,}000(0.04)^{15}}{1 - 0.04}$$

$$\frac{33000}{0.96}$$

$$34375$$

Score 2: The student made a conceptual error interpreting the 4% increase.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33{,}000 - 33{,}000 \cdot 1.04^n}{1 - 1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_n = \frac{33{,}000 - 33{,}000 \cdot 1.04^{15}}{1 - 1.04} = \frac{59{,}431}{} = 1{,}485{,}775$$

Score 2: The student only correctly wrote the geometric series formula.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$$S_n = \frac{33000 - (1.04)^N}{1.04}$$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$$S_{15} = \frac{33000 - (1.04)^{15}}{1.04}$$

$$= \frac{33000 + 1.80094}{1.04}$$

$$= \#31732.50$$

Score 1: The student made a computational error in the second part, having received no credit for the first part.

Question 34

34 Alexa earns \$33,000 in her first year of teaching and earns a 4% increase in each successive year.

Write a geometric series formula, S_n, for Alexa's total earnings over n years.

$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ $S_n = \frac{a_1 - a_1 (0.04)(t)}{1 - 0.04}$

Use this formula to find Alexa's total earnings for her first 15 years of teaching, to the *nearest cent*.

$S_n = \frac{33{,}000(0.04)(15)}{.96} =$

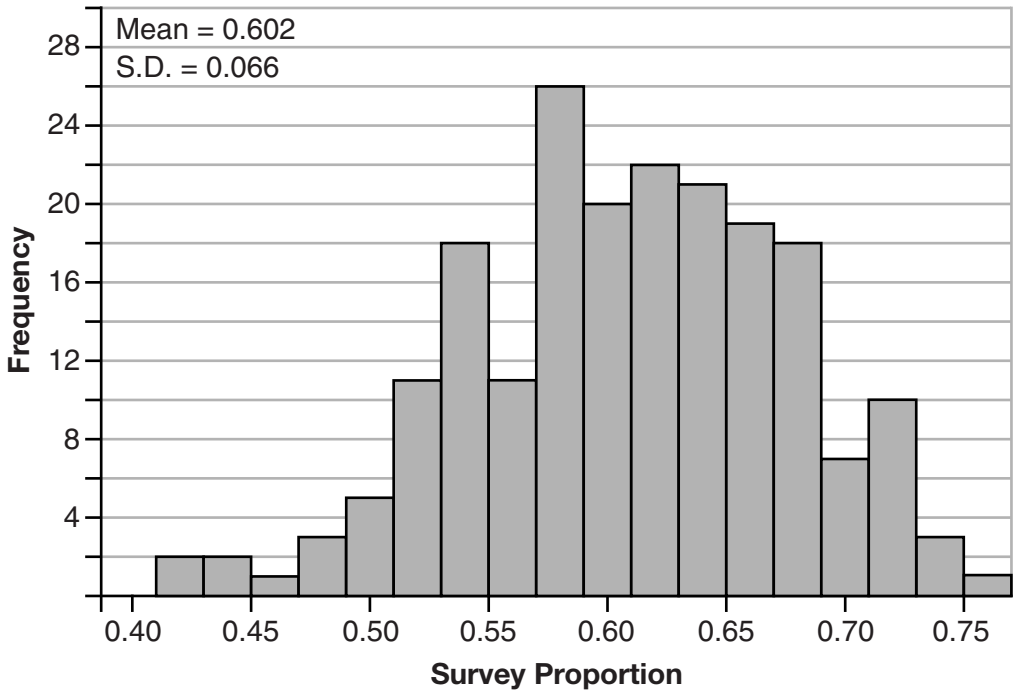
$S_n \approx \$535$

Score 0: The student made multiple errors.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

0.602 ± 0.132

From ~~0.60~~ 0.47 to 0.73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

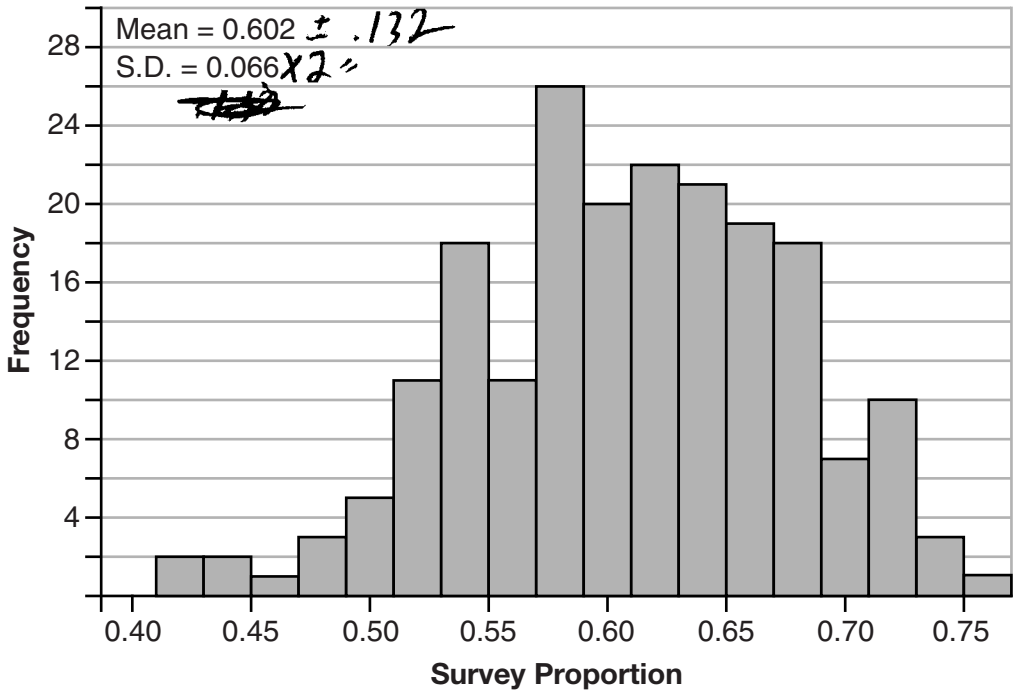
.50 or less occurs 13 out of 200 times which is possible

Score 4: The student gave a complete and correct response.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

