I

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA I

Wednesday, June 19, 2019 — 1:15 to 4:15 p.m., only

Student Name Steve Watson

School Name www.jmap.org

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II**, **III**, and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Perfect Squares

1 The expression $w^4 - 36$ is equivalent to

(1) $(w^2 - 18)(w^2 - 18)$

(2) $(w^2 + 18)(w^2 - 18)$

(3) $(w^2 - 6)(w^2 - 6)$

(4) $(w^2 + 6)(w^2 - 6)$

Use this space for computations.

$a^2 - b^2 = (a+b)(a-b)$

$w^4 - 36 = (w^2+6)(w^2-6)$

2 If $f(x) = 4x + 5$, what is the value of $f(-3)$?

(1) -2

(2) -7

(3) 17

(4) 4

$f(x) = 4x + 5$

$f(-3) = 4(-3) + 5$

$f(-3) = -12 + 5$

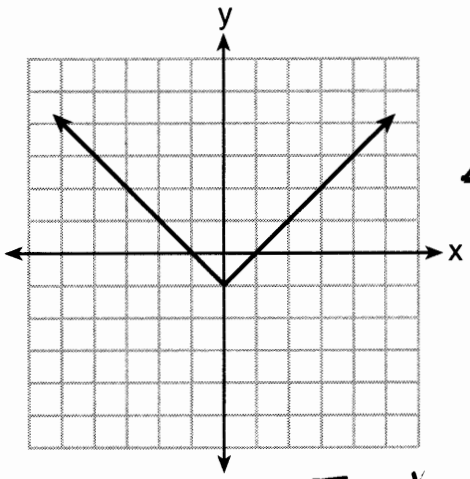
$f(-3) = -7$

3 Which relation is *not* a function?

Every value of x is paired with one and only one value of y.

x	y
–10	–2
–6	2
–2	6
–2	6
1	9
5	13

(1) → Function



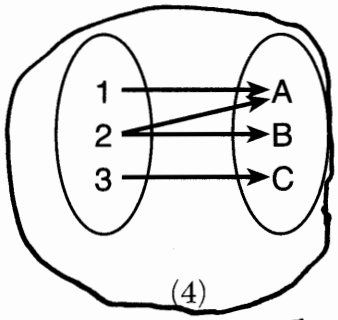
This graph passes the ~~straight~~ (vertical) line test.

(3) → Function

This is a linear equation

$3x + 2y = 4$

(2) → Function



The value 2 maps onto both A + B

(4)

Not a Function

Definition: A function has one and only one value of y for every value of x.

4 Given: $f(x) = (x - 2)^2 + 4$
$g(x) = (x - 5)^2 + 4$

moves function 2 units right
moves function 5 units right.

When compared to the graph of $f(x)$, the graph of $g(x)$ is

(1) shifted 3 units to the left (3) shifted 5 units to the left
(2) shifted 3 units to the right (4) shifted 5 units to the right

Use this space for computations.

HINT: Check using a graphing calculator.

5 Students were asked to write $6x^5 + 8x - 3x^3 + 7x^7$ in standard form. Shown below are four student responses.

Anne: $7x^7 + 6x^5 - 3x^3 + 8x$
Bob: $-3x^3 + 6x^5 + 7x^7 + 8x$
Carrie: $8x + 7x^7 + 6x^5 - 3x^3$
Dylan: $8x - 3x^3 + 6x^5 + 7x^7$

Standard form requires that exponents go from bigger to smaller.

Which student is correct?

(1) Anne (3) Carrie
(2) Bob (4) Dylan

6 The function f is shown in the table below.

x	f(x)
0	1
1	3
2	9
3	27

ΔX: 1, 1, 1 $\quad$ ΔY: 2, 6, 18

$Slope = \frac{\Delta Y}{\Delta X} = \frac{rise}{run} =$ rate of change

$\frac{2}{1} = 2$
$\frac{6}{1} = 6$
$\frac{18}{1} = 18$

The rate of change is increasing (growing)

Which type of function best models the given data?

(1) exponential growth function
(2) exponential decay function
(3) linear function with positive rate of change
(4) linear function with negative rate of change

This is not a constant rate of change, so it cannot be linear.

