The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA I (Common Core)

Wednesday, June 17, 2015 — 1:15 to 4:15 p.m., only

Student Name: *Steve Watson*

School Name: *JMAP.ORG*

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The cost of airing a commercial on television is modeled by the function $C(n) = 110n + 900$, where n is the number of times the commercial is aired. Based on this model, which statement is true?

(1) The commercial costs $0 to produce and $110 per airing up to $900.

(2) The commercial costs $110 to produce and $900 each time it is aired.

(3) The commercial costs $900 to produce and $110 each time it is aired.

(4) The commercial costs $1010 to produce and can air an unlimited number of times.

2 The graph below represents a jogger's speed during her 20-minute jog around her neighborhood.

9 - 12

Not Distance
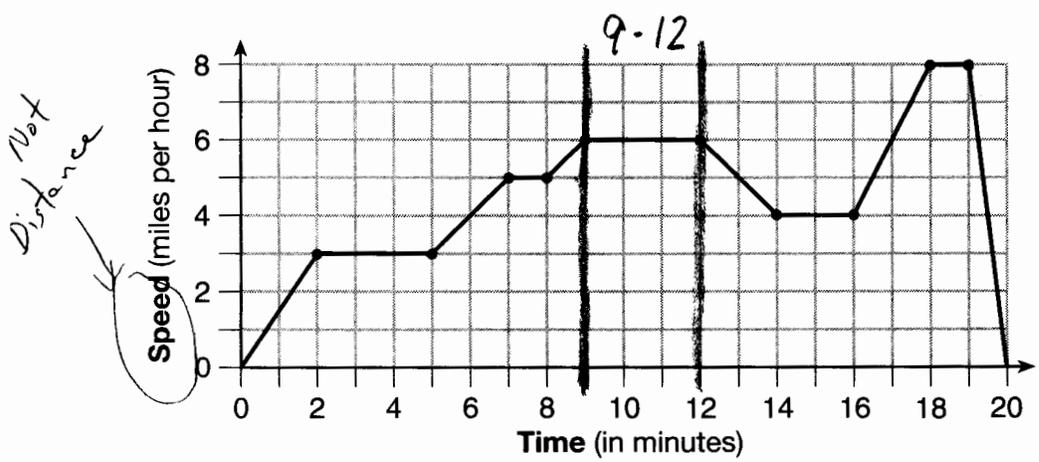
Time increase Speed remains the same

Which statement best describes what the jogger was doing during the 9–12 minute interval of her jog?

(1) She was standing still.

(2) She was increasing her speed.

(3) She was decreasing her speed.

(4) She was jogging at a constant rate.

3 If the area of a rectangle is expressed as $x^4 - 9y^2$, then the product of the length and the width of the rectangle could be expressed as

(1) $(x - 3y)(x + 3y)$

(3) $(x^2 - 3y)(x^2 - 3y)$

(2) $(x^2 - 3y)(x^2 + 3y)$

(4) $(x^4 + y)(x - 9y)$

Handwritten: $(x^2 + 3y)(x^2 - 3y)$

4 Which table represents a function?

x	2	4	2	4
f(x)	3	5	7	9

(1)

x	3	5	7	9
f(x)	2	4	2	4

(3)

x	0	−1	0	1
f(x)	0	1	−1	0

(2)

x	0	1	−1	0
f(x)	0	−1	0	1

(4)

5 Which inequality is represented in the graph below?
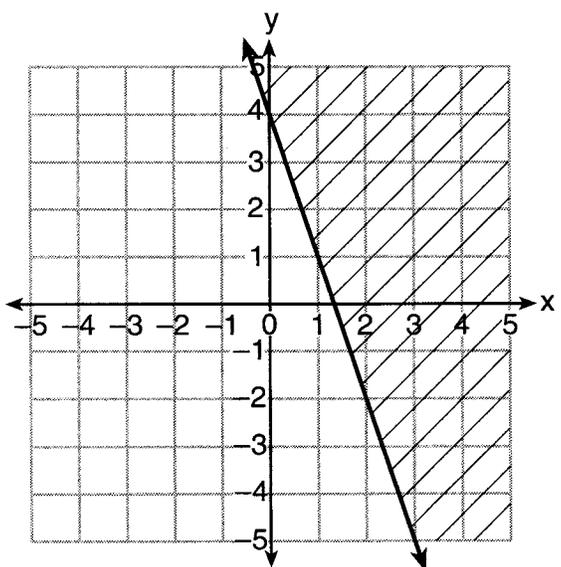
Handwritten notes: Shading is above the line, so y ≥. Line is solid, so y ≥. b = 4. y-intercept is 4. slope is rise/run = −3/1 = −3. m = −3. y = mx + b. y = −3x + 4

