

ALGEBRA I (COMMON CORE)

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA I (Common Core)

Thursday, January 28, 2016 — 1:15 to 4:15 p.m., only

Student Name: Steve Watson

School Name: www.jmap.org

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

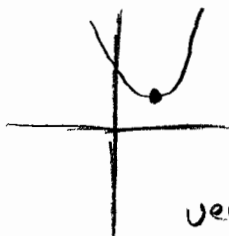
Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

1 In the function $f(x) = (x - 2)^2 + 4$, the minimum value occurs when x is

- (1) -2 (3) -4
 2 (4) 4

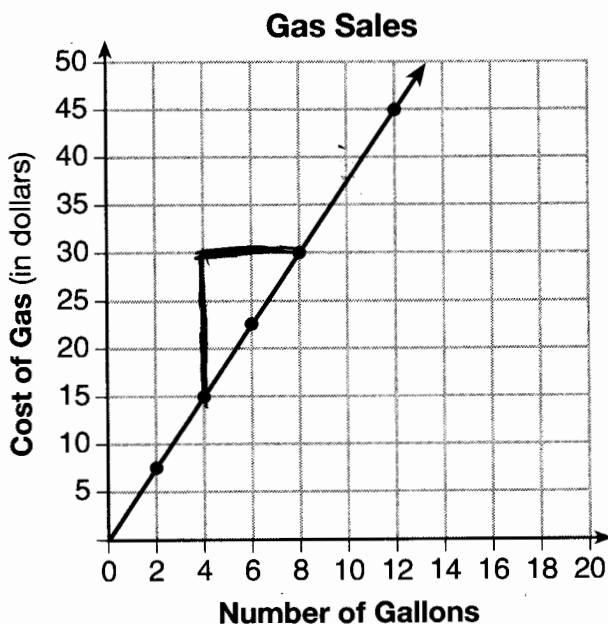
Input function in graphing calculator.



Use this space for computations.

x	$f(x)$
-1	13
0	8
1	5
2	4
3	5
4	8
5	13

2 The graph below was created by an employee at a gas station.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30 - 15}{8 - 4} = \frac{15}{4} = 3.75$$

$(8, 30)$
 $(4, 15)$

Which statement can be justified by using the graph?

- (1) If 10 gallons of gas was purchased, \$35 was paid.
 (2) For every gallon of gas purchased, \$3.75 was paid.
 (3) For every 2 gallons of gas purchased, \$5.00 was paid.
 (4) If zero gallons of gas were purchased, zero miles were driven.

Use this space for computations.

Input function in graphing calculator

x	y
3	27.163
5	10.107

$$\begin{array}{r} 27.163 \\ - 10.107 \\ \hline 17.056 \end{array}$$

3 For a recently released movie, the function $y = 119.67(0.61)^x$ models the revenue earned, y , in millions of dollars each week, x , for several weeks after its release.

Based on the equation, how much more money, in millions of dollars, was earned in revenue for week 3 than for week 5?

- (1) 37.27
- (2) 27.16
- (3) 17.06
- (4) 10.11

4 Given the following expressions:

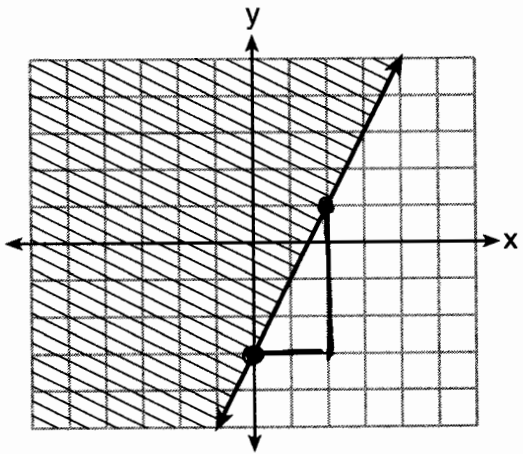
- I. $-\frac{5}{8} + \frac{3}{5} = \frac{-1}{40}$ Rational
- II. $\frac{1}{2} + \sqrt{2}$ Yes
- III. $(\sqrt{5}) \cdot (\sqrt{5}) = \sqrt{25} = 5 = \frac{5}{1}$ Rational
- IV. $3 \cdot (\sqrt{49}) = 3 \cdot 7 = 21 = \frac{21}{1}$ Rational

Which expression(s) result in an irrational number?