7 Which expression results in a rational number?

(1) $\sqrt{2} \cdot \sqrt{18}$

(2) $5 \cdot \sqrt{5}$

(3) $\sqrt{2} + \sqrt{2}$

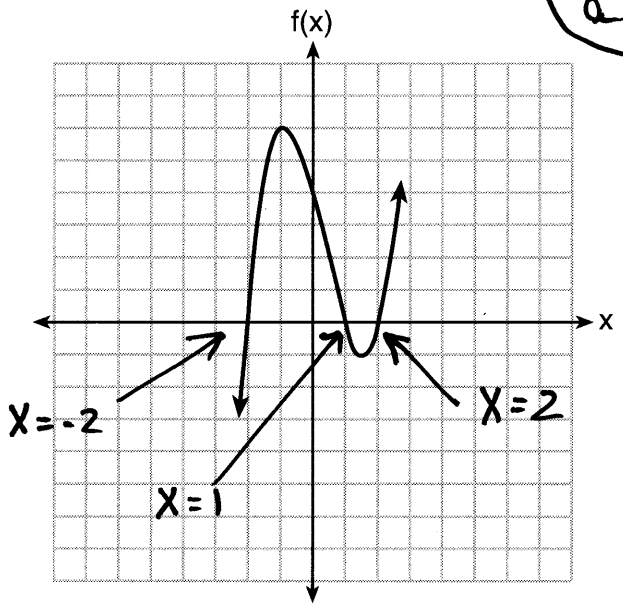
(4) $3\sqrt{2} + 2\sqrt{3}$

Use this space for computations.

$\sqrt{2} \cdot \sqrt{18} = \sqrt{36} = 6 = \frac{6}{1}$ ✓

A rational number can be expressed in fraction form as the ratio of two integers.

8 A polynomial function is graphed below.



Zeros	Factors
$x = -2$	$x + 2$
$x = 1$	$x - 1$
$x = 2$	$x - 2$

$f(x) = (x-1)(x-2)(x+2)$

$f(x) = (x-1)(x^2-4)$

Which function could represent this graph?

~~(1) $f(x) = (x+1)(x^2+2)$~~

(3) $f(x) = (x-1)(x^2-4)$

~~(2) $f(x) = (x-1)(x^2-2)$~~

~~(4) $f(x) = (x+1)(x^2+4)$~~

HINT: Check with a graphing calculator.

9 When solving $p^2 + 5 = 8p - 7$, Kate wrote $p^2 + 12 = 8p$. The property she used is

(1) the associative property

(2) the commutative property

(3) the distributive property

(4) the addition property of equality

$p^2 + 5 = 8p - 7$

(add 7 to both sides) $+7 \qquad +7$

$p^2 + 12 = 8p$

Kate added 7 to both sides of the equation.

10 David wanted to go on an amusement park ride. A sign posted at the entrance read "You must be greater than 42 inches tall and no more than 57 inches tall for this ride." Which inequality would model the height, x, required for this amusement park ride?

(1) $42 < x \leq 57$

(2) $42 > x \geq 57$

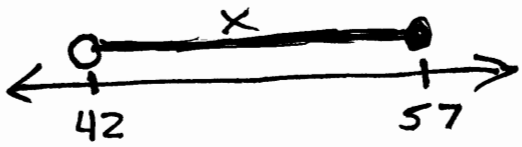
(3) $42 < x$ or $x \leq 57$

(4) $42 > x$ or $x \geq 57$

$x > 42$

Use this space for computations.

$x \leq 57$



$42 < x \leq 57$

11 Which situation can be modeled by a linear function?

(1) The population of bacteria triples every day. exponential growth

(2) The value of a cell phone depreciates at a rate of 3.5% each year. exponential decay

(3) An amusement park allows 50 people to enter every 30 minutes. constant rate of change

(4) A baseball tournament eliminates half of the teams after each round. exponential decay

12 Jenna took a survey of her senior class to see whether they preferred pizza or burgers. The results are summarized in the table below.

	Pizza	Burgers
Male	23	42
Female	31	26
	Total	68

Females Total $\frac{26}{68} = \frac{x\%}{100\%}$

$2600 = 68x$

$38.2\% = x$

Of the people who preferred burgers, approximately what percentage were female?