0.47 - 0.73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

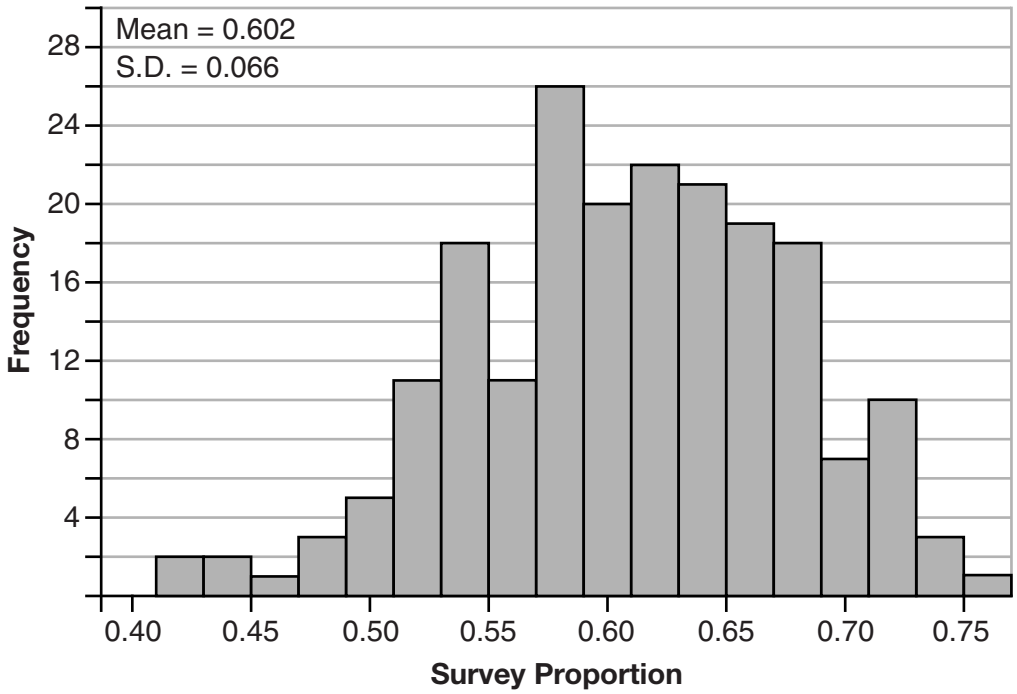
.50 is within this interval so its possible to get a split vote.

Score 4: The student gave a complete and correct response.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

.602 + 2(0.06)

.602 − 2(.06)

(.47 − .73)

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

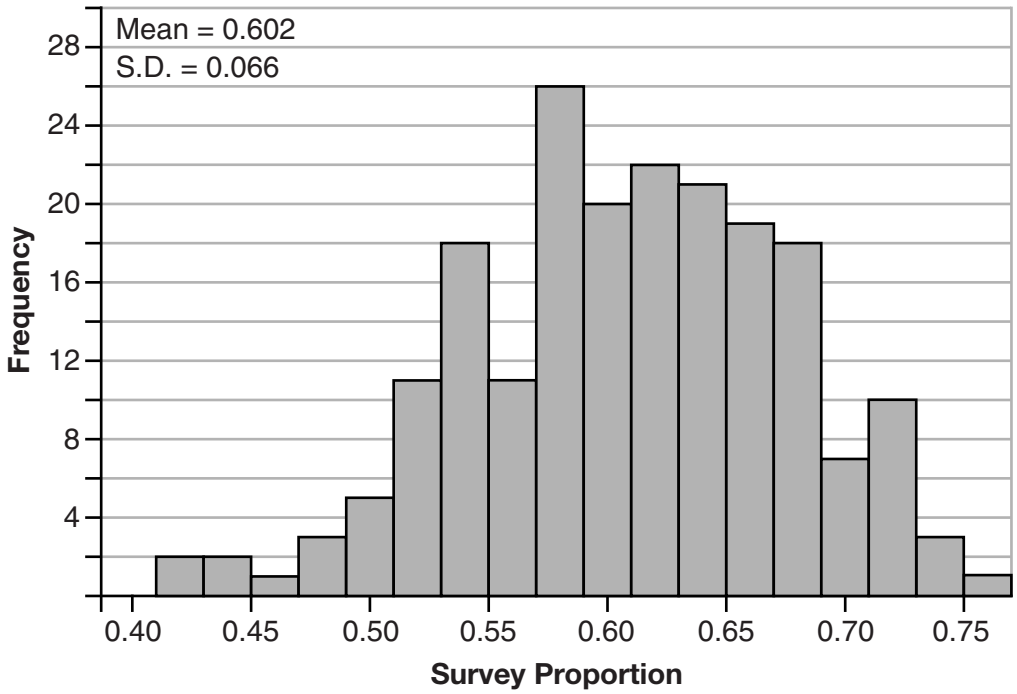
.50 is in the interval (.47 − .73) so even though its close to the end it could happen. .50 = 50% vote

Score 4: The student gave a complete and correct response.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

$0.602 + 2 \cdot 0.066 = 0.734$
$0.602 - 2 \cdot 0.066 = 0.47$

0.47 to 0.73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

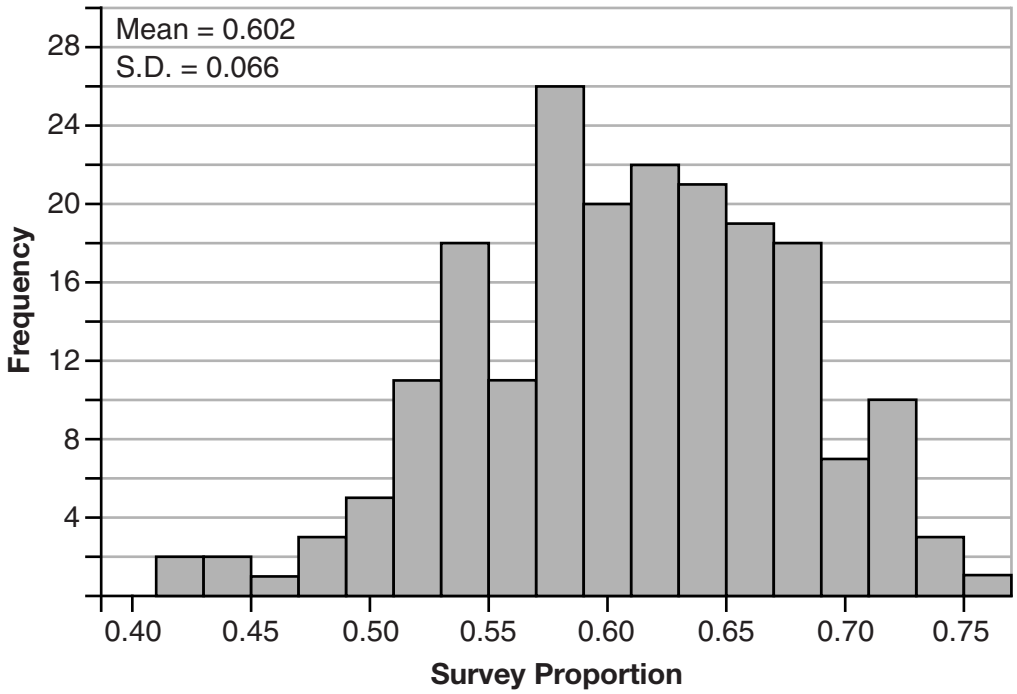
It is not a concern since .50 falls within the interval.