(1) $y \geq -3x + 4$

(3) $y \geq -4x - 3$

(2) $y \leq -3x + 4$

(4) $y \leq -4x - 3$

6 Mo's farm stand sold a total of 165 pounds of apples and peaches. She sold apples for \$1.75 per pound and peaches for \$2.50 per pound. If she made \$337.50, how many pounds of peaches did she sell?
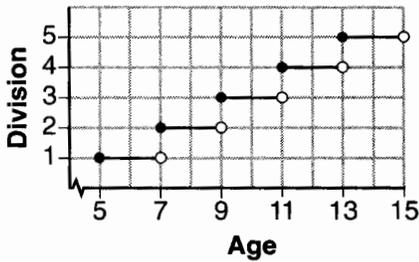
(1) 11

(●) 65

(2) 18

(4) 100

Handwritten: a p

$a + p = 165$

$1.75a + 2.50p = 337.50$

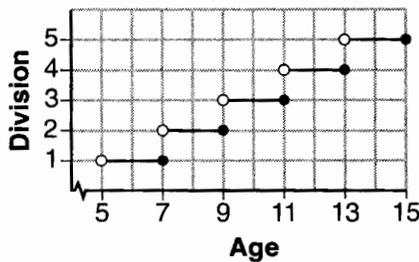$\begin{bmatrix} 1 & 1 \\ 1.75 & 2.5 \end{bmatrix} \begin{bmatrix} 165 \\ 337.5 \end{bmatrix}$

7 Morgan can start wrestling at age 5 in Division 1. He remains in that division until his next odd birthday when he is required to move up to the next division level. Which graph correctly represents this information?
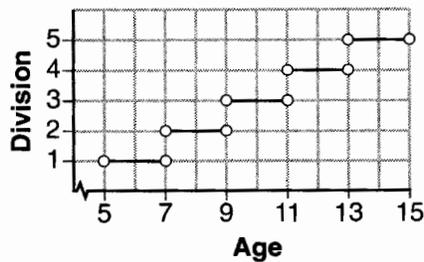

(●)

(3)

(2)

(4)

*Handwritten:* Q. old birthday?

A. Morgan moves to the next division.

8 Which statement is *not* always true?

(1) The sum of two rational numbers is rational.

● The product of two irrational numbers is rational.

(3) The sum of a rational number and an irrational number is irrational.

(4) The product of a nonzero rational number and an irrational number is irrational.

9 The graph of the function $f(x) = \sqrt{x + 4}$ is shown below.
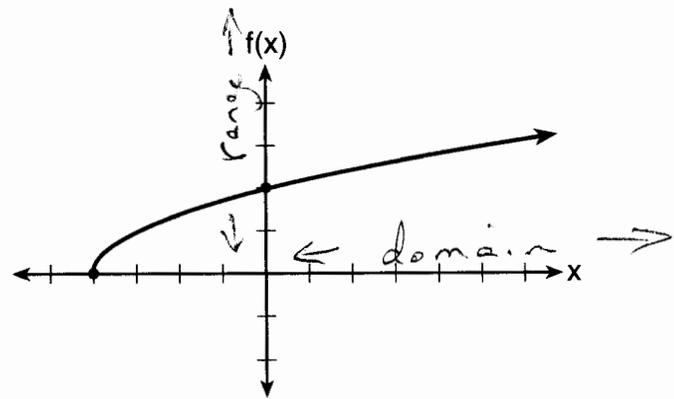
The domain of the function is

(1) $\{x | x > 0\}$

(2) $\{x | x \geq 0\}$

(3) $\{x | x > -4\}$

● $\{x | x \geq -4\}$

10 What are the zeros of the function $f(x) = x^2 - 13x - 30$?

(1) -10 and 3

(2) 10 and -3

(3) -15 and 2

● 15 and -2

11 Joey enlarged a 3-inch by 5-inch photograph on a copy machine. He enlarged it four times. The table below shows the area of the photograph after each enlargement.

Enlargement	0	1	2	3	4
Area (square inches)	15	18.8	23.4	29.3	36.6

3.8   4.6   5.9   7.3

What is the average rate of change of the area from the original photograph to the fourth enlargement, to the *nearest tenth*?