- (1) II, only
- (2) III, only
- (3) I, III, IV
- (4) II, III, IV

$$\begin{array}{r} 0.50000 \\ 1.41421 \dots \\ \hline 1.91421 \dots \\ \text{Irrational} \end{array}$$

5 Which inequality is represented by the graph below?



- (1) $y \leq 2x - 3$
- (2) $y \geq 2x - 3$
- (3) $y \leq -3x + 2$
- (4) $y \geq -3x + 2$

$y = mx + b$
 y intercept = -3
 eliminate choices (3) and (4)
 $m = \frac{4}{2} = 2$
 Shading is above the line,
 and the line is solid,
 so use $y \geq$
 The inequality is $y \geq 2x - 3$

Use this space for computations.

6 Michael borrows money from his uncle, who is charging him simple interest using the formula $I = Prt$. To figure out what the interest rate, r , is, Michael rearranges the formula to find r . His new formula is r equals

(1) $\frac{I-P}{t}$

$\frac{I}{Pt}$

(2) $\frac{P-I}{t}$

(4) $\frac{Pt}{I}$

$$I = Prt$$

$$\frac{I}{Pt} = \frac{Prt}{Pt}$$

$$\frac{I}{Pt} = r$$

7 Which equation is equivalent to $y - 34 = x(x - 12)$?

(1) $y = (x - 17)(x + 2)$

(3) $y = (x - 6)^2 + 2$

(2) $y = (x - 17)(x - 2)$

$y = (x - 6)^2 - 2$

$$A = Prt$$

$$y - 34 = x(x - 12)$$

$$y - 34 = x^2 - 12x$$

$$y = x^2 - 12x + 34$$

$$x^2 - 12x + 34 = 0$$

$$x^2 - 12x + (6)^2 = -34 + (6)^2$$

$$(x - 6)^2 = -2$$

$$(x - 6)^2 - 2 = 0$$

8 The equation $A = 1300(1.02)^7$ is being used to calculate the amount of money in a savings account. What does 1.02 represent in this equation?

(1) 0.02% decay

(3) 2% decay

(2) 0.02% growth

2% growth

$$.02 = 2\%$$

9 The zeros of the function $f(x) = 2x^2 - 4x - 6$ are

3 and -1

(3) -3 and 1

(2) 3 and 1

(4) -3 and -1

$$f(x) = 2x^2 - 4x - 6$$

$$2x^2 - 4x - 6 = 0$$

$$2x^2 - 4x = 6$$

$$x^2 - 2x = 3$$

$$x^2 - 2x + (-1)^2 = 3 + (-1)^2$$

$$(x - 1)^2 = 4$$

$$x - 1 = \pm 2$$

$$x = 1 \pm 2$$

$$x = 1 + 2 = 3$$

$$x = 1 - 2 = -1$$

10 When $(2x - 3)^2$ is subtracted from $5x^2$, the result is

(1) $x^2 - 12x - 9$

$x^2 + 12x - 9$

(2) $x^2 - 12x + 9$

(4) $x^2 + 12x + 9$

$$(2x - 3)(2x - 3) = 4x^2 - 6x - 6x + 9$$

$$4x^2 - 12x + 9$$

$$5x^2 + 0x + 0$$

$$- (4x^2 - 12x + 9)$$

$$\hline x^2 + 12x - 9$$

120 ft²

Use this space for computations.

120 (1.5) = 180

11 Joe has a rectangular patio that measures 10 feet by 12 feet. He wants to increase the area by 50% and plans to increase each dimension by equal lengths, x . Which equation could be used to determine x ?

- (1) ~~(10 + x)(12 + x) = 120~~ (3) (15 + x)(18 + x) = 180
- (10 + x)(12 + x) = 180 (4) (15)(18) = 120 + x²

$(10+x)(12+x) = 180$

$x^3 - 13x^2 - 30x$
 $x(x^2 - 13x - 30)$
 $x(x+2)(x-15)$

12 When factored completely, $x^3 - 13x^2 - 30x$ is

- (1) $x(x + 3)(x - 10)$ $x(x + 2)(x - 15)$
- (2) $x(x - 3)(x - 10)$ (4) $x(x - 2)(x + 15)$

13 The table below shows the cost of mailing a postcard in different years. During which time interval did the cost increase at the greatest average rate?

