The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA 2/TRIGONOMETRY

Tuesday, June 21, 2011—1:15 to 4:15 p.m., only

Student Name: Mr. Sibol

School Name: HSCR

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 27 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [54]

Use this space for computations.

1 A doctor wants to test the effectiveness of a new drug on her patients. She separates her sample of patients into two groups and administers the drug to only one of these groups. She then compares the results. Which type of study *best* describes this situation?

(1) census
(2) survey
(3) observation
(4) controlled experiment

2 If $f(x) = \frac{x}{x^2 - 16}$, what is the value of $f(-10)$?

(1) $-\frac{5}{2}$
(2) $-\frac{5}{42}$
(3) $\frac{5}{58}$
(4) $\frac{5}{18}$

$\frac{-10}{(-10)^2-16} = \frac{-10}{84} = -\frac{5}{42}$

3 An auditorium has 21 rows of seats. The first row has 18 seats, and each succeeding row has two more seats than the previous row. How many seats are in the auditorium?

(1) 540
(2) 567
(3) 760
(4) 798

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{21}{2}[2(18) + (21-1)2]$

$= 798$

Use this space for computations.

4 Expressed as a function of a positive acute angle, $\cos(-305°)$ is equal to

(1) $-\cos 55°$
(2) $\cos 55°$
(3) $-\sin 55°$
(4) $\sin 55°$

5 The value of x in the equation $4^{2x+5} = 8^{3x}$ is

(1) 1
(2) 2
(3) 5
(4) -10

$(2^2)^{2x+5} = (2^3)^{3x}$
$2^{4x+10} = 2^{9x}$
$4x+10 = 9x$
$10 = 5x$
$2 = x$

6 What is the value of x in the equation $\log_5 x = 4$?

(1) 1.16
(2) 20
(3) 625
(4) 1,024

$x = 5^4 = 625$

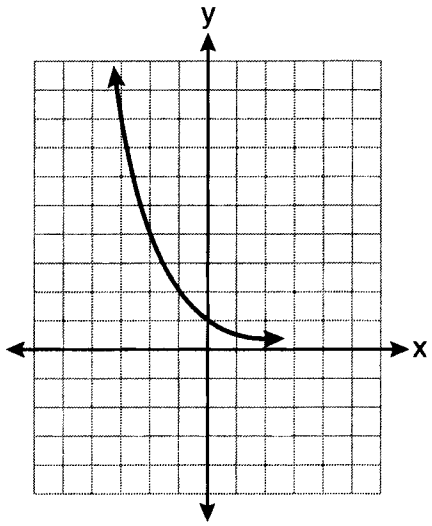
7 The expression $\sqrt[4]{16x^2y^7}$ is equivalent to

(1) $2x^{\frac{1}{2}}y^{\frac{7}{4}}$
(2) $2x^8y^{28}$
(3) $4x^{\frac{1}{2}}y^{\frac{7}{4}}$
(4) $4x^8y^{28}$

$16^{\frac{1}{4}} x^{\frac{2}{4}} y^{\frac{7}{4}}$
$2x^{\frac{1}{2}}y^{\frac{7}{4}}$

Use this space for computations.

8 Which equation is represented by the graph below?



(1) $y = 5^x$
(2) $y = 0.5^x$
(3) $y = 5^{-x}$
(4) $y = 0.5^{-x}$

9 What is the fifteenth term of the geometric sequence $-\sqrt{5}$, $\sqrt{10}$, $-2\sqrt{5}$, . . . ?

(1) $-128\sqrt{5}$
(2) $128\sqrt{10}$
(3) $-16384\sqrt{5}$
(4) $16384\sqrt{10}$

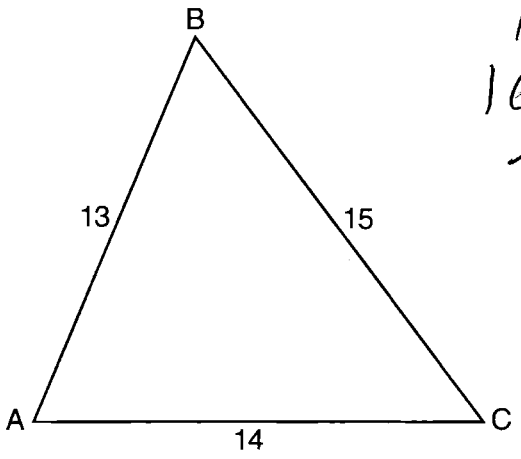
$a_n = -\sqrt{5}(-\sqrt{2})^{n-1}$

$a_{15} = -\sqrt{5}(-\sqrt{2})^{15-1}$

$= -\sqrt{5}(128)$

Use this space for computations.