Score 3: The student determined a correct interval, but provided contradictory statistical evidence.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

$0.602 \pm 2 \cdot 0.066$ → .47, .734

.47 to .73

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

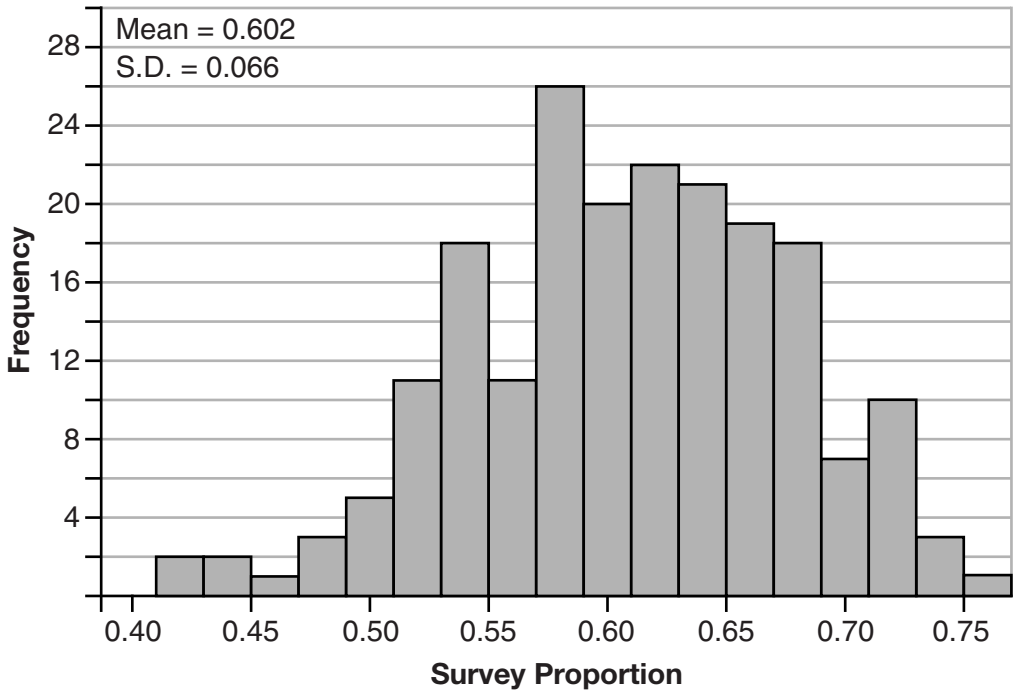
The graph shows 0.50

Score 2: The student gave no statistical explanation.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

Interval is .602 ± .066 or .536 - .668

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

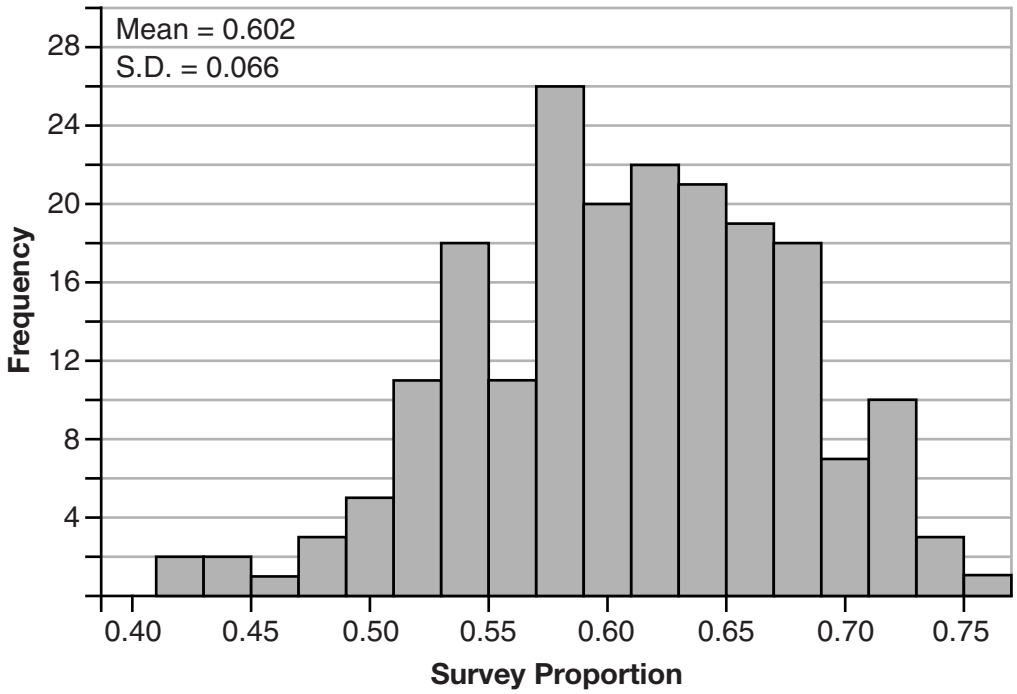
There is concern because .50 is not within this interval

Score 1: The student used only one standard deviation in the interval, rounded incorrectly, and provided contradictory statistical evidence.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

.066 + 2(.602) = 1.27

.066 − 2(.602) = −1.138

−1.138 to 1.27

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

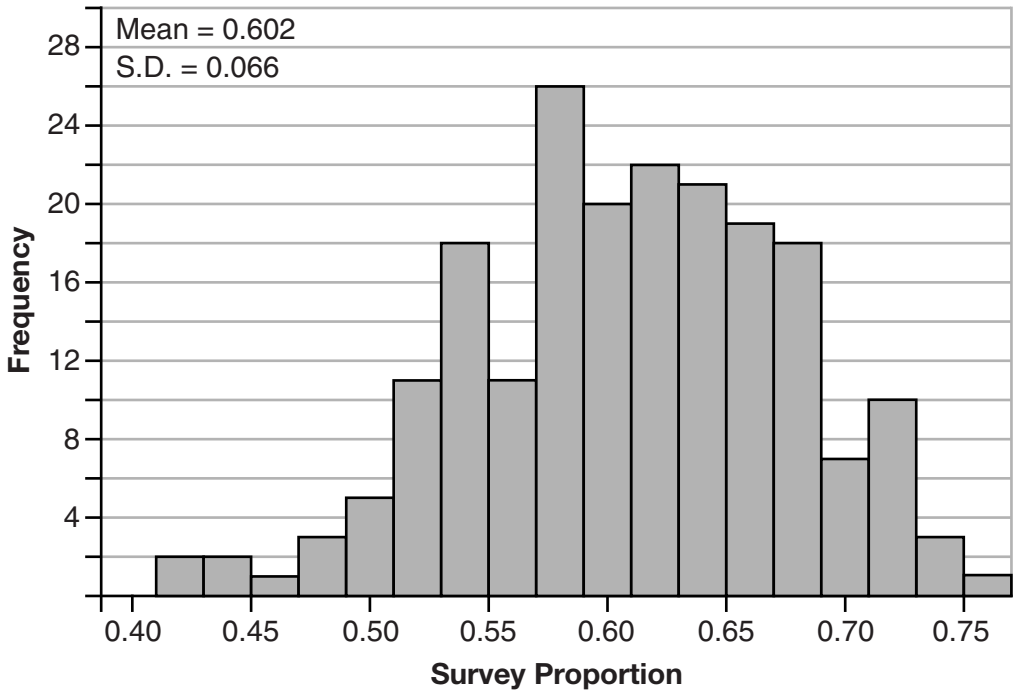
Since 60% of the students voted and 50% is close to 60%, they could be concerned that 50% is possible because voting can vary.