(1) 4.3   ● 5.4

(2) 4.5   (4) 6.0

else $\dfrac{36.6 - 15}{4}$

$\dfrac{21.}{4}$

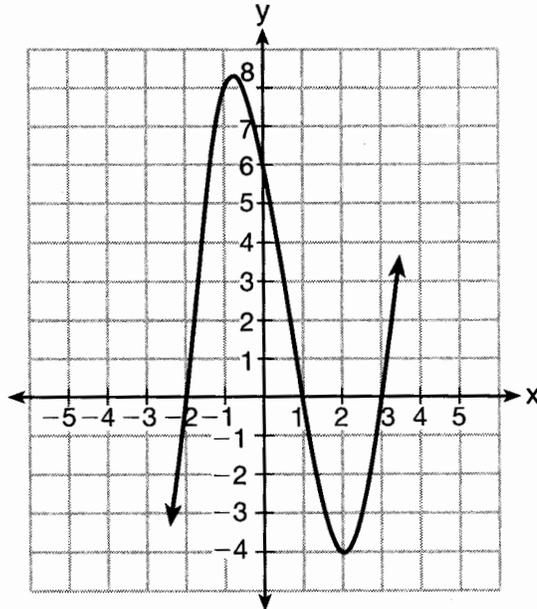*5.4* ✓

12 Which equation(s) represent the graph below?

✗ I $y = (x + 2)(x^2 - 4x - 12)$

✓ II $y = (x - 3)(x^2 + x - 2)$

✗ III $y = (x - 1)(x^2 - 5x - 6)$

factors

$(x+2)\ (x-6)(x+2)$

$(x-3)\ (x+2)\ (x-1)$

$(x-1)\ (x-6)\ (x+1)$

zeros

$-2, 6, -2$

$3, -2, 1$

$1, 6, -1$

zeros $-2, 1, 3$

(1) I, only   (3) I and II

● II, only   (4) II and III

13 A laboratory technician studied the population growth of a colony of bacteria. He recorded the number of bacteria every other day, as shown in the partial table below.

t (time, in days)	0	2	4
f(t) (bacteria)	25	15,625	9,765,625

Which function would accurately model the technician's data?

(1) $f(t) = 25^t$

(2) $f(t) = 25^{t+1}$

(3) $f(t) = 25t$

(4) $f(t) = 25(t + 1)$

14 Which quadratic function has the largest maximum?

$h(x) = (3 - x)(2 + x)$

$k(x) = -5x^2 - 12x + 4$

(1)

(3)

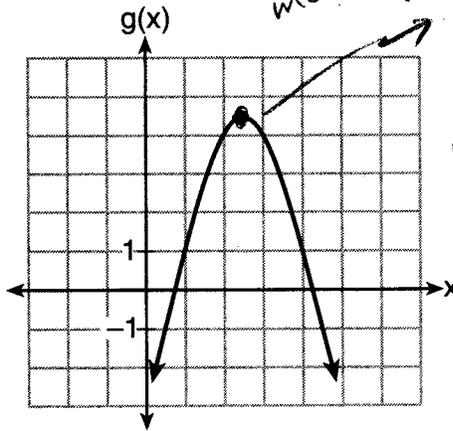| x	f(x)
−1	−3
0	5
1	9
2	9
3	5
4	−3

(2)

g(x)

(4)

15 If $f(x) = 3^x$ and $g(x) = 2x + 5$, at which value of x is $f(x) < g(x)$?

(1) −1

(3) −3

(2) 2

(4) 4

[7]

[OVER]

16 Beverly did a study this past spring using data she collected from a cafeteria. She recorded data weekly for ice cream sales and soda sales. Beverly found the line of best fit and the correlation coefficient, as shown in the diagram below.

Beverly's Cafeteria Study
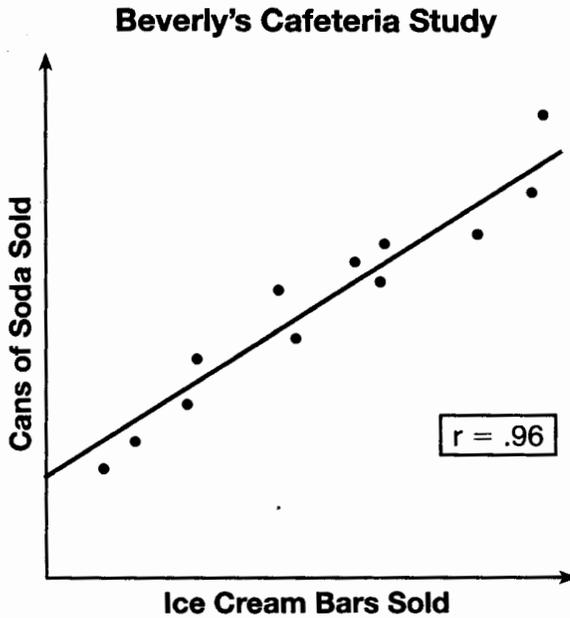
Given this information, which statement(s) can correctly be concluded?

I.   Eating more ice cream causes a person to become thirsty.
II. Drinking more soda causes a person to become hungry.
III. There is a strong correlation between ice cream sales and soda sales.

(1) I, only   (3) I and III

(2) III, only   (4) II and III

17 The function $V(t) = 1350(1.017)^t$ represents the value $V(t)$, in dollars, of a comic book t years after its purchase. The yearly rate of appreciation of the comic book is

(1) 17%   (3) 1.017%

(2) 1.7%   (4) 0.017%

original value

rate of appreciation

$1 + .017$

$.017 = 1.7\%$

18 When directed to solve a quadratic equation by completing the square, Sam arrived at the equation $\left(x - \frac{5}{2}\right)^2 = \frac{13}{4}$. Which equation could have been the original equation given to Sam?