(1) 21.3

(2) 38.2

(3) 45.6

(4) 61.9

13 When $3a + 7b > 2a - 8b$ is solved for a, the result is

(1) $a > -b$

(2) $a < -b$

(3) $a < -15b$

(4) $a > -15b$

$3a + 7b > 2a - 8b$

$-2a \quad -2a$

$a + 7b > -8b$

$-7b \quad -7b$

$a > -15b$

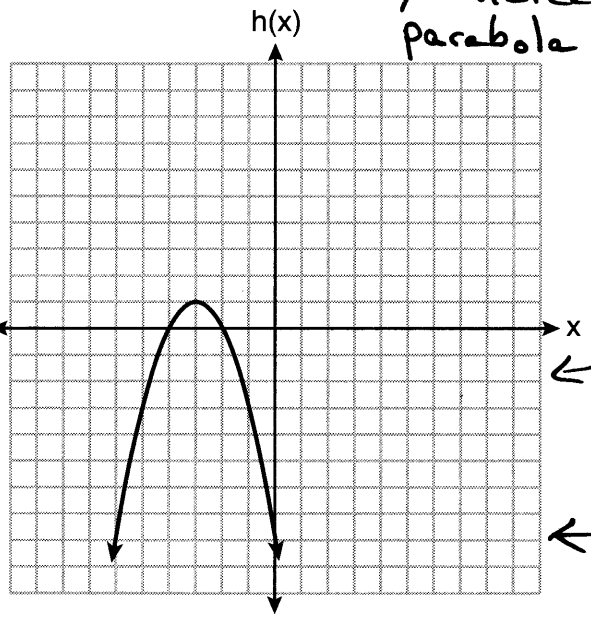
Use this space for computations.

14 Three functions are shown below.

Linear → A: $g(x) = -\frac{3}{2}x + 4$ → y-intercept

Quadratic → B: $f(x) = (x + 2)(x + 6)$ → zeros are -2 and -6
y-intercept is 12 (from graphing calculator)
parabola opens upward, so it has a minimum.

C:



Quadratic → ← zeros are -2 and -4

← y-intercept is -8

Which statement is true?

~~(1)~~ B and C have the same zeros. $\{-2,-6\} \neq \{-2,-4\}$

~~(2)~~ A and B have the same y-intercept. $4 \neq 12$

(3) B has a minimum and C has a maximum.

(4) C has a maximum and ~~A has a minimum.~~ linear equations do not have minimums.

2a-7

15 Nicci's sister is 7 years less than twice Nicci's age, a. The sum of Nicci's age and her sister's age is 41. Which equation represents this relationship?

(1) $a + (7 - 2a) = 41$

(2) $a + (2a - 7) = 41$

(3) $2a - 7 = 41$

(4) $a = 2a - 7$

Let S represent the sister's age

$S = 2a - 7$

Let a represent Nicci's age

$S + a = 41$

Substitute 2a-7 for S

$(2a - 7) + a = 41$

Use this space for computations.

16 The population of a small town over four years is recorded in the chart below, where 2013 is represented by $x = 0$. [Population is rounded to the nearest person]

	0	1	2	3
Year	2013	2014	2015	2016
Population	3810	3943	4081	4224

Exponential

The population, $P(x)$, for these years can be modeled by the function $P(x) = ab^x$, where b is rounded to the nearest thousandth. Which statements about this function are true?

I. $a = 3810$

II. $a = 4224$

III. $b = 0.035$

IV. $b = 1.035$

(1) I and III

(2) I and IV

(3) II and III

(4) II and IV

Use exponential regression and a graphing calculator.

Inputs L1	Inputs L2	Output
0	3810	$y = ab^x$
1	3943	$a = 3809.8$
2	4081	$b = 1.035$
3	4224	

Hints:

① Use the distributive property to find the middle term (11w)

② Put the original expression and each answer choice in a graphing calculator and inspect the graphs.

17 When written in factored form, $4w^2 - 11w - 3$ is equivalent to

(1) $(2w + 1)(2w - 3)$ $2w - 6w$

(2) $(2w - 1)(2w + 3)$ $-2w + 6w$

(3) $(4w + 1)(w - 3)$ $w - 12w = -11w$

(4) $(4w - 1)(w + 3)$ $-w + 12w$

18 Which ordered pair does *not* represent a point on the graph of $y = 3x^2 - x + 7$?

(1) $(-1.5, 15.25)$

(2) $(0.5, 7.25)$

(3) $(1.25, 10.25)$

(4) $(2.5, 23.25)$

Hint

Put equation in graphing calculator and set table to show increments of .25

19 Given the following three sequences:

I. 2, 4, 6, 8, 10... increases by 2 each time

II. 2, 4, 8, 16, 32... doubles each time (requires multiplication and not addition)

III. $a, a + 2, a + 4, a + 6, a + 8...$ increases by 2 each time.

Which ones are arithmetic sequences?

(1) I and II, only

(2) I and III, only

(3) II and III, only

(4) I, II, and III

Use this space for computations.