Score 1: The student used the standard deviation as the center and rounded incorrectly. The student gave an incomplete explanation.

Question 35

35 Fifty-five students attending the prom were randomly selected to participate in a survey about the music choice at the prom. Sixty percent responded that a DJ would be preferred over a band. Members of the prom committee thought that the vote would have 50% for the DJ and 50% for the band.

A simulation was run 200 times, each of sample size 55, based on the premise that 60% of the students would prefer a DJ. The approximate normal simulation results are shown below.



Using the results of the simulation, determine a plausible interval containing the middle 95% of the data. Round all values to the *nearest hundredth.*

.602 + .132 = .734 (.602, .734)

Members of the prom committee are concerned that a vote of all students attending the prom may produce a 50% – 50% split. Explain what statistical evidence supports this concern.

.5 is in the interval.

Score 0: The student made multiple conceptual and computational errors.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

$g(-2) = -32 - 20 + 3$
$= -49$

$g(4) = 179$

$\frac{80 - 1.25}{4 - (-2)} = \frac{78.75}{6}$

$= 13.125$

$= \frac{105}{8}$.

$\frac{179 - (-49)}{4 - (-2)}$

$= \frac{228}{6}$

$= 38$

g(x) b/c its rate of change over [-2, 4] is 38, and f(x)'s rate of change is 13.125.

Score 4: The student gave a complete and correct response.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

g(x) has a greater avg rate of change on the interval [-2,4] because g(x) went from -49 to 179 which is a greater change than from 1.25 to 80.

Score 4: The student gave a complete and correct response.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

X Y

x	f(x)
−4	0.3125
−3	0.625
x_1 −2	y_1 1.25
−1	2.5
0	5
1	10
2	20
3	40
x_2 4	y_2 80
5	160
6	320

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{80 - 1.25}{4 - (-2)}$

$\frac{78.75}{7} = 11.25$ rate

$g(x) = 4x^3 - 5x^2 + 3$

X	Y
−2	−49
−1	−26
0	3
1	2
2	15
3	66
4	179

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{179 - (-49)}{4 - (-2)}$

$\frac{228}{7} = 32.57142857$ rate

$g(x) = 4x^3 - 5x^2 + 3$ has the greater average rate of change on the interval $[-2, 4]$

Score 3: The student made a computational error when calculating the denominators.

Question 36

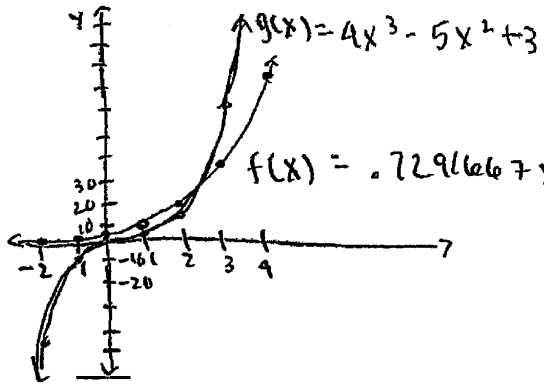
36 Which function shown below has a greater average rate of change on the interval [−2, 4]? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

x	g(x)
−2	−49
−1	−6
0	3
1	2
2	15
3	66
4	179

Function $g(x) = 4x^3 - 5x^2 + 3$ has a greater average rate of change in the interval [-2, 4]. This is because when both functions are graphed, g(x)'s slope is steeper than f(x)'s.



$f(x) = .729166x^3 + 1.27976x^2 + 1.80059x + 5$

Score 2: The student made a conceptual error by creating an appropriate model for $f(x)$, but wrote an appropriate explanation for that model.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

$y = 5 \cdot 2^x$

$g(x)=4x^3-5x^2+3$ has a greater average rate of change because between the interval $[-2,4]$ it went from $(-2,-49)$ to $(4,179)$ for the chart, the function would be $f(x)=5\cdot 2^x$. In this case, it went from $(-2,1.25)$ to $(4,80)$. Between the two functions, $g(x)=4x^3-5x^2+3$ had the greater average rate of change.

Score 2: The student found $g(-2)$ and $g(4)$ correctly, but made no comparison of the average rates of change.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

78.75

$g(x) = 4x^3 - 5x^2 + 3$

$g(-2) = -8(4) - 5(4) + 3$

$= -32 - 20 + 3$

$= -52 + 3$

$= -49$

$g(4) = 4(64) - 5(16) + 3$

$= 252 - 80 + 3$

$= 0169$

$169 > 78.75$

$g(x) = 4x^3 - 5x^2 + 3$ has a greater change.

Score 1: The student made an error finding the average rates of change by not dividing by Δx, and made one computational error.

Question 36

36 Which function shown below has a greater average rate of change on the interval $[-2, 4]$? Justify your answer.

x	f(x)
−4	0.3125
−3	0.625
−2	1.25
−1	2.5
0	5
1	10
2	20
3	40
4	80
5	160
6	320

$g(x) = 4x^3 - 5x^2 + 3$

↑this function has a greater ~~area~~ average change on the interval [-2,4] because this function is a geometric sequence which doubles its x-values.

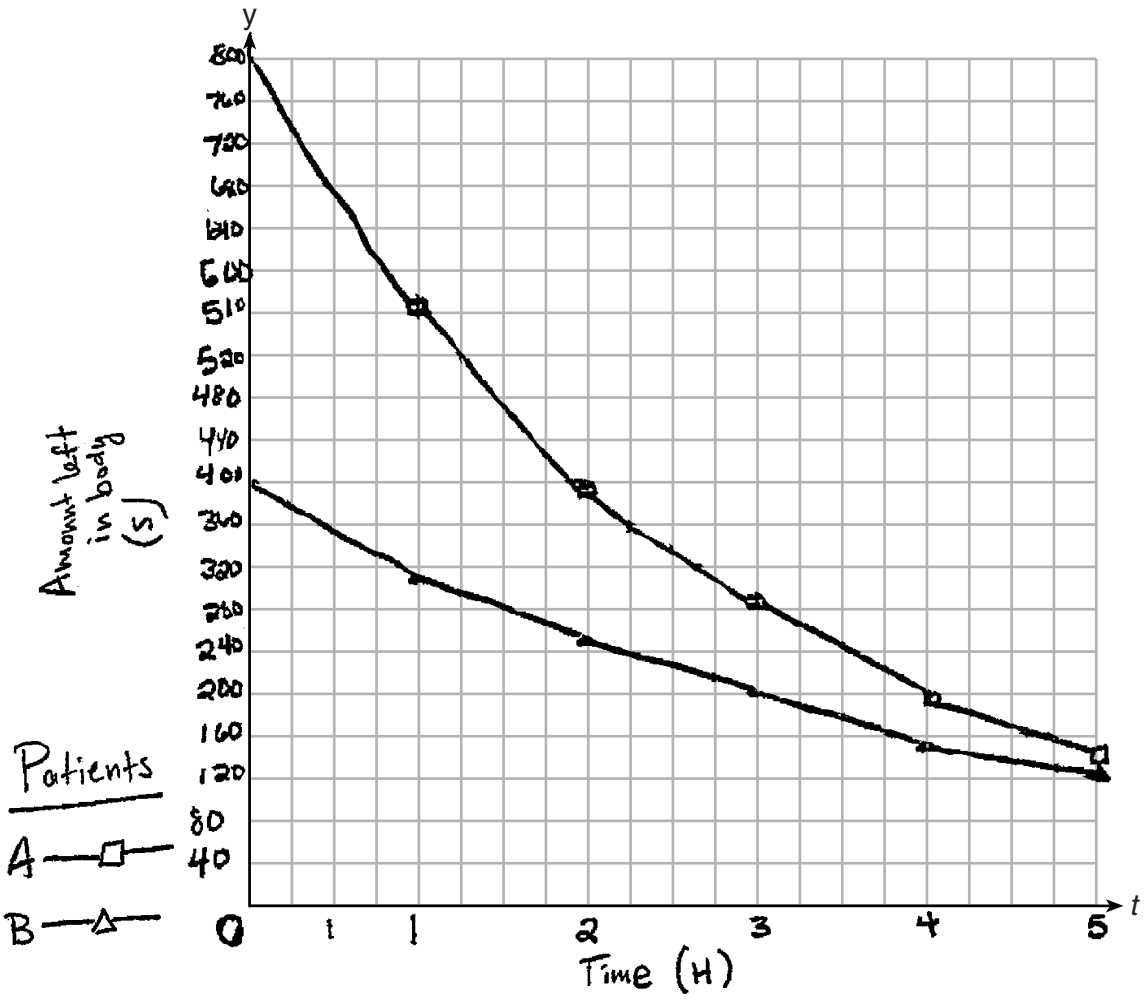
Score 0: The student did not calculate an average rate of change and wrote an irrelevant explanation.