(1) $x^2 + 5x + 7 = 0$

(2) $x^2 + 5x + 3 = 0$

(3) $x^2 - 5x + 7 = 0$

● $x^2 - 5x + 3 = 0$

b must be negative

negative b

$x^2 - \frac{5}{x} = -7$

$\left(x - \frac{5}{2}\right)^2 = -7 + \left(\frac{-5}{2}\right)^2$

$\left(x - \frac{5}{2}\right)^2 = \frac{-28}{4} + \frac{25}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{-3}{4}$

$\left(x - \frac{5}{2}\right)^2 = \frac{-3}{4}$

19 The distance a free falling object has traveled can be modeled by the equation $d = \frac{1}{2}at^2$, where a is acceleration due to gravity and t is the amount of time the object has fallen. What is t in terms of a and d?

(1) $t = \sqrt{\dfrac{da}{2}}$

● $t = \sqrt{\dfrac{2d}{a}}$

(3) $t = \left(\dfrac{da}{d}\right)^2$

(4) $t = \left(\dfrac{2d}{a}\right)^2$

$d = \frac{1}{2}at^2$

$2d = at^2$

$\dfrac{2d}{a} = t^2$

$\sqrt{\dfrac{2d}{a}} = t$

20 The table below shows the annual salaries for the 24 members of a professional sports team in terms of millions of dollars.

0.5	0.5	0.6	0.7	0.75	0.8
1.0	1.0	1.1	1.25	1.3	1.4
1.4	1.8	2.5	3.7	3.8	4
4.2	4.6	5.1	6	6.3	7.2

$+ \boxed{10}$

The team signs an additional player to a contract worth 10 million dollars per year. Which statement about the median and mean is true?

(1) Both will increase.

(2) Only the median will increase.

● Only the mean will increase.

(4) Neither will change.

more than any current player

# of players goes from 24 to 25

Current median is $\frac{1.4+1.4}{2} = 1.4$

New median will be 1.4

The mean will increase.

21 A student is asked to solve the equation $4(3x - 1)^2 - 17 = 83$. The student's solution to the problem starts as

$$4(3x - 1)^2 = 100$$
$$(3x - 1)^2 = 25$$

A correct next step in the solution of the problem is

● $3x - 1 = \pm 5$ (3) $9x^2 - 1 = 25$

(2) $3x - 1 = \pm 25$ (4) $9x^2 - 6x + 1 = 5$

[handwritten: $\sqrt{(3x-1)^2} = \sqrt{25}$; $3x - 1 = \pm 5$]

22 A pattern of blocks is shown below.

Term 1   Term 2   Term 3   Term 4
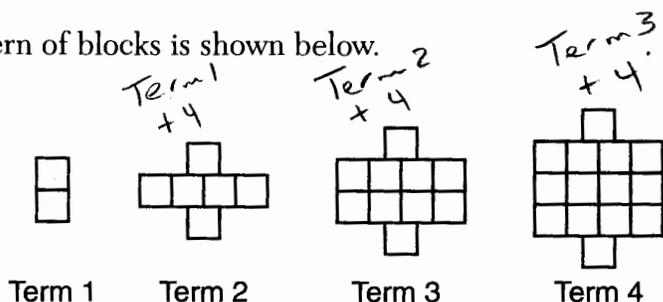
If the pattern of blocks continues, which formula(s) could be used to determine the number of blocks in the nth term?

I	II	III
$a_n = n + 4$	$a_1 = 2$ $a_n = a_{n-1} + 4$	$a_n = 4n - 2$

(1) I and II   ● II and III

(2) I and III   (4) III, only

23 What are the solutions to the equation $x^2 - 8x = 24$?

(1) $x = 4 \pm 2\sqrt{10}$ ●

(2) $x = -4 \pm 2\sqrt{10}$

(3) $x = 4 \pm 2\sqrt{2}$

(4) $x = -4 \pm 2\sqrt{2}$

$\$2.76 \leq cost \leq \$3.24$

24 Natasha is planning a school celebration and wants to have live music and food for everyone who attends. She has found a band that will charge her $750 and a caterer who will provide snacks and drinks for $2.25 per person. If her goal is to keep the average cost per person between $2.75 and $3.25, how many people, p, must attend?