20 A grocery store sells packages of beef. The function $C(w)$ represents the cost, in dollars, of a package of beef weighing w pounds. The most appropriate domain for this function would be

(1) ~~integers~~ (3) positive ~~integers~~

(2) rational numbers (4) positive rational numbers

- Eliminate integers because beef can be sold in parts of a pound.
- There is no need for anything but positive numbers.

21 The roots of $x^2 - 5x - 4 = 0$ are

(1) 1 and 4 (3) –1 and –4

(2) $\frac{5 \pm \sqrt{41}}{2}$ (4) $\frac{-5 \pm \sqrt{41}}{2}$

$x^2-5x-4=0$

$x^2-5x=4$

$x^2-5x+\left(\frac{5}{2}\right)^2=4+\left(\frac{-5}{2}\right)^2$

$\left(x-\frac{5}{2}\right)^2=4+\frac{25}{4}$

$\left(x-\frac{5}{2}\right)^2=\frac{41}{4}$

$x-\frac{5}{2}=\pm\frac{\sqrt{41}}{2}$

$x=\frac{5\pm\sqrt{41}}{2}$

22 The following table shows the heights, in inches, of the players on the opening-night roster of the 2015-2016 New York Knicks.

84	80	87	75	77	79	80	74	76	80	80	82	82

The population standard deviation of these data is approximately

(1) 3.5 ~~(3) 79.7~~ } much too large

~~(2) 13~~ } too large ~~(4) 80~~ } much too large

Hint

Calculate 1-Variable stats in a graphing calculator.

$\sigma X = 3.4947$

23 A population of bacteria can be modeled by the function $f(t) = 1000(0.98)^t$, where t represents the time since the population started decaying, and $f(t)$ represents the population of the remaining bacteria at time t. What is the rate of decay for this population?

(1) 98% (3) 0.98%

(2) 2% (4) 0.02%

$A = P(1 \pm r)^t$

$f(t) = 1000(0.98)^t$

$1 + r = 0.98$

$r = -0.02$

This is a decimal and must be converted to a %.

24 Bamboo plants can grow 91 centimeters per day. What is the approximate growth of the plant, in inches per hour?

(1) 1.49 (3) 9.63

(2) 3.79 (4) 35.83

$\frac{91 \text{ cm}}{1 \text{ day}} \left(\frac{\div 2.54 \text{ cm/inch}}{\times 24 \text{ hrs/day}}\right) \Rightarrow \frac{35.83 \text{ inches}}{24 \text{ hours}}$

$\frac{\text{inches}}{\text{hours}} \quad \frac{35.83}{24} = \frac{x}{1}$

$24x = 35.83$

$x = 1.4927$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Solve algebraically for x:

$$-\frac{2}{3}(x+12)+\frac{2}{3}x=-\frac{5}{4}x+2$$

$-\frac{2}{3}(x+12)+\frac{2}{3}x=-\frac{5}{4}x+2$

$-\frac{2}{3}x-\frac{24}{3}+\frac{2}{3}x=-\frac{5}{4}x+2$

$-8 = -\frac{5}{4}x+2$

$-2 \qquad -2$

$-10 = -\frac{5}{4}x$

M(4) $-40 = -5x$

D(-5) $\boxed{8 = x}$

Check

$-\frac{2}{3}(8+12)+\frac{2}{3}(8)=-\frac{5}{4}(8)+2$

$-\frac{2}{3}(20)+\frac{16}{3}=\frac{-40}{4}+2$

$\frac{-40}{3}+\frac{16}{3}=-10+2$

$\frac{-24}{3}=-8$

$-8=-8$ ✓

26 If $C = G - 3F$, find the trinomial that represents C when $F = 2x^2 + 6x - 5$ and $G = 3x^2 + 4$.

$$C = G - 3F$$

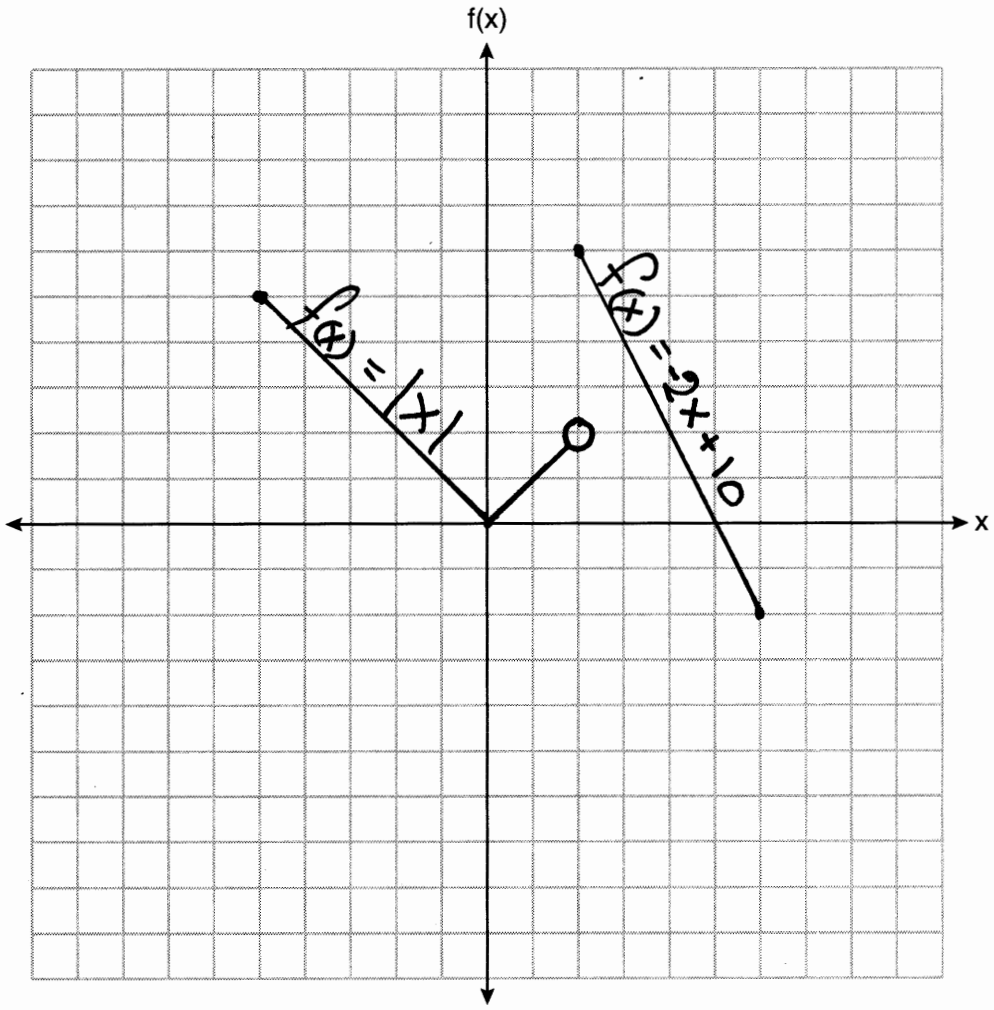
$$C = (3x^2 + 4) - 3(2x^2 + 6x - 5)$$

$$C = 3x^2 + 4 - 6x^2 - 18x + 15$$

$$\boxed{C = -3x^2 - 18x + 19}$$

27 Graph the following piecewise function on the set of axes below.

$$f(x) = \begin{cases} |x|, & -5 \le x < 2 \\ -2x + 10, & 2 \le x \le 6 \end{cases}$$



28 Solve $5x^2 = 180$ algebraically.

$5x^2 = 180$

D(5) $x^2 = 36$

$x = \pm\sqrt{36}$

$\boxed{x = \pm 6}$

Check

$5(6)^2 = 180$

$5(36) = 180$

$180 = 180$ ✓

$5(-6)^2 = 180$

$5(36) = 180$

$180 = 180$ ✓

29 A blizzard occurred on the East Coast during January, 2016. Snowfall totals from the storm were recorded for Washington, D.C. and are shown in the table below.

Washington, D.C.	
Time	**Snow** (inches)
1 a.m.	1
3 a.m.	5
6 a.m.	11
12 noon	33
3 p.m.	36

11 hours → 32 inches

9 hours → 25 inches

Which interval, 1 a.m. to 12 noon or 6 a.m. to 3 p.m., has the greatest rate of snowfall, in inches per hour? Justify your answer.

1 a.m. to 12 noon

$\frac{\text{inches}}{\text{hours}} \mid \frac{32}{11}$ or 2.91 inches/hour

6 a.m. to 3 p.m.

$\frac{\text{inches}}{\text{hours}} \mid \frac{25}{9}$ or 2.78 inches/hour

The greatest rate of snowfall occured from **1 a.m. to 12 noon.**

30 The formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Solve the equation for h in terms of V, r, and π.

$$V = \frac{1}{3}\pi r^2 h$$

M(3) $\quad 3V = \pi r^2 h$

Dπr^2 $\quad \boxed{\frac{3V}{\pi r^2} = h}$

31 Given the recursive formula:

$$a_1 = 3$$
$$a_n = 2(a_{n-1} + 1)$$

State the values of a_2, a_3, and a_4 for the given recursive formula.