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

P_A $A(t) = 800\text{mg}(e)^{-.347t}$ P_B $B(t) = 400\text{mg}(e)^{-.231t}$

Graph each function on the set of axes below.



Score 6: The student gave a complete and correct response.

Question 37

To the *nearest hour*, t, when does the amount of the given drug remaining in patient B begin to exceed the amount of the given drug remaining in patient A?

Hour 6 because when the equation is solved the amount left in B is 100 while A has about 99.7.

The doctor will allow patient A to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient A will have to wait to take another 800 milligram dose of the drug.

$800(.15) = 120mg$

$H_5 = 141$

$\frac{120}{800} = \frac{800mg(e)^{-.347(t)}}{800}$

$.15 = (e)^{-.347(t)}$

$\log_e .15 = \log_e e^{-.347(t)}$

$\frac{-1.897}{-.347} = \frac{-.347(t)}{-.347}$

$5.5 hrs = t$

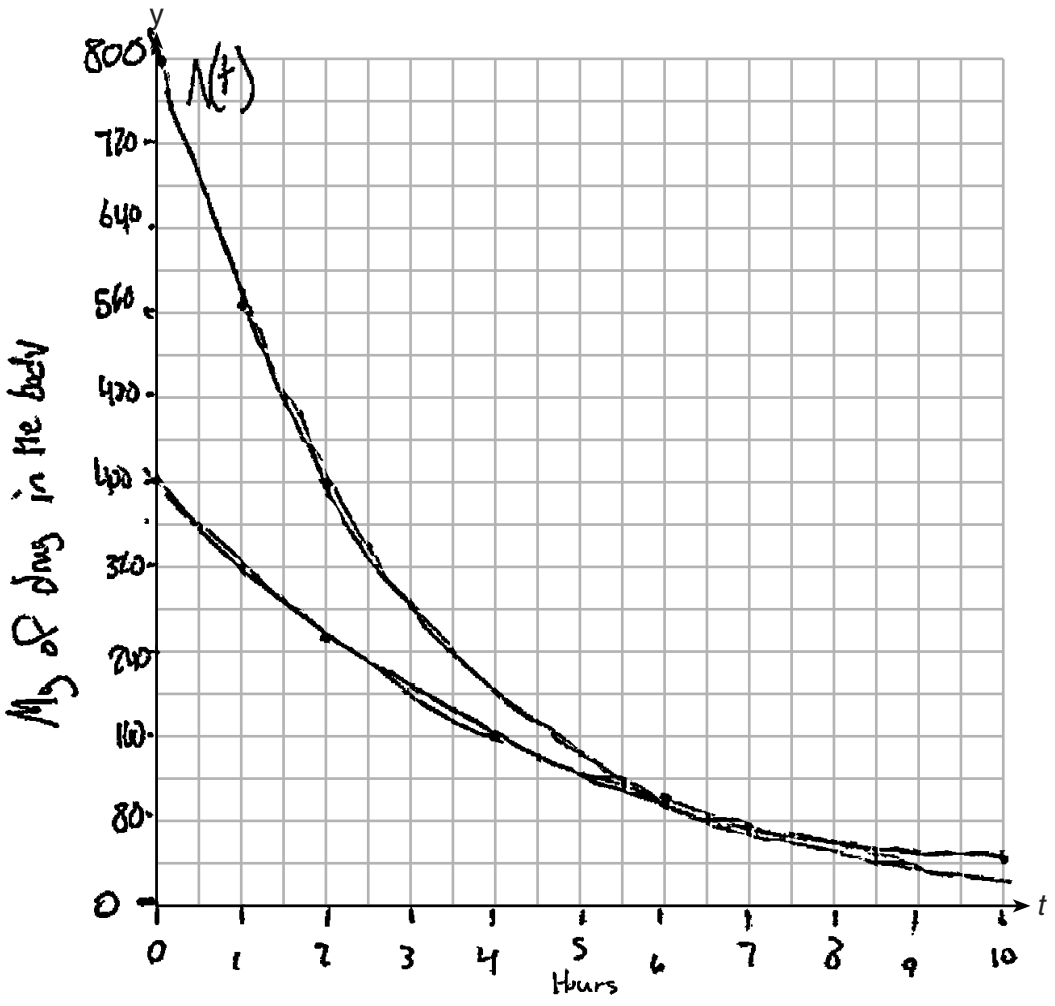
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$N(t) = 800(e)^{-.347t}$

$400(e)^{-.231t}$

Graph each function on the set of axes below.



Score 5: The student did not indicate which function models which patient.