(1) $225 < p < 325$

(2) $325 < p < 750$

(3) $500 < p < 1000$

(4) $750 < p < 1500$ ●

#23

$x^2 - 8X = 24$

$x^2 - 8x - 24 = 0$

$a = 1 \quad b = -8 \quad c = -24$

$X = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$X = \dfrac{8 \pm \sqrt{64 - 4(1)(-24)}}{2(1)}$

$X = \dfrac{8 \pm \sqrt{64 + 96}}{2}$

$X = \dfrac{8 \pm \sqrt{160}}{2}$

$X = \dfrac{8 \pm \sqrt{16}\sqrt{10}}{2}$

$X = \dfrac{8 \pm 4\sqrt{10}}{2}$

$X = 4 \pm 2\sqrt{10}$

#24 Expenses $= 750 + 2.25p$

Assume that Natasha wants income to meet or exceed expenses.

If each person pays $2\frac{25}{}$, then 50¢ will go towards the $750 charge and $\dfrac{750}{.50} = \boxed{1500}$ people must attend.

If each person pays $3.25, then $1.00 will go towards the $750 and $\dfrac{750}{1} = \boxed{750}$ people must attend.

$750 < p < 1500$

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 Graph the function $y = |x - 3|$ on the set of axes below.

X	$|x-3|$
0	3
1	2
2	1
3	0
4	1
5	2
6	3
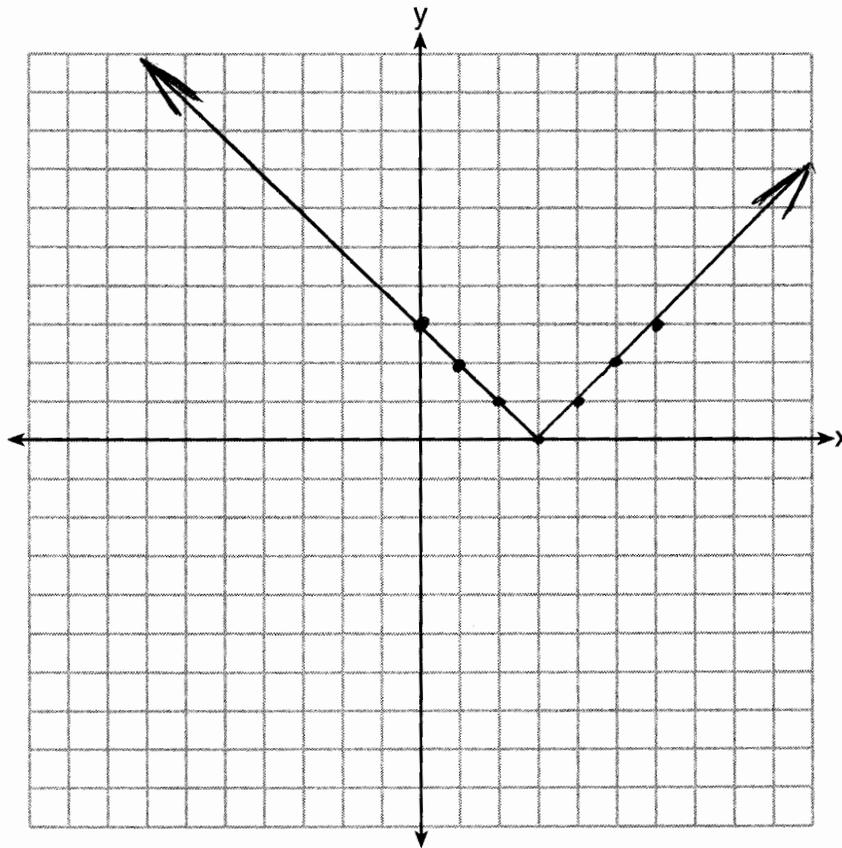
Explain how the graph of $y = |x - 3|$ has changed from the related graph $y = |x|$.

The graph of $y = |x-3|$ moves the graph of $y = |x|$ three units to the right.

26 Alex is selling tickets to a school play. An adult ticket costs $6.50 and a student ticket costs $4.00. Alex sells *x* adult tickets and 12 student tickets. Write a function, *f(x)*, to represent how much money Alex collected from selling tickets.

Total money = $6.50 times X adult tickets

plus $4.00 times 12 student tickets.

$f_{(x)}$

$4 \times 12 = 48$

$f(x) = \$6.5x + \$48$

27 John and Sarah are each saving money for a car. The total amount of money John will save is given by the function $f(x) = 60 + 5x$. The total amount of money Sarah will save is given by the function $g(x) = x^2 + 46$. After how many weeks, x, will they have the same amount of money saved? Explain how you arrived at your answer.