Step 1 - Set up a table (optional)

n	1	2	3	4
a_n	3	8	18	38

Step 2 - Solve for a_2

$a_2 = 2(3+1)$

$a_2 = 2(4)$

$\boxed{a_2 = 8}$

Step 3 - Solve for a_3

$a_3 = 2(8+1)$

$a_3 = 2(9)$

$\boxed{a_3 = 18}$

Step 4 - Solve for a_4

$a_4 = 2(18+1)$

$a_4 = 2(19)$

$\boxed{a_4 = 38}$

$a_1 = 3$

$a_2 = 8$

$a_3 = 18$

$a_4 = 38$

Answer

32 Determine and state the vertex of $f(x) = x^2 - 2x - 8$ using the method of completing the square.

$$X^2 - 2X - 8 = 0$$

$$X^2 - 2X = 8$$

$$X^2 - 2X + \left(\frac{-2}{2}\right)^2 = 8 + \left(\frac{-2}{2}\right)^2$$

$$(X-1)^2 = 8 + 1$$

$$(X-1)^2 = 9$$

$$(X-1)^2 - 9 = 0$$

Change sign

keep sign

The vertex is at $\boxed{(1, -9)}$

Hint: Check with graphing calculator.

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 A school plans to have a fundraiser before basketball games selling shirts with their school logo. The school contacted two companies to find out how much it would cost to have the shirts made. Company *A* charges a \$50 set-up fee and \$5 per shirt. Company *B* charges a \$25 set-up fee and \$6 per shirt.

Write an equation for Company *A* that could be used to determine the total cost, *A*, when *x* shirts are ordered. Write a second equation for Company *B* that could be used to determine the total cost, *B*, when *x* shirts are ordered.

Let x represent the # of shirts

$A(x) = 5x + 50$

$B(x) = 6x + 25$

Determine algebraically and state the *minimum* number of shirts that must be ordered for it to be cheaper to use Company *A*.

