The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA 2/TRIGONOMETRY

Thursday, January 29, 2015 — 9:15 a.m to 12:15 p.m., only

Student Name: _Mr. Sibol_

School Name: _JMAP_

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 27 questions in this part. Each correct answer will receive 2 credits. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [54]

1 In $\triangle FGH$, $f = 6$, $g = 9$, and m$\angle H = 57$. Which statement can be used to determine the numerical value of h?

(1) $h^2 = 6^2 + 9^2 - 2(9)(h) \cos 57°$

(2) $h^2 = 6^2 + 9^2 - 2(6)(9) \cos 57°$

(3) $6^2 = 9^2 + h^2 - 2(9)(h) \cos 57°$

(4) $9^2 = 6^2 + h^2 - 2(6)(h) \cos 57°$

2 The table of values below can be modeled by which equation?

x	y
−2	5
−1	4
0	3
1	4
2	5

(1) $f(x) = |x + 3|$

(2) $f(x) = |x| + 3$

(3) $f(y) = |y + 3|$

(4) $f(y) = |y| + 3$

3 The equation $\log_a x = y$ where $x > 0$ and $a > 1$ is equivalent to

(1) $x^y = a$

(2) $y^a = x$

(3) $a^y = x$

(4) $a^x = y$

4 Which expression is equivalent to the sum of the sequence 6, 12, 20, 30?

(1) $\displaystyle\sum_{n=4}^{7} 2^n - 10$

(3) $\displaystyle\sum_{n=2}^{5} 5n - 4$

(2) $\displaystyle\sum_{n=3}^{6} \frac{2n^2}{3}$

(4) $\displaystyle\sum_{n=2}^{5} n^2 + n$

5 An investment is earning 5% interest compounded quarterly. The equation $A = P\left(1 + \dfrac{r}{n}\right)^{nt}$ represents the total amount of money, A, where P is the original investment, r is the interest rate, t is the number of years, and n represents the number of times per year the money earns interest.

Which graph could represent this investment over at least 50 years?

(1)

(3)

(2)

(4)
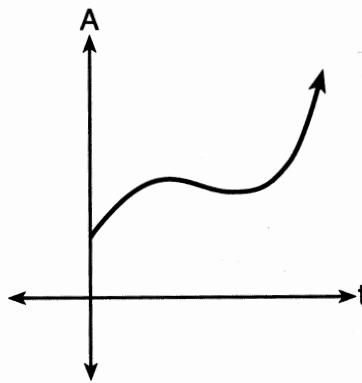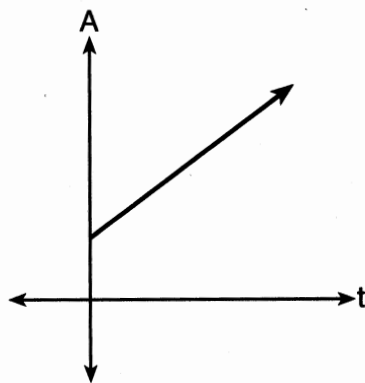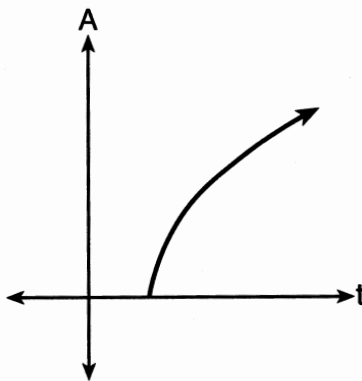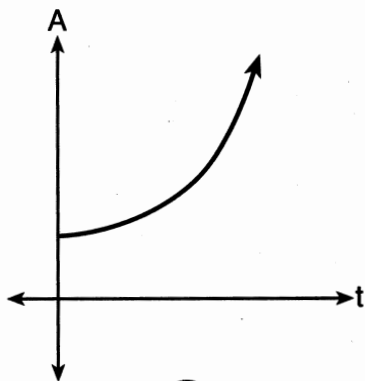
6 Which equation has real, rational, and unequal roots?

(1) $x^2 + 10x + 25 = 0$

(2) $x^2 - 5x + 4 = 0$ $\quad b^2 - 4ac = (-5)^2 - 4(1)(4)$

(3) $x^2 - 3x + 1 = 0$ $\qquad 25 - 16$

(4) $x^2 - 2x + 5 = 0$ $\qquad\quad 9$

7 Which statement is true about the graphs of f and g shown below?
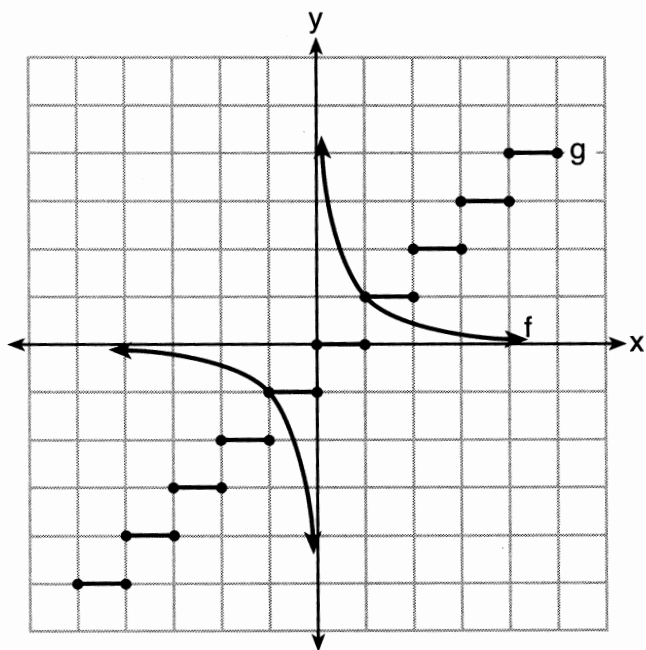
(1) f is a relation and g is a function.