Year	1898	1971	1985	2006	2012
Cost (¢)	1	6	14	24	35

$m = \frac{y_2 - y_1}{x_2 - x_1}$

- (1) 1898–1971 (3) 1985–2006
- (2) 1971–1985 2006–2012

(1) $m = \frac{6-1}{1971-1898} = \frac{5}{73}$

(3) $m = \frac{24-14}{2006-1985} = \frac{10}{21}$

(2) $m = \frac{14-6}{1985-1971} = \frac{8}{14}$

(4) $m = \frac{35-24}{2012-2006} = \frac{11}{6}$

14 When solving the equation $x^2 - 8x - 7 = 0$ by completing the square, which equation is a step in the process?

- (1) $(x - 4)^2 = 9$ (3) $(x - 8)^2 = 9$
- (2) $(x - 4)^2 = 23$ (4) $(x - 8)^2 = 23$

$x^2 - 8x - 7 = 0$
 $x^2 - 8x = 7$
 $x^2 - 8x + (-4)^2 = 7 + (-4)^2$
 $(x-4)^2 = 23$

15 A construction company uses the function $f(p)$, where p is the number of people working on a project, to model the amount of money it spends to complete a project. A reasonable domain for this function would be

- (1) positive integers 1, 2, 3, 4, 5, ...
- (2) positive real numbers - no need for fractions
- (3) both positive and negative integers - no need for negative integers
- (4) both positive and negative real numbers - no need for negative real numbers

16 Which function is shown in the table below?

Use this space for computations.

x	f(x)
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

Check in graphing calculator

x	y
-2	.11111 = $\frac{1}{9}$
-1	.3333 = $\frac{1}{3}$
0	1
1	3
2	9
3	27

- ~~(1)~~ $f(x) = 3x$
~~(2)~~ $f(x) = x + 3$

- (3) $f(x) = -x^3$
 (4) $f(x) = 3^x$

17 Given the functions $h(x) = \frac{1}{2}x + 3$ and $j(x) = |x|$, which value of x makes $h(x) = j(x)$?

- (1) -2
 (2) 2
 (3) 3
 (4) -6

Use graphing calculator

x	h(x)	j(x)
-6	0	6
-2	2	2
2	4	2
3	4.5	3

$h(-2) = j(-2) \rightarrow$

18 Which recursively defined function represents the sequence 3, 7, 15, 31, ...?

- (1) $f(1) = 3, f(n+1) = 2f(n) + 3$
 (2) $f(1) = 3, f(n+1) = 2f(n) - 1$
 (3) $f(1) = 3, f(n+1) = 2f(n) + 1$
 (4) $f(1) = 3, f(n+1) = 3f(n) - 2$

Each term is twice the preceding term + 1
 $f(1) = 3 \checkmark$
 $f(2) = 2(3) + 1 = 7 \checkmark$
 $f(3) = 2(7) + 1 = 15 \checkmark$
 $f(4) = 2(15) + 1 = 31 \checkmark$

19 The range of the function defined as $y = 5^x$ is

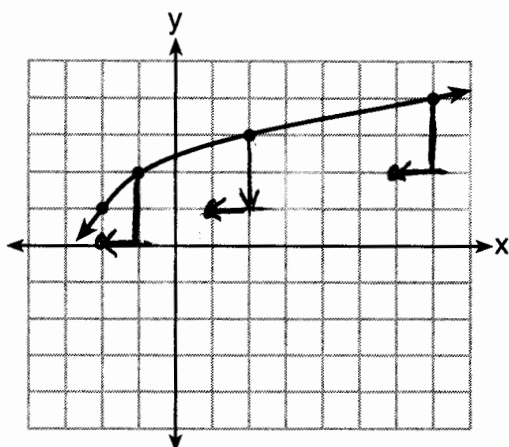
- (1) $y < 0$
 (2) $y > 0$
 (3) $y \leq 0$
 (4) $y \geq 0$

Use graphing calculator.
 y is always positive and never reaches zero

Use this space for computations.