$A(x) < B(x)$

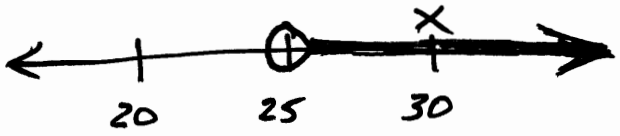
$5x + 50 < 6x + 25$

$-5x \quad -5x$

$50 < x + 25$

$-25 \quad -25$

$25 < x$



Answer
26 shirts

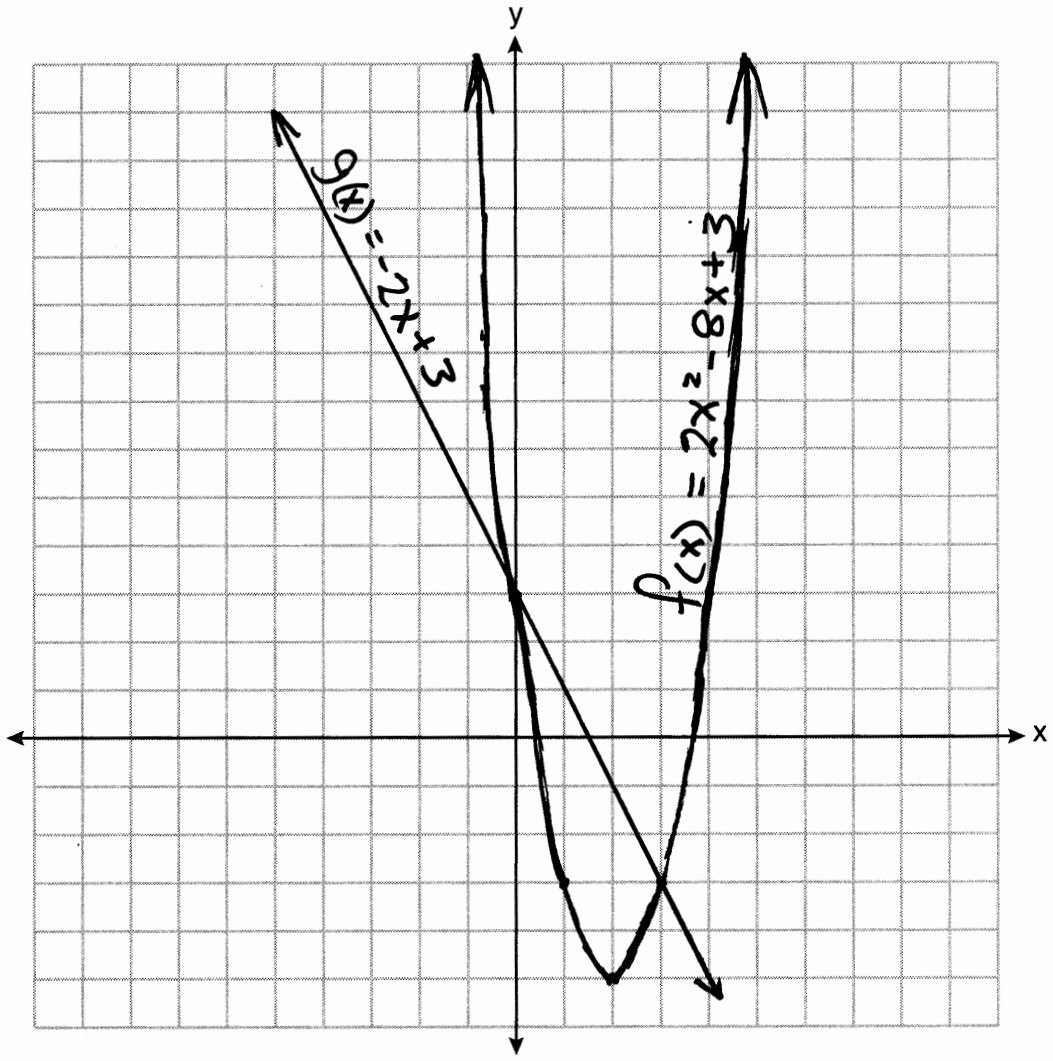
Check

$A(26) = 5(26) + 50$ $\qquad$ $B(26) = 6(26) + 25$

$A(26) = 180$ $\qquad$ $B(26) = 181$

34 Graph $y = f(x)$ and $y = g(x)$ on the set of axes below.

$$f(x) = 2x^2 - 8x + 3$$
$$g(x) = -2x + 3$$



Determine and state all values of x for which $f(x) = g(x)$.

x	f(x)	g(x)
0	3	3
3	-3	-3

$x = \{0, 3\}$

35 The table below shows the number of hours ten students spent studying for a test and their scores.

Hours Spent Studying (x)	0	1	2	4	4	4	6	6	7	8
Test Scores (y)	35	40	46	65	67	70	82	88	82	95

Write the linear regression equation for this data set. Round all values to the *nearest hundredth*.

Input data in graphing calculator.
Calculate linear regression.

Output

$Y = aX + b$

$a = 7.7422$

$b = 34.2727$

$\boxed{Y = 7.79x + 34.27}$

State the correlation coefficient of this line, to the *nearest hundredth*.

Use graphing calculator to find correlation coefficient.