The problem wants to know when $f(x)$ will equal $g(x)$.

$$f(x) = 60 + 5x$$
$$g(x) = x^2 + 46$$

Let $f(x) = g(x)$

$$60 + 5x = x^2 + 46$$
$$0 = x^2 - 5x + 46 - 60$$
$$0 = x^2 - 5x - 14$$
$$0 = (x - 7)(x + 2)$$

$$\underline{X - 7 = 0} \qquad X + 2 = 0 \qquad \swarrow \text{reject}$$
$$\boxed{X = 7} \qquad\qquad X = -2$$

They will have the same amount of money after 7 weeks. I set both functions equal to one another, solved for X, and rejected the negative solution of the quadratic because -2 weeks makes no sense

subtract

28 If the difference $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$ is multiplied by $\frac{1}{2}x^2$, what is the result, written in standard form?

$$3x^2 - 2x + 5$$
$$\underline{-x^2 - 3x + 2}$$
$$2x^2 - 5x + 7$$

← *Notice that I changed the signs and added the second expression*

$$\frac{1}{2}x^2\left(2x^2 - 5x + 7\right)$$

$$\boxed{x^4 - \frac{5}{2}x^3 + \frac{7}{2}x^2}$$

Principal   *rate* $1.6\% = .016$

29 Dylan invested $600 in a savings account at a 1.6% annual interest rate. He made no deposits or withdrawals on the account for 2 years. The interest was compounded annually. Find, to the *nearest cent*, the balance in the account after 2 years.

amount   *time*

$$\text{Amount} = (\text{Principal})(1 + \text{rate})^{\text{time}}$$

$$A = (\$600)(1 + .016)^2$$

$$A = (\$600)(1.016)^2$$

$$A = \$619.3536 \quad \text{nearest cent}$$

$$\boxed{\$619.35}$$

30 Determine the smallest integer that makes $-3x + 7 - 5x < 15$ true.

$$-3x + 7 - 5x < 15$$

$$-8x + 7 \quad\quad < 15$$
$$\quad\quad -7 \quad\quad\quad -7$$

$$\frac{-8x}{-8} \quad\quad \xrightarrow{\text{change}} \frac{< 8}{-8}$$

$$x \quad\quad\quad > -1$$

$$\boxed{0} \text{ is the smallest integer that}$$
$$\text{makes } x > -1 \text{ true.}$$

31 The residual plots from two different sets of bivariate data are graphed below.
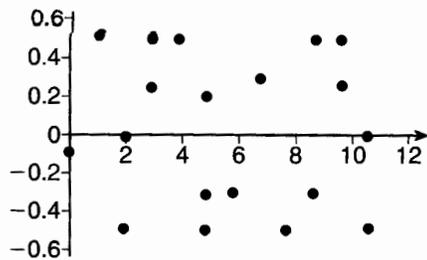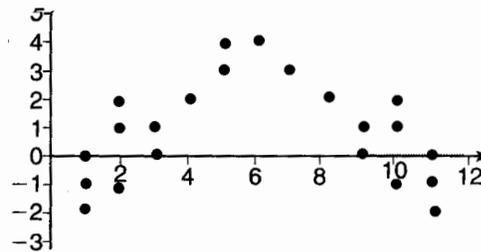
Graph A **Graph B**

Explain, using evidence from graph A and graph B, which graph indicates that the model for the data is a good fit.

Domain of X

Range of Y

Graph A shows the range of the residuals to be approximately ± .5. The range of residuals in Graph B is much larger, meaning the model is not as good a fit.

$$w = \tfrac{1}{2}\ell$$

32 A landscaper is creating a rectangular flower bed such that the width is half of the length. The area of the flower bed is 34 square feet. Write and solve an equation to determine the width of the flower bed, to the *nearest tenth of a foot*.

$$A = 34 \qquad w = \frac{\ell}{2}$$
$$\ell$$

$$A = \ell w$$
$$34 = \ell\left(\frac{\ell}{2}\right)$$
$$34 = \frac{\ell^2}{2}$$
$$68 = \ell^2$$
$$\sqrt{68} = \ell$$
$$8.246 \approx \ell$$
$$\frac{8.246}{2} \approx \frac{\ell}{2} = w$$
$$4.123 \approx w$$

$$\boxed{4.1 \text{ feet}}$$

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Albert says that the two systems of equations shown below have the same solutions.

First System	Second System
$8x + 9y = 48$	$8x + 9y = 48$
$12x + 5y = 21$	$-8.5y = -51$

Determine and state whether you agree with Albert. Justify your answer.

$$8x + 9y = 48$$
$$12x + 5y = 21$$

$$\begin{matrix} A \\ \begin{bmatrix} 8 & 9 \\ 12 & 5 \end{bmatrix} \end{matrix} \quad \begin{matrix} B \\ \begin{bmatrix} 48 \\ 21 \end{bmatrix} \end{matrix}$$

$$8x + 9y = 48$$
$$0x - 8.5y = -51$$

$$\begin{matrix} A \\ \begin{bmatrix} 8 & 9 \\ 0 & -8.5 \end{bmatrix} \end{matrix} \quad \begin{matrix} B \\ \begin{bmatrix} 48 \\ -51 \end{bmatrix} \end{matrix}$$

Output

$$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} -.75 \\ 6 \end{bmatrix}$$

Output

$$\begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} -.75 \\ 6 \end{bmatrix}$$

Yes, Albert is correct. Both systems have solutions of $x = -\frac{3}{4}$ and $y = 6$.