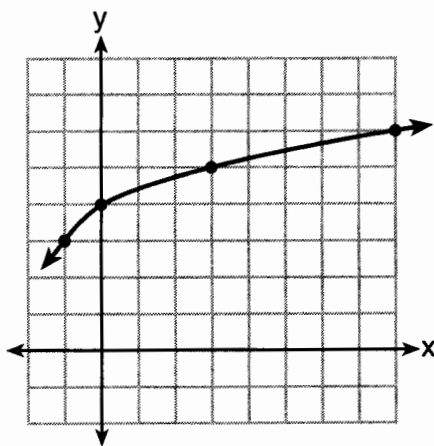
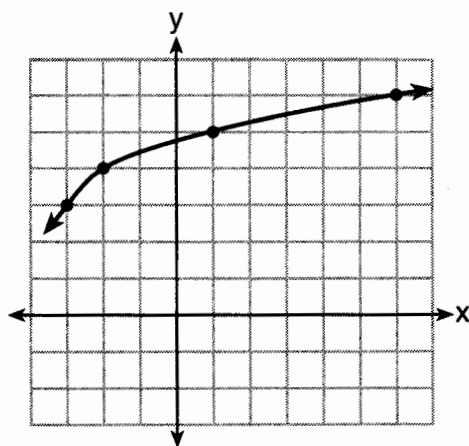
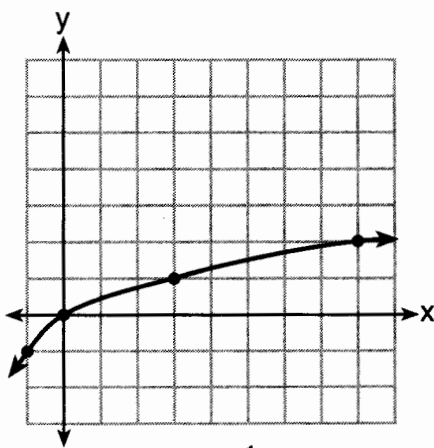
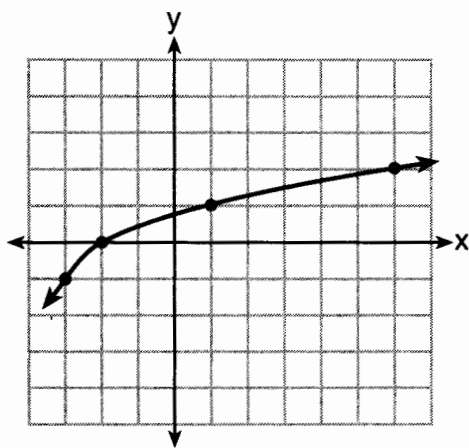
20 The graph of $y = f(x)$ is shown below.

- $(-2, 1)$
- $(-1, 2)$
- $(2, 3)$
- $(7, 4)$



$(x+1) \rightarrow$ moves 1 to left
 $-2 \rightarrow$ move 2 down

What is the graph of $y = f(x + 1) - 2$?



Use this space for computations.