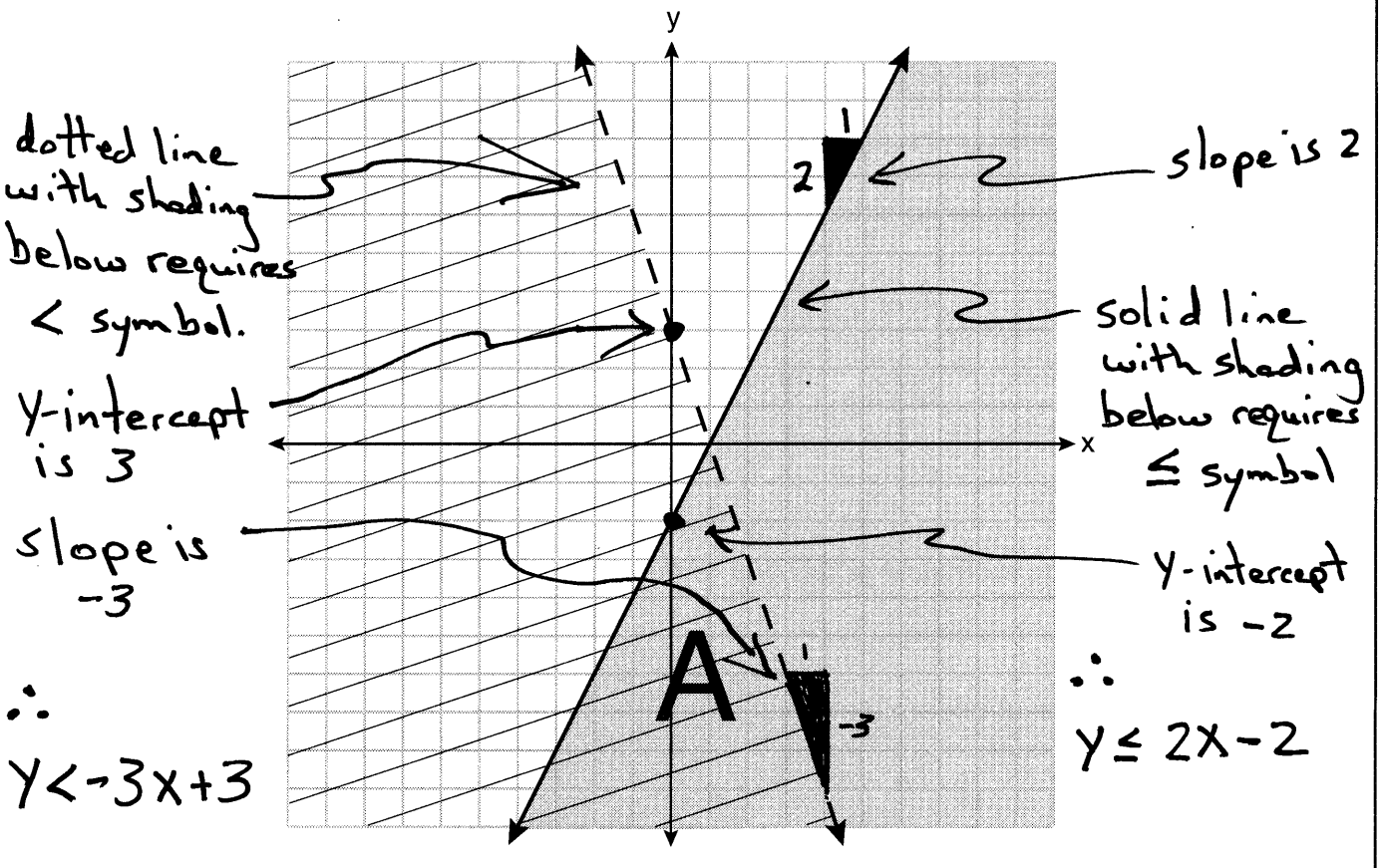
$r = .98157$

$\boxed{r = .98}$

Explain what the correlation coefficient suggests in the context of the problem.

There is a very strong positive correlation between time spent studying for a test and scores on the test.

36 A system of inequalities is graphed on the set of axes below.



State the system of inequalities represented by the graph.

$y < -3x + 3$

$y \leq 2x - 2$

State what region *A* represents.

The solution set of the system.

State what the entire gray region represents.

The solution set of $y \leq 2x - 2$.

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 When visiting friends in a state that has no sales tax, two families went to a fast-food restaurant for lunch. The Browns bought 4 cheeseburgers and 3 medium fries for \$16.53. The Greens bought 5 cheeseburgers and 4 medium fries for \$21.11.

Using c for the cost of a cheeseburger and f for the cost of medium fries, write a system of equations that models this situation.

Browns $4c + 3f = 16.53$

Greens $5c + 4f = 21.11$

The Greens said that since their bill was \$21.11, each cheeseburger must cost \$2.49 and each order of medium fries must cost \$2.87 each. Are they correct? Justify your answer.

$5c + 4f \stackrel{?}{=} 21.11$

$5(2.49) + 4(2.87) \stackrel{?}{=} 21.11$

$12.45 + 11.48 \stackrel{?}{=} 21.11$

$23.93 \neq 21.11$

No. The Greens are not correct.

Using your equations, algebraically determine both the cost of one cheeseburger and the cost of one order of medium fries.

$(4c + 3f = 16.53)\ 5 \Rightarrow 20c + 15f = 82.65$

$(5c + 4f = 21.11)\ 4 \Rightarrow 20c + 16f = 84.44$

$-f = -1.79$

$f = \$1.79$

$4c + 3f = 16.53$

$4c + 3(1.79) = 16.53$

$4c + 5.37 = 16.53$

$-5.37 \quad -5.37$

$4c = 11.16$

$c = \$2.79$

Check:

$4(2.79) + 3(1.79) = 16.53$

$16.53 = 16.53$ ✓

$5(2.79) + 4(1.79) = 21.11$

$21.11 = 21.11$ ✓