10 In $\triangle ABC$, $a = 15$, $b = 14$, and $c = 13$, as shown in the diagram below. What is the $m\angle C$, to the *nearest degree*?



$13^2 = 15^2 + 14^2 - 2(15)(14)\cos C$

$169 = 421 - 420\cos C$

$-252 = -420\cos C$

$\frac{252}{420} = \cos C$

$53 \approx C$

(1) 53
(2) 59
(3) 67
(4) 127

11 What is the period of the function $f(\theta) = -2\cos 3\theta$?

(1) π
(2) $\frac{2\pi}{3}$
(3) $\frac{3\pi}{2}$
(4) 2π

$\frac{2\pi}{3}$

12 What is the range of $f(x) = (x + 4)^2 + 7$?

(1) $y \geq -4$
(2) $y \geq 4$
(3) $y = 7$
(4) $y \geq 7$

The vertex of this parabola is $(-4, 7)$

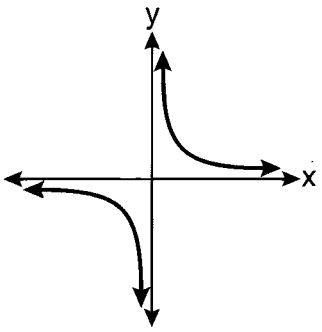
Use this space for computations.

13 Ms. Bell's mathematics class consists of 4 sophomores, 10 juniors, and 5 seniors. How many different ways can Ms. Bell create a four-member committee of juniors if each junior has an equal chance of being selected?

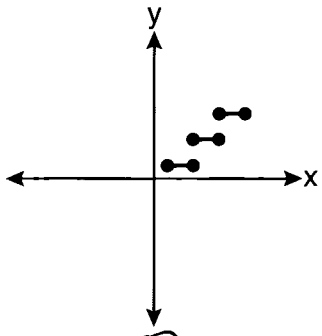
(1) 210
(3) 5,040
(2) 3,876
(4) 93,024

$_{10}C_4$

14 Which graph represents a relation that is *not* a function?



(1)



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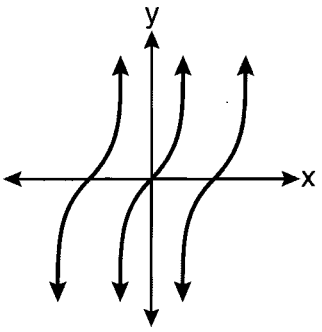

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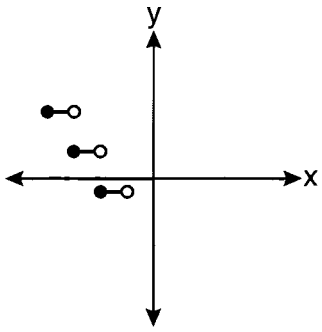


(4)

Use this space for computations.

15 The value of tan 126°43′ to the *nearest ten-thousandth* is

(1) −1.3407
(2) −1.3408
(3) −1.3548
(4) −1.3549

16 The expression $\frac{4}{5-\sqrt{13}}$ is equivalent to

(1) $\frac{4\sqrt{13}}{5\sqrt{13}-13}$
(2) $\frac{4(5-\sqrt{13})}{38}$
(3) $\frac{5+\sqrt{13}}{3}$
(4) $\frac{4(5+\sqrt{13})}{38}$

$\frac{4}{5-\sqrt{13}} \cdot \frac{5+\sqrt{13}}{5+\sqrt{13}} = \frac{4(5+\sqrt{13})}{25-13} = \frac{5+\sqrt{13}}{3}$

OR

$\frac{4}{5-\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{4\sqrt{13}}{5\sqrt{13}-13}$

17 Akeem invests $25,000 in an account that pays 4.75% annual interest compounded continuously. Using the formula $A = Pe^{rt}$, where A = the amount in the account after t years, P = principal invested, and r = the annual interest rate, how many years, to the *nearest tenth*, will it take for Akeem's investment to triple?