21 Which pair of equations could *not* be used to solve the following equations for x and y ?

$$\begin{aligned} 4x + 2y &= 22 \\ -2x + 2y &= -8 \end{aligned}$$

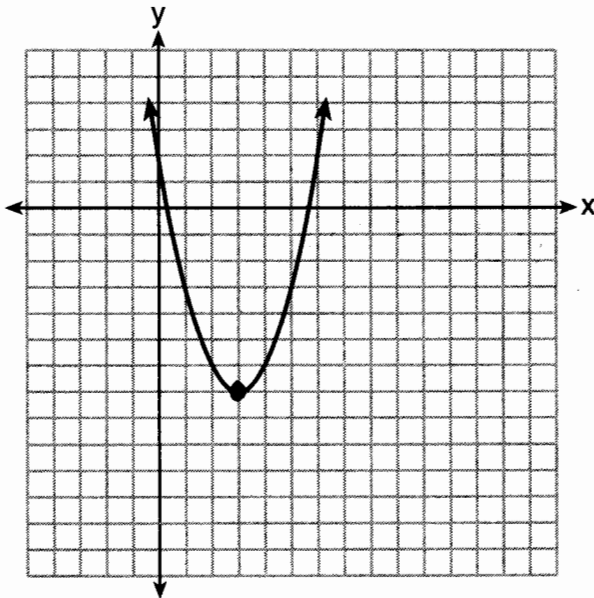
(1) $4x + 2y = 22$ ✓
 $2x - 2y = 8$ ✓

(3) $12x + 6y = 66$ ✓
 $6x - 6y = 24$ ✓

(2) $4x + 2y = 22$ ✓
 $-4x + 4y = -16$ ✓

● $8x + 4y = 44$ ✓
 $-8x + 8y = -8$

22 The graph representing a function is shown below.



Minimum = -7

Which function has a minimum that is *less* than the one shown in the graph? vertex = (3, -2)

-2 (1) $y = x^2 - 6x + 7$

● (2) $y = x^2 - 2x - 10$

-6 (3) $y = |x + 3| - 6$

(4) $y = |x - 8| + 2$

(-3, -6)

(8, 2)

23 Grisham is considering the three situations below.

- I. For the first 28 days, a sunflower grows at a rate of 3.5 cm per day.
- II. The value of a car depreciates at a rate of 15% per year after it is purchased.
- III. The amount of bacteria in a culture triples every two days during an experiment.

Use this space for computations.

$$h = 3.5d$$

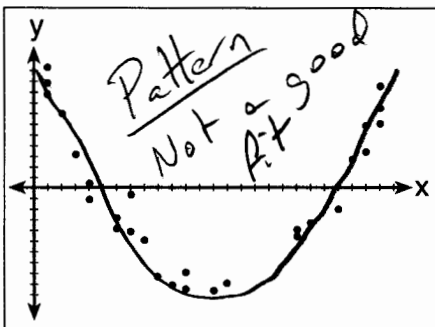
$$V = P(1-.15)^t$$

$$A = P(3)^{\frac{d}{2}}$$

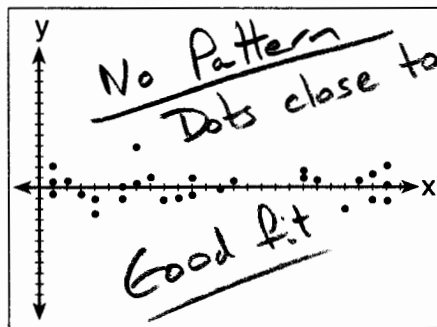
Which of the statements describes a situation with an equal difference over an equal interval?

- (1) I, only
- (2) II, only
- (3) I and III
- (4) II and III

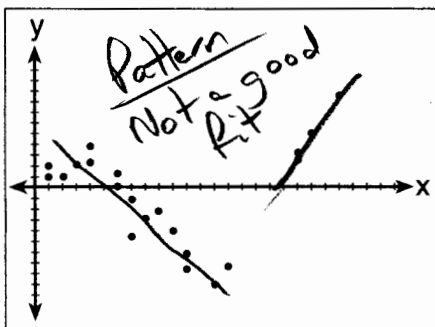
24 After performing analyses on a set of data, Jackie examined the scatter plot of the residual values for each analysis. Which scatter plot indicates the best linear fit for the data?



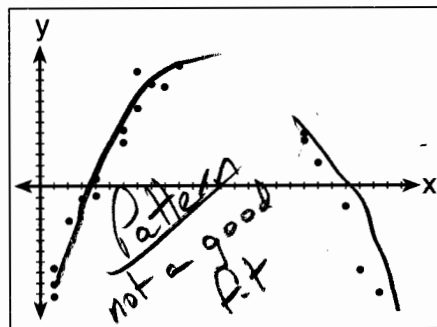
(1)



(2)



(3)



(4)

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

25 The function, $t(x)$, is shown in the table below.

x	t(x)
-3	10
-1	7.5
1	5
3	2.5
5	0

Determine whether $t(x)$ is linear or exponential. Explain your answer.

Linear

Step 1 - Input table in Stat edit

Step 2 - Turn Stat Plot on.