Question 37

To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

Hour 6

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

$\frac{120}{800} = \frac{800(e)^{-.347t}}{800}$

$.15 = e^{-.347t}$

$\log_e .15 = -.347t$

$\frac{\ln .15}{-.347} = \frac{-.347t}{-.347}$

$5.5 = t$

5.5 hours

Question 37

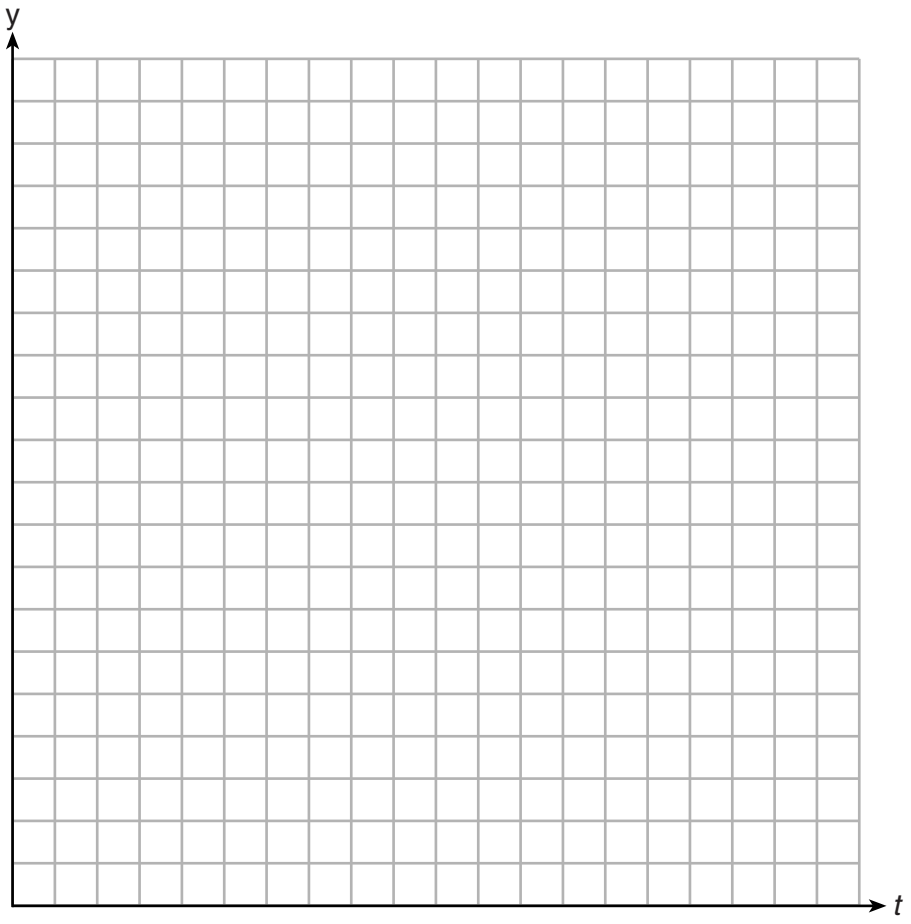
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$A(t) = 800mg(e)^{-.347t}$

$B(t) = 400mg(e)^{-.231t}$

Graph each function on the set of axes below.



Score 4: The student did not graph either function.

Question 37

To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

After 6 hours. I see this after graphing both functions on my calculator and looking at the table. I could then see that at 6 hours Patient a would have 99.74 mg of drug, while patient b would have 100.03. This is probably because, despite starting with more drug, Patient A's decay rate is also greater.

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

$\frac{15}{100} = \frac{x}{800}$

$\frac{15 \cdot 800}{100} = \frac{100x}{100}$

120 mg left $= x$

Using my graphing calculator... I know that Patient A would have to wait approximately 5.5 hours or 5 hours and 30 minutes.

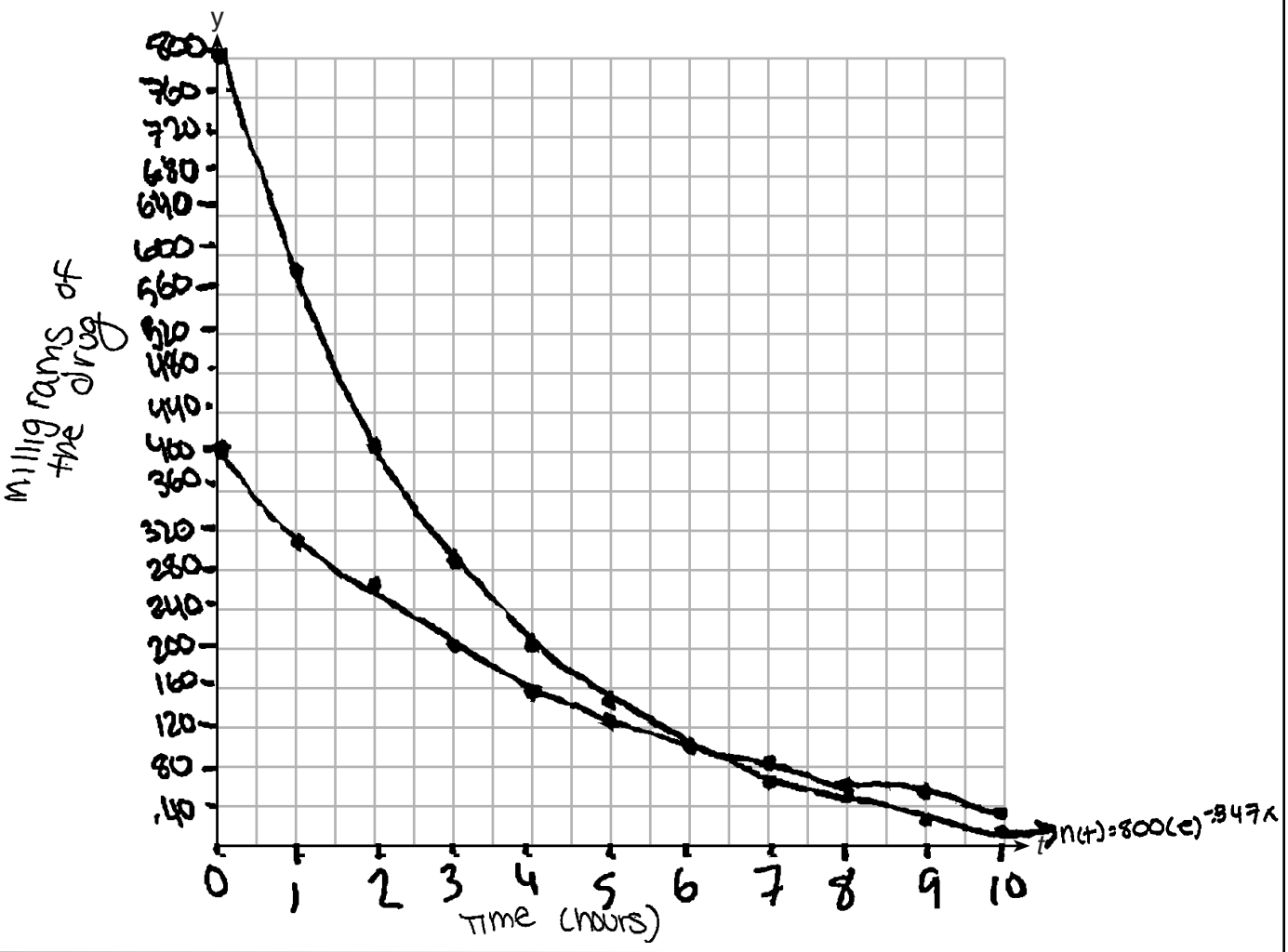
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-.347x}$

$400(e)^{-.231x}$

Graph each function on the set of axes below.



Score 3: The student drew a correct graph and gave a correct answer of 5.5 hours.