(1) 10.0
(3) 23.1
(2) 14.6
(4) 24.0

$75{,}000 = 25{,}000e^{.0475t}$

$3 = e^{.0475t}$

$\ln 3 = \ln e^{.0475t}$

$\frac{\ln 3}{.0475} = \frac{.0475t \cdot \ln e}{.0475}$

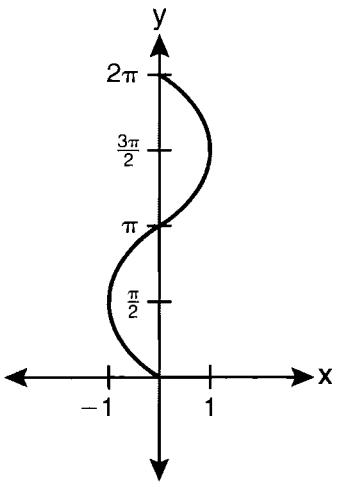
18 The value of the expression $\sum_{r=3}^{5}(-r^2 + r)$ is

(1) −38
(3) 26
(2) −12
(4) 62

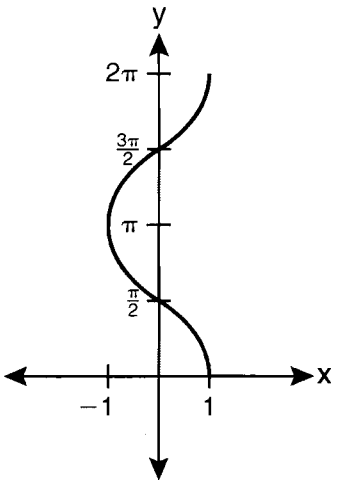
r	$-r^2+r$	
3	$-9+3$	-6
4	$-16+4$	-12
5	$-25+5$	-20
		-38

Use this space for computations.

19 Which graph shows $y = \cos^{-1} x$?



(1)



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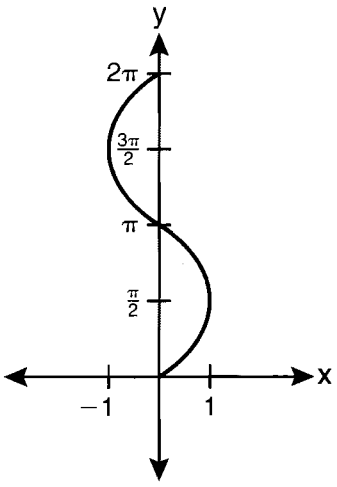
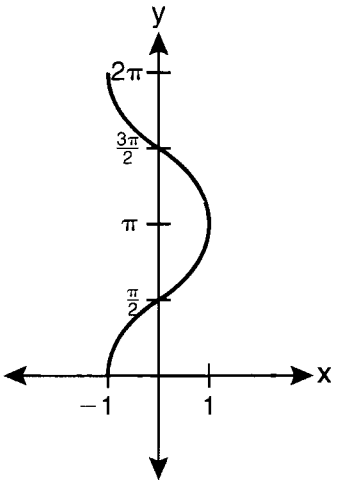


(2)



(4)

Use this space for computations.