Step 3 - Use Zoom Stat to inspect scatter plot

The scatterplot shows a straight line

Check

Use Stat Calc LinReg to get equation

$$y = ax + b$$

$$a = -1.25$$

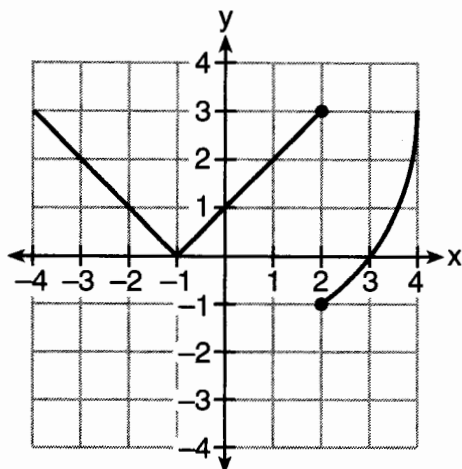
$$b = 6.25$$

$$y = -1.25x + 6.25$$

Input into equation editor and check table

-3	10
-1	7.5
1	5
3	2.5
5	0

26 Marcel claims that the graph below represents a function.



State whether Marcel is correct. Justify your answer.

Marcel is wrong.

When x equals 2, there are two values of y .

A function has one and only one value of y for every value of x .

27 Solve the equation for y .

$$(y - 3)^2 = 4y - 12$$

$$(y - 3)^2 = 4y - 12$$

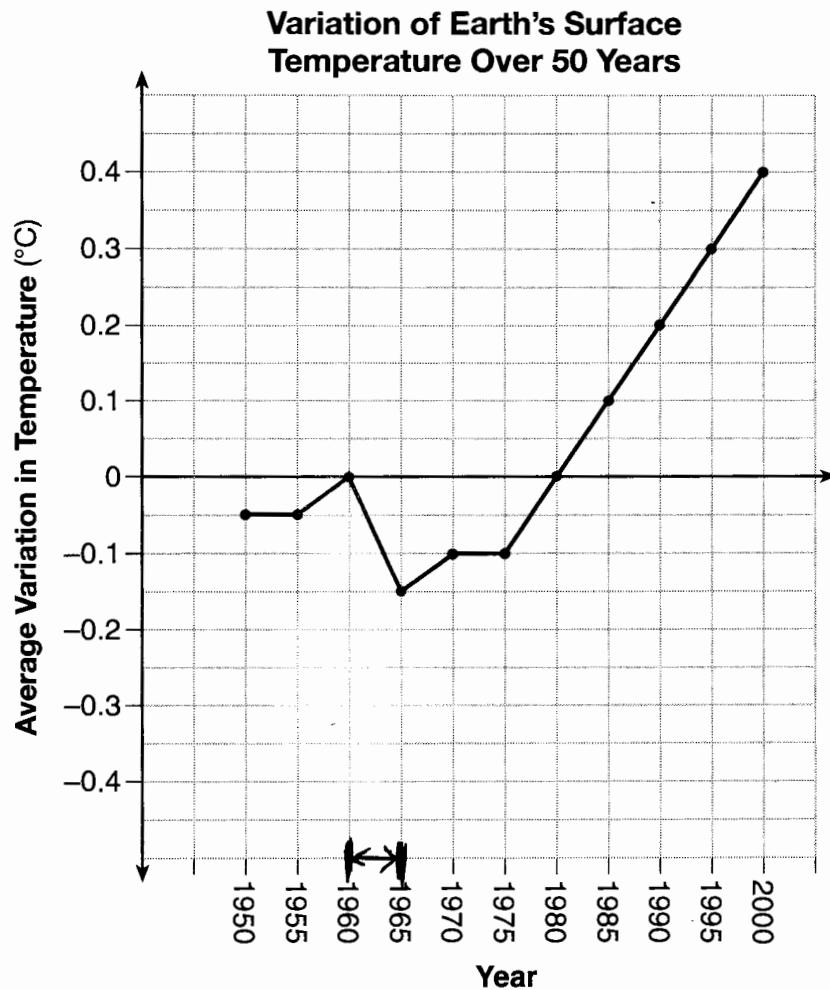
$$y^2 - 6y + 9 = 4y - 12$$

$$y^2 - 10y + 21 = 0$$

$$(y - 7)(y - 3) = 0$$

$$y = 7 \text{ and } y = 3$$

28 The graph below shows the variation in the average temperature of Earth's surface from 1950–2000, according to one source.