Question 37

To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

Hour 7

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

5.5 hours

Question 37

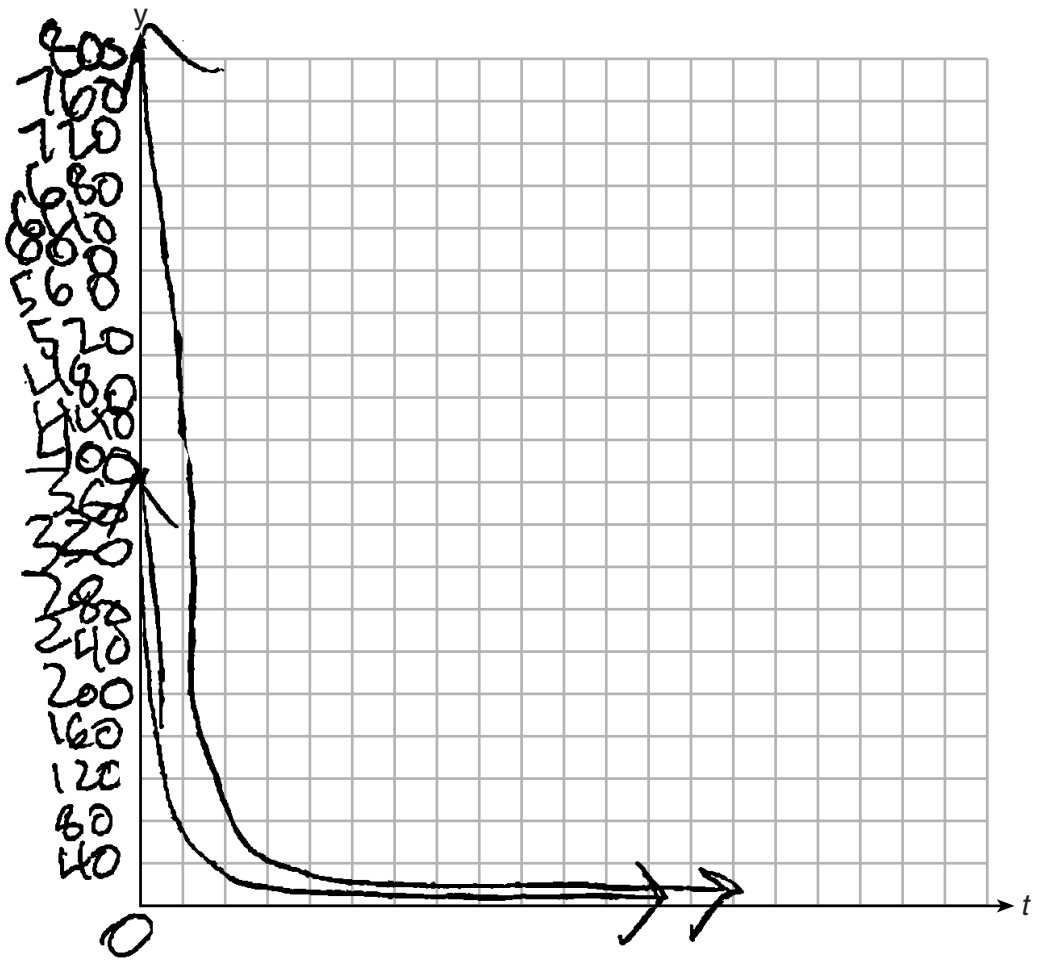
37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-0.347t}$

$400(e)^{-0.231t}$

Graph each function on the set of axes below.



Score 2: The student correctly identified 6 hours and 5.5 hours.

Question 37

To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

6 hours because that is what N(t) = when you plug 6 in for t

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

5.5 hours

Question 37

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

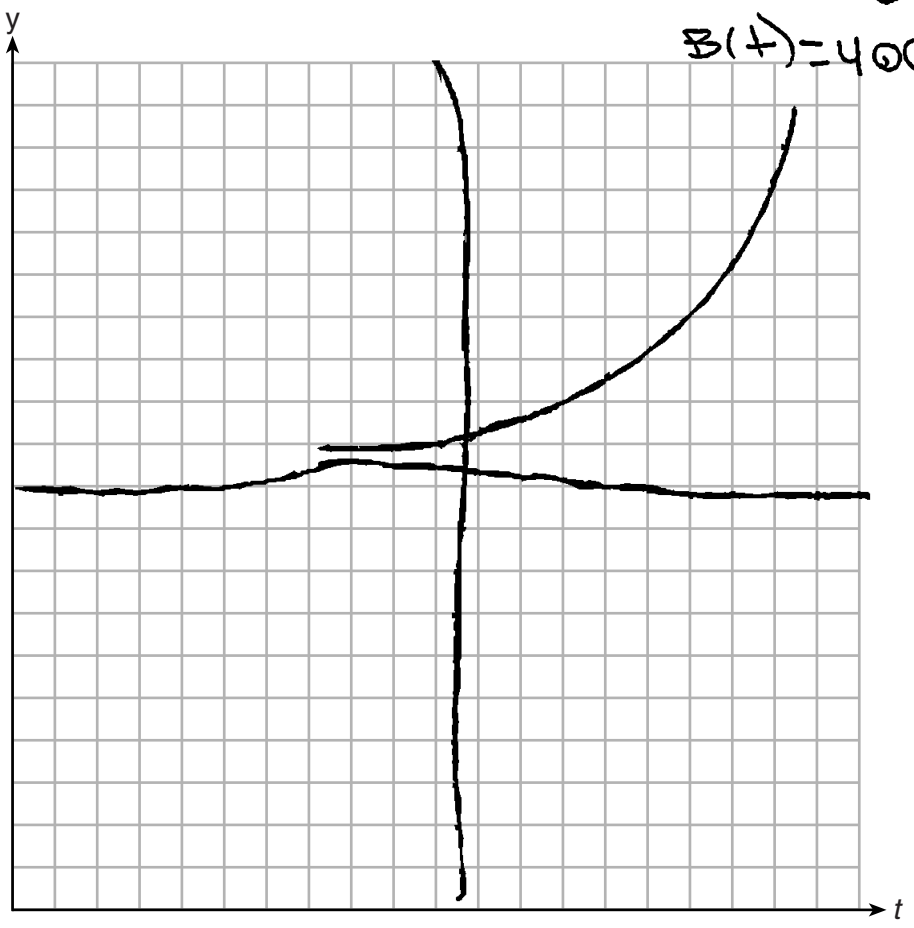
Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800A$

$A(t) = 800(e)^{-.347t}$

$B(t) = 400(e)^{-.231t}$

Graph each function on the set of axes below.



Score 1: The student created and labeled correct functions.