20 If $r = \sqrt[3]{\frac{A^2B}{C}}$, then $\log r$ can be represented by

(1) $\frac{1}{6}\log A + \frac{1}{3}\log B - \log C$

(2) $3(\log A^2 + \log B - \log C)$

(3) $\frac{1}{3}\log(A^2 + B) - C$

(4) $\frac{2}{3}\log A + \frac{1}{3}\log B - \frac{1}{3}\log C$

$\log \sqrt[3]{\frac{A^2B}{C}}$

$\log \left(\frac{A^2B}{C}\right)^{1/3}$

$\frac{1}{3}\log \frac{A^2B}{C}$

$\frac{1}{3}\log A^2 + \frac{1}{3}\log B - \frac{1}{3}\log C$

$\frac{2}{3}\log A + \frac{1}{3}\log B - \frac{1}{3}\log C$

21 The solution set of $\sqrt{3x + 16} = x + 2$ is

(1) $\{-3, 4\}$

(2) $\{-4, 3\}$

(3) $\{3\}$

(4) $\{-4\}$

$3x+16 = (x+2)^2$

$3x+16 = x^2+4x+4$

$0 = x^2+x-12$

$0 = (x+4)(x-3)$

$x = -4 \quad x = 3$

$\sqrt{3x+16} \neq -4+2$

22 Brian correctly used a method of completing the square to solve the equation $x^2 + 7x - 11 = 0$. Brian's first step was to rewrite the equation as $x^2 + 7x = 11$. He then added a number to both sides of the equation. Which number did he add?

(1) $\frac{7}{2}$

(2) $\frac{49}{4}$

(3) $\frac{49}{2}$

(4) 49

$\left(\frac{7}{2}\right)^2 = \frac{49}{4}$

23 The expression $\frac{\sin^2\theta + \cos^2\theta}{1 - \sin^2\theta}$ is equivalent to

(1) $\cos^2\theta$

(2) $\sin^2\theta$

(3) $\sec^2\theta$

(4) $\csc^2\theta$

Use this space for computations.

$\frac{1}{\cos^2\theta} = \sec^2\theta$

24 The number of minutes students took to complete a quiz is summarized in the table below.

Minutes	14	15	16	17	18	19	20
Number of Students	5	3	x	5	2	10	1

$26+x$

$70 \quad 45 \quad 16x \quad 85 \quad 36 \quad 190 \quad 20 = 446+16x$

If the mean number of minutes was 17, which equation could be used to calculate the value of x?

(1) $17 = \frac{119 + x}{x}$

(2) $17 = \frac{119 + 16x}{x}$

(3) $17 = \frac{446 + x}{26 + x}$

(4) $17 = \frac{446 + 16x}{26 + x}$

25 What is the radian measure of the smaller angle formed by the hands of a clock at 7 o'clock?

(1) $\frac{\pi}{2}$

(2) $\frac{2\pi}{3}$

(3) $\frac{5\pi}{6}$

(4) $\frac{7\pi}{6}$

$\frac{5}{12} \cdot 2\pi = \frac{5\pi}{6}$

Use this space for computations.

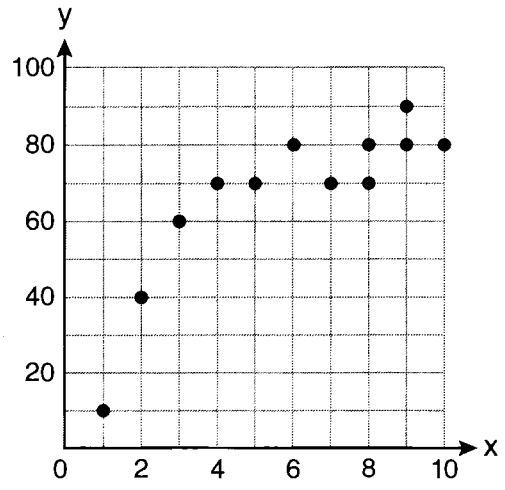
26 What is the coefficient of the fourth term in the expansion of $(a - 4b)^9$?

(1) $-5{,}376$
(2) -336
(3) 336
(4) 5,376

$_9C_3\, a^6(-4b)^3$

$-5376a^6b^3$

27 Samantha constructs the scatter plot below from a set of data.



Based on her scatter plot, which regression model would be most appropriate?

(1) exponential
(2) linear
(3) logarithmic
(4) power

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

28 Express the product of $\left(\frac{1}{2}y^2 - \frac{1}{3}y\right)$ and $\left(12y + \frac{3}{5}\right)$ as a trinomial.