During which years did the temperature variation change the most per unit time? Explain how you determined your answer.

The temperature variation changed the most from 1960 to 1965. The rate of change for this interval is greater than for any other interval.

29 The cost of belonging to a gym can be modeled by $C(m) = 50m + 79.50$, where $C(m)$ is the total cost for m months of membership.

State the meaning of the slope and y -intercept of this function with respect to the costs associated with the gym membership.

$$y = \overset{\text{slope}}{m} x + \overset{y\text{-intercept}}{b}$$
$$C(m) = 50m + 79.50$$

The slope is 50, which represents a monthly charge of $\$50$.

The y -intercept is $\$79.50$, which represents a one-time fee for membership.

- 30 A statistics class surveyed some students during one lunch period to obtain opinions about television programming preferences. The results of the survey are summarized in the table below.

	Comedy	Drama
Male	70	35
Female	48	42

Based on the sample, predict how many of the school's 351 males would prefer comedy. Justify your answer.

$$70 + 35 = 105 \text{ males were surveyed.}$$

70 out of 105 males prefer comedy

$$\frac{70}{105} = \frac{2}{3} = 66.\bar{6}\%$$

$$\frac{2}{3}(351) = \boxed{234}$$

31 Given that $a > b$, solve for x in terms of a and b :

$$b(x - 3) \geq ax + 7b$$

$$bx - 3b \geq ax + 7b$$

$$bx - ax \geq 7b + 3b$$

$$x(b - a) \geq 10b$$

If $a > b$, then $(b - a)$ is negative.

$$x \leq \frac{10b}{b - a}$$

32 Jacob and Jessica are studying the spread of dandelions. Jacob discovers that the growth over t weeks can be defined by the function $f(t) = (8) \cdot 2^t$. Jessica finds that the growth function over t weeks is $g(t) = 2^{t+3}$.

Calculate the number of dandelions that Jacob and Jessica will each have after 5 weeks.

$$f(5) = 8(2)^5$$

$$f(5) = 8(32)$$

$$f(5) = 256$$

Jacob

$$g(5) = 2^{(5+3)}$$

$$g(5) = 2^8$$

$$g(5) = 256$$

Jessica

Based on the growth from both functions, explain the relationship between $f(t)$ and $g(t)$.

$$f(t) = g(t) \text{ for all values of } t$$

$$f(t) = 8 \cdot 2^t$$

$$f(t) = 2^3 \cdot 2^t$$

$$f(t) = 2^{(3+t)}, \text{ which is the same as } g(t) = 2^{(t+3)}$$

Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

33 Let $h(t) = -16t^2 + 64t + 80$ represent the height of an object above the ground after t seconds. Determine the number of seconds it takes to achieve its maximum height. Justify your answer.

$h(t) = -16t^2 + 64t + 80$

Input function rule in graphing calculator

t	$h(t)$
-1	0
0	80
1	128
2	144
3	128
4	80
5	0

Vertex

The object achieves maximum height after 2 seconds

State the time interval, in seconds, during which the height of the object *decreases*. Explain your reasoning.

The height of the object decreases during the interval $2 < t < 5$.

34 Fred's teacher gave the class the quadratic function $f(x) = 4x^2 + 16x + 9$.

a) State two different methods Fred could use to solve the equation $f(x) = 0$.

Quadratic formula

Complete the square

b) Using one of the methods stated in part a, solve $f(x) = 0$ for x , to the nearest tenth.

Complete the square

$$4x^2 + 16x + 9 = 0$$

$$4x^2 + 16x = -9$$

$$x^2 + 4x = -\frac{9}{4}$$

$$\underbrace{x^2 + 4x + (2)^2}_{(x+2)^2} = -\frac{9}{4} + (2)^2$$

$$(x+2)^2 = -\frac{9}{4} + 4$$

$$(x+2)^2 = -\frac{9}{4} + \frac{16}{4}$$

$$(x+2)^2 = \frac{7}{4}$$

$$x+2 = \pm \frac{\sqrt{7}}{2}$$

$$x = -2 \pm \frac{\sqrt{7}}{2}$$

$$x = -2 - \frac{\sqrt{7}}{2} = \boxed{-3.3}$$

$$x = -2 + \frac{\sqrt{7}}{2} = \boxed{-0.7}$$

Check

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-16 \pm \sqrt{16^2 - 4(4)(9)}}{8}$$