34 The equation to determine the weekly earnings of an employee at The Hamburger Shack is given by $w(x)$, where x is the number of hours worked.

$$w(x) = \begin{cases} 10x, & 0 \leq x \leq 40 \\ 15(x - 40) + 400, & x > 40 \end{cases}$$

Determine the difference in salary, *in dollars*, for an employee who works 52 hours versus one who works 38 hours.

The equation used for 52 hours is

$w(x) = 15(x-40)+400$

$w(52) = 15(52-40)+400$

$w(52) = 15(12)+400$

$w(52) = 180+400$

$w(52) = \$580$

The equation used for 38 hours is

$w(x) = 10x \rightarrow w(x) = \$380$

$w(x) = 10(38)$

The difference is $\$200$

Determine the number of hours an employee must work in order to earn $445. Explain how you arrived at this answer.

$w(x) = 10x$

$445 = 10(x)$

$44.5 = x$

$\}$ This equation cannot be use because $x > 40$

$w(x) = 15(x-40)+400$

$445 = 15x - 600 + 400$

$445 = 15x - 200$

$645 = 15x$

$\dfrac{645}{15} = x$

$43 = x$

43 Hours

I substitute the amount earned into the second equation and solved for x, which represents the number of hours worked.

35 An on-line electronics store must sell at least $2500 worth of printers and computers per day. Each printer costs $50 and each computer costs $500. The store can ship a maximum of 15 items per day.

Let p represent the # of printers
Let c represent the # of computers

$$p + c \leq 15$$

$$50p + 500c \geq 2500$$

On the set of axes below, graph a system of inequalities that models these constraints.

Number of Computers (vertical axis)

Number of Printers (horizontal axis)

(3,8)

$p + c \leq 15$

$50p + 500c \geq 2500$
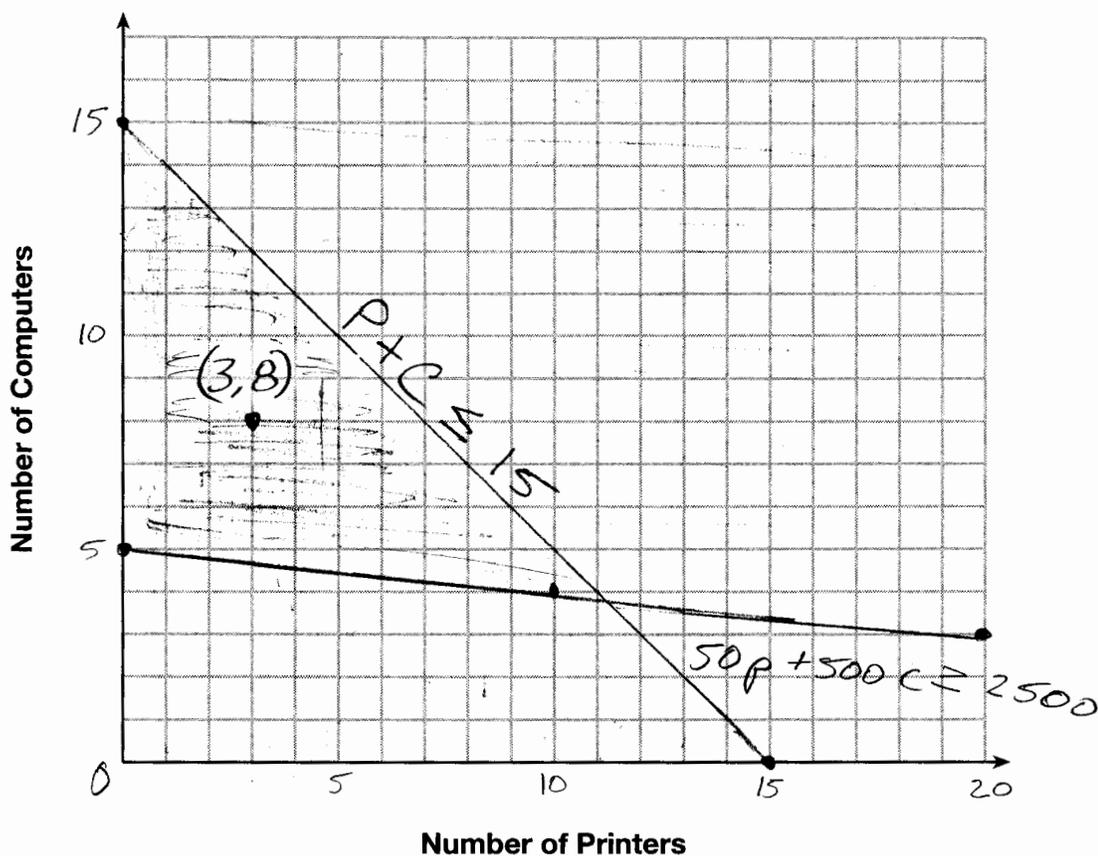
Determine a combination of printers and computers that would allow the electronics store to meet all of the constraints. Explain how you obtained your answer.