$6y^3 + \frac{3}{10}y^2 - 4y^2 - \frac{1}{5}y$

$6y^3 - \frac{37}{10}y^2 - \frac{1}{5}y$

29 In a study of 82 video game players, the researchers found that the ages of these players were normally distributed, with a mean age of 17 years and a standard deviation of 3 years. Determine if there were 15 video game players in this study over the age of 20. Justify your answer.

over 20 is more than 1 sd above mean

.159 · 82 = 13.038

No

30 Write a quadratic equation such that the sum of its roots is 6 and the product of its roots is -27.

$\frac{-b}{a} = 6 \qquad \frac{c}{a} = -27$

assume $a = 1$, $-b = 6$ $\qquad c = -27$

$b = -6$

$x^2 - 6x - 27 = 0$

31 Evaluate $e^{x \ln y}$ when $x = 3$ and $y = 2$.

$e^{3 \ln 2} = e^{\ln 2^3} = e^{\ln 8} = 8$

32 If $f(x) = x^2 - 6$, find $f^{-1}(x)$.

$y = x^2 - 6$

$x = y^2 - 6$

$x + 6 = y^2$

$\pm\sqrt{x+6} = y$

$\pm\sqrt{x+6} = f^{-1}(x)$

Not a Function

33 Factor the expression $12t^8 - 75t^4$ completely.

$3t^4(4t^4-25)$

$3t^4(2t^2+5)(2t^2-5)$

34 Simplify the expression $\frac{3x^{-4}y^5}{(2x^3y^{-7})^{-2}}$ and write the answer using only positive exponents.

$$\frac{3y^5(2x^3y^{-7})^2}{x^4}$$

$$\frac{3y^5(4x^6y^{-14})}{x^4}$$

$$\frac{12x^6y^{-9}}{x^4}$$

$$\frac{12x^2}{y^9}$$

35 If $f(x) = x^2 - 6$ and $g(x) = 2^x - 1$, determine the value of $(g \circ f)(-3)$.

$F(-3) = (-3)^2 - 6 = 3$

$g(3) = 2^3 - 1 = 7$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 Express as a single fraction the exact value of sin 75°.

$\sin 75 = \sin(45+30) = \sin 45\cos 30 + \cos 45 \sin 30$

$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$

$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

37 Solve the inequality $-3|6 - x| < -15$ for x. Graph the solution on the line below.

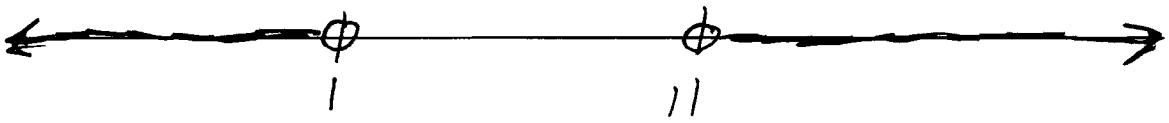
$\frac{-3|6-x|}{-3} < \frac{-15}{-3}$

$|6-x| > 5$

$6-x > 5$ $\quad$ $6-x < -5$

$1 > x$ $\quad$ $11 < x$

$x < 1$ or $x > 11$



38 The probability that a professional baseball player will get a hit is $\frac{1}{3}$. Calculate the exact probability that he will get *at least* 3 hits in 5 attempts.

$r = 3 \quad {}_5C_3 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^2 = \frac{40}{243}$

$r = 4 \quad {}_5C_4 \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^1 = \frac{16}{243}$

$r = 5 \quad {}_5C_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^0 = \frac{1}{243}$

$\frac{51}{243}$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

39 Solve the following system of equations algebraically:

$$5 = y - x$$
$$4x^2 = -17x + y + 4$$

$y = x + 5$

$y = 4x^2 + 17x - 4$

$4x^2 + 17x - 4 = x + 5$

$4x^2 + 16x - 9 = 0$

$(2x + 9)(2x - 1) = 0$

$2x + 9 = 0$ $\qquad$ $2x - 1 = 0$

$x = -\frac{9}{2}$ $\qquad$ $x = \frac{1}{2}$

$y = -\frac{9}{2} + 5$ $\qquad$ $y = \frac{1}{2} + 5$

$= \frac{1}{2}$ $\qquad$ $y = \frac{11}{2}$

$\left(-\frac{9}{2}, \frac{1}{2}\right)$ $\qquad$ $\left(\frac{1}{2}, \frac{11}{2}\right)$