$$x = \frac{-16 \pm \sqrt{112}}{8}$$

$$x = \frac{-16 \pm 10.58}{8}$$

$$x = -3.3 \checkmark$$

$$x = -0.7 \checkmark$$

- 35 Erica, the manager at Stellarbeans, collected data on the daily high temperature and revenue from coffee sales. Data from nine days this past fall are shown in the table below.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9
x High Temperature, t	54	50	62	67	70	58	52	46	48
y Coffee Sales, $f(t)$	\$2900	\$3080	\$2500	\$2380	\$2200	\$2700	\$3000	\$3620	\$3720

State the linear regression function, $f(t)$, that estimates the day's coffee sales with a high temperature of t . Round all values to the nearest integer.

$$y = ax + b$$

$$a = -58$$

$$b = 6182$$

$$y = -58x + 6182$$

$$f(t) = -58t + 6182$$

State the correlation coefficient, r , of the data to the nearest hundredth. Does r indicate a strong linear relationship between the variables? Explain your reasoning.

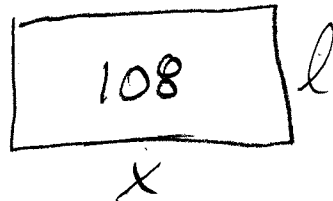
$$r = -.9441304218$$

$$r = -.94$$

r indicates a strong relationship because $|r|$ is close to 1.

36 A contractor has 48 meters of fencing that he is going to use as the perimeter of a rectangular garden. The length of one side of the garden is represented by x , and the area of the garden is 108 square meters.

Determine, algebraically, the dimensions of the garden in meters.



$$P = 2x + 2l$$

$$\text{Eq. 1 } 48 = 2x + 2l$$

$$\text{Eq. 2 } xl = 108$$

$$\text{Eq. 2a } l = \frac{108}{x}$$

Use Substitution Method

$$48 = 2x + 2\left(\frac{108}{x}\right)$$

$$48 = 2x + \frac{216}{x}$$

$$48x = 2x^2 + 216$$

$$2x^2 - 48x + 216 = 0$$

$$2x^2 - 48x = -216$$

$$x^2 - 24x + (-12)^2 = -108 + (12)^2$$

$$(x-12)^2 = 36$$

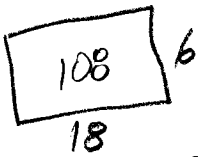
$$x-12 = \pm 6$$

$$x = 12 \pm 6$$

$$x = 18$$

$$l = \frac{108}{18} = 6$$

Check



$$6 \times 18 = 108 \checkmark$$

$$48 = 2x + 2l$$

$$48 = 2(18) + 2(6)$$

$$48 = 36 + 12$$

$$48 = 48 \checkmark$$

The garden is 18 x 6 meters

Answer

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

37 The Reel Good Cinema is conducting a mathematical study. In its theater, there are 200 seats. Adult tickets cost \$12.50 and child tickets cost \$6.25. The cinema's goal is to sell at least \$1500 worth of tickets for the theater.

let A represent # of adult tickets / let C represent # of child tickets
Write a system of linear inequalities that can be used to find the possible combinations of adult tickets, x , and child tickets, y , that would satisfy the cinema's goal.

$$A + C \leq 200$$

$$x + y \leq 200$$

$$y \leq 200 - x$$

$$12.50A + 6.25C \geq 1500$$

$$12.5x + 6.25y \geq 1500$$

$$y \geq \frac{1500 - 12.5x}{6.25}$$

Graph the solution to this system of inequalities on the set of axes on the next page. Label the solution with an S .

Marta claims that selling 30 adult tickets and 80 child tickets will result in meeting the cinema's goal. Explain whether she is correct or incorrect, based on the graph drawn.

Marta is incorrect

$(30, 80)$ is not in the solution set.

X	Y ₁	Y ₂
0	200	240
120		0
200	0	
40	160	160