3 printers and 8 computers meet the constraints
Any combination of printers and computers, represented by coordinates (p,c) that are in the solution set of the system of inequalities will work.

36 An application developer released a new app to be downloaded. The table below gives the number of downloads for the first four weeks after the launch of the app.

Number of Weeks	1	2	3	4	$\times$
Number of Downloads	120	180	270	405	y

Write an exponential equation that models these data.

$$y = a \cdot b^x$$
$$a = 80$$
$$b = 1.5$$

$$y = 80(1.5)^{\#weeks}$$

or

$$y = 80(1.5)^x$$

Use this model to predict how many downloads the developer would expect in the 26th week if this trend continues. Round your answer to the *nearest download*.

$$y = 80(1.5)^{26}$$
$$y = 80(37876.75244)$$
$$y = \boxed{3,030,140}$$

Would it be reasonable to use this model to predict the number of downloads past one year? Explain your reasoning.

No. Apps come and go. There will probably be a new and better app in one year an nobody will be downloading this app anymore.

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be written in pencil. [6]

37 A football player attempts to kick a football over a goal post. The path of the football can be modeled by the function $h(x) = -\frac{1}{225}x^2 + \frac{2}{3}x$, where x is the horizontal distance from the kick, and $h(x)$ is the height of the football above the ground, when both are measured in feet.

On the set of axes below, graph the function $y = h(x)$ over the interval $0 \le x \le 150$.
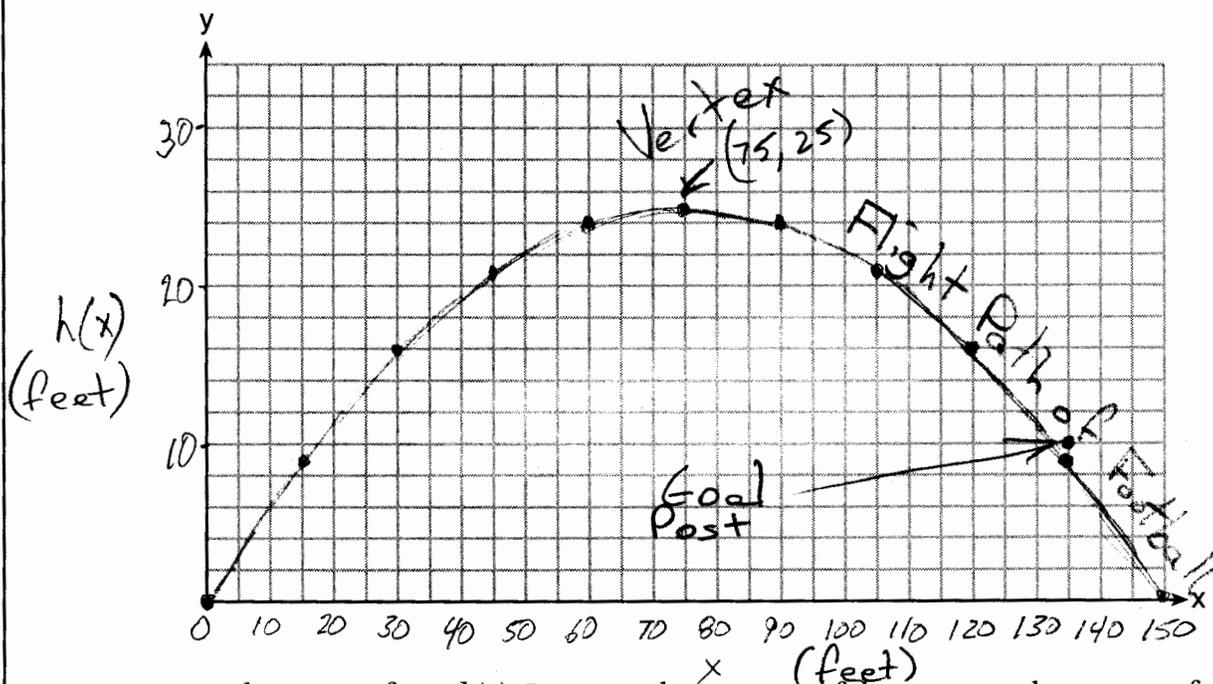
Determine the vertex of $y = h(x)$. Interpret the meaning of this vertex in the context of the problem.

Vertex is at (75,25). This is the highest point in the flight of the football.

The goal post is 10 feet high and 45 yards away from the kick. Will the ball be high enough to pass over the goal post? Justify your answer.

No. The flight of the football goes beneath the goal post.

3(45 yards) = 135 feet