Question 37

To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

$.15 = 800(e)^{-.347t}$

$.15 = 800(e)^{-.347\ln t}$

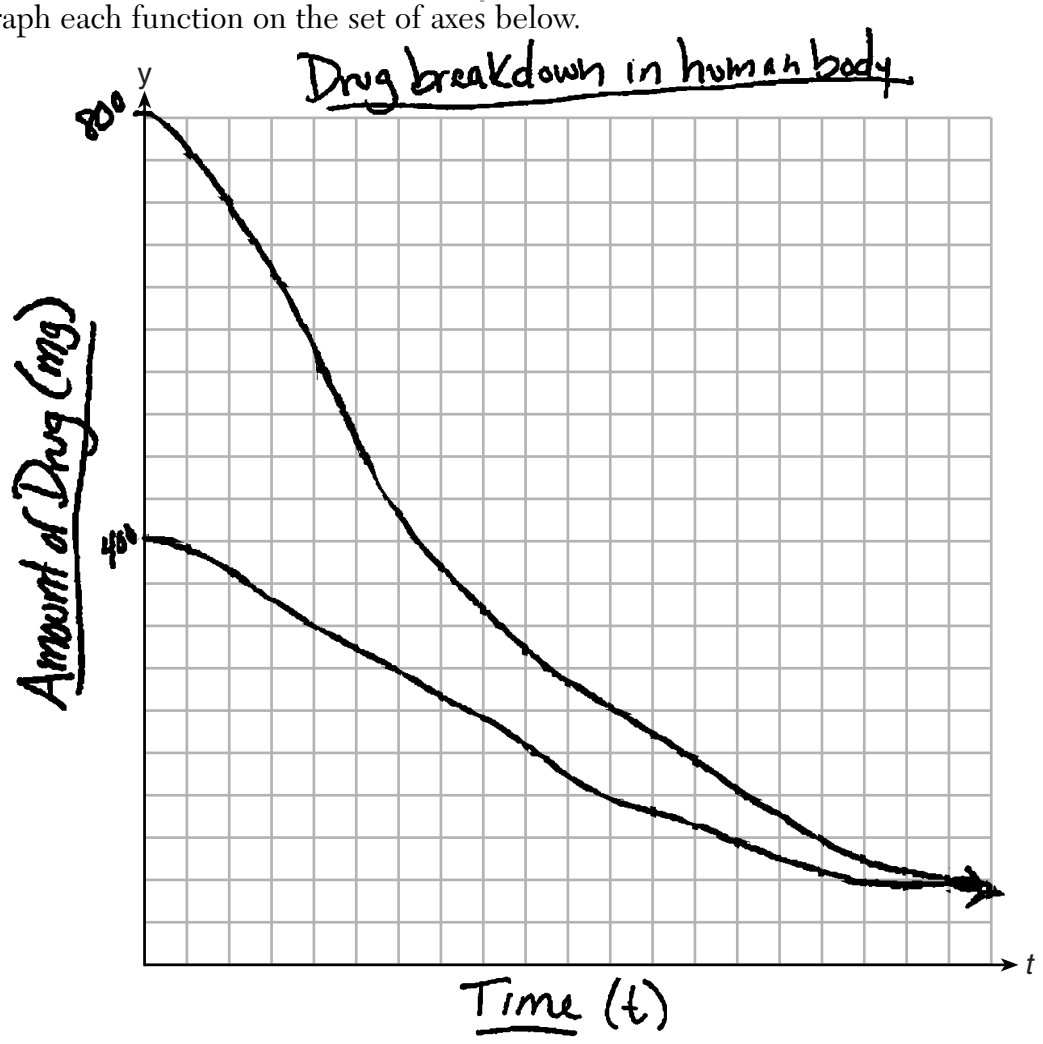
Question 37

37 Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function $N(t) = N_0(e)^{-rt}$, where $N(t)$ is the amount left in the body, N_0 is the initial dosage, r is the decay rate, and t is time in hours. Patient A, $A(t)$, is given 800 milligrams of a drug with a decay rate of 0.347. Patient B, $B(t)$, is given 400 milligrams of another drug with a decay rate of 0.231.

Write two functions, $A(t)$ and $B(t)$, to represent the breakdown of the respective drug given to each patient.

$800(e)^{-.347t}$
400

Graph each function on the set of axes below.



Score 0: The student did not complete enough correct work in any part to receive credit.

Question 37

To the *nearest hour, t*, when does the amount of the given drug remaining in patient *B* begin to exceed the amount of the given drug remaining in patient *A*?

Hour 5

The doctor will allow patient *A* to take another 800 milligram dose of the drug once only 15% of the original dose is left in the body. Determine, to the *nearest tenth of an hour,* how long patient *A* will have to wait to take another 800 milligram dose of the drug.

$120 = 800(e)^{-.347t}$

The State Education Department / The University of the State of New York

Regents Examination in Algebra II (Common Core) – June 2016

Chart for Converting Total Test Raw Scores to Final Exam Scores (Scale Scores)

(Use for the June 2016 exam only.)

Raw Score	Scale Score	Performance Level
86	**100**	5
85	**99**	5
84	**98**	5
83	**97**	5
82	**97**	5
81	**96**	5
80	**95**	5
79	**94**	5
78	**94**	5
77	**93**	5
76	**92**	5
75	**91**	5
74	**91**	5
73	**90**	5
72	**89**	5
71	**89**	5
70	**88**	5
69	**88**	5
68	**87**	5
67	**87**	5
66	**86**	5
65	**86**	5
64	**85**	5
63	**84**	4
62	**84**	4
61	**84**	4
60	**83**	4
59	**83**	4
58	**82**	4
57	**82**	4
56	**82**	4
55	**81**	4
54	**81**	4
53	**81**	4
52	**80**	4
51	**80**	4
50	**80**	4
49	**79**	4
48	**79**	4
47	**79**	4
46	**78**	4
45	**78**	4
44	**77**	3
43	**77**	3
42	**77**	3
41	**76**	3
40	**76**	3
39	**76**	3
38	**75**	3
37	**75**	3
36	**74**	3
35	**73**	3
34	**73**	3
33	**72**	3
32	**72**	3
31	**71**	3
30	**70**	3
29	**69**	3
28	**68**	3
27	**67**	3
26	**66**	3
25	**65**	3
24	**64**	2
23	**63**	2
22	**61**	2
21	**60**	2
20	**58**	2
19	**55**	2
18	**54**	1
17	**53**	1
16	**51**	1
15	**49**	1
14	**47**	1
13	**44**	1
12	**42**	1
11	**39**	1
10	**37**	1
9	**34**	1
8	**31**	1
7	**27**	1
6	**24**	1
5	**20**	1
4	**17**	1
3	**13**	1
2	**9**	1
1	**4**	1
0	**0**	1

To determine the student's final examination score (scale score), find the student's total test raw score in the column labeled "Raw Score" and then locate the scale score that corresponds to that raw score. The scale score is the student's final examination score. Enter this score in the space labeled "Scale Score" on the student's answer sheet.

Schools are not permitted to rescore any of the open-ended questions on this exam after each question has been rated once, regardless of the final exam score. Schools are required to ensure that the raw scores have been added correctly and that the resulting scale score has been determined accurately.

Because scale scores corresponding to raw scores in the conversion chart change from one administration to another, it is crucial that for each administration the conversion chart provided for that administration be used to determine the student's final score. The chart above is usable only for this administration of the Regents Examination in Algebra II (Common Core).