(2) f is a function and g is a relation.

(3) Both f and g are functions.

(4) Neither f nor g is a function.

8 The common ratio of the sequence $-\dfrac{1}{2}, \dfrac{3}{4}, -\dfrac{9}{8}$ is

$\dfrac{\frac{3}{4}}{-\frac{1}{2}} = -\dfrac{3}{2}$

(1) $-\dfrac{3}{2}$

(2) $-\dfrac{2}{3}$

(3) $-\dfrac{1}{2}$

(4) $-\dfrac{1}{4}$

9 How many different ways can teams of four members be formed from a class of 20 students?

(1) 5   (③) 4,845

(2) 80   (4) 116,280

$$_{20}C_4$$

10 If $\sin A = \frac{3}{8}$, what is the value of $\cos 2A$?

(1) $-\frac{9}{64}$ (③) $\frac{23}{32}$

(2) $\frac{1}{4}$ (4) $\frac{55}{64}$

$= 1 - 2\sin^2 A$

$1 - 2\left(\frac{3}{8}\right)^2$

$1 - \frac{9}{32}$

$\frac{23}{32}$

11 When factored completely, the expression $x^3 - 2x^2 - 9x + 18$ is equivalent to

(1) $(x^2 - 9)(x - 2)$ (3) $(x - 2)^2(x - 3)(x + 3)$

((2)) $(x - 2)(x - 3)(x + 3)$ (4) $(x - 3)^2(x - 2)$

$x^2(x-2) - 9(x-2)$

$(x^2-9)(x-2)$

$(x+3)(x-3)(x-2)$

12 When $-3 - 2i$ is multiplied by its conjugate, the result is

(1) -13 (3) 5

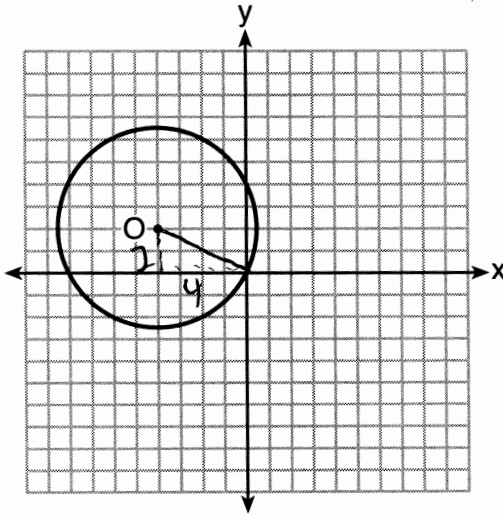(2) -5 ((4)) 13

$(-3-2i)(-3+2i)$

$9 - 4i^2$

$9 + 4$

13

13 A circle with center O and passing through the origin is graphed below.

Handwritten: $2^2 + 4^2 = r^2$
$20 = r^2$

What is the equation of circle O?

(1) $x^2 + y^2 = 2\sqrt{5}$

(3) $(x + 4)^2 + (y - 2)^2 = 2\sqrt{5}$

(2) $x^2 + y^2 = 20$

(4) $(x + 4)^2 + (y - 2)^2 = 20$ ⟵ *circled*

14 Which expression is equivalent to $(5^{-2}a^3b^{-4})^{-1}$?

Handwritten: $5^2 \cdot a^{-3} b^4$

$\dfrac{25b^2}{a^3}$

(1) $\dfrac{10b^4}{a^3}$

(3) $\dfrac{a^3}{25b^4}$

(2) $\dfrac{25b^4}{a^3}$ ⟵ *circled*

(4) $\dfrac{a^2}{125b^5}$

15 Which trigonometric expression does *not* simplify to 1?

(1) $\sin^2 x(1 + \cot^2 x)$

(3) $\cos^2 x(\tan^2 x - 1)$ ⟵ *circled*

(2) $\sec^2 x(1 - \sin^2 x)$

(4) $\cot^2 x(\sec^2 x - 1)$

Handwritten: $\cos^2 x\left(\dfrac{\sin^2 x}{\cos^2 x}\right) - \cos^2 x$
$\sin^2 x - \cos^2 x \neq 1$

16 What is the product of $\sqrt[3]{4a^2b^4}$ and $\sqrt[3]{16a^3b^2}$?

(1) $4ab^2\sqrt[3]{a^2}$ (3) $8ab^2\sqrt[3]{a^2}$

(2) $4a^2b^3\sqrt[3]{a}$ (4) $8a^2b^3\sqrt[3]{a}$

$\sqrt[3]{64a^5b^6}$

$4ab^2\sqrt[3]{a^2}$

17 What is the product of the roots of $4x^2 - 5x = 3$?

(1) $\dfrac{3}{4}$ (3) $-\dfrac{3}{4}$

(2) $\dfrac{5}{4}$ (4) $-\dfrac{5}{4}$

$4x^2 - 5x - 3 = 0$

$\dfrac{c}{a} = \dfrac{-3}{4}$

18 How many different 11-letter arrangements are possible using the letters in the word "ARRANGEMENT"?

(1) 2,494,800   (3) 19,958,400

(2) 4,989,600   (4) 39,916,800

$\dfrac{_{11}P_{11}}{2!\,2!\,2!\,2!} = 2,494,800$

19 What is the third term in the expansion of $(2x - 3)^5$?

(1) $720x^3$ (3) $-540x^2$

(2) $180x^3$ (4) $-1080x^2$

$_5C_2\,(2x)^3(-3)^2$

$10 \cdot 8x^3 \cdot 9$

$720x^3$

20 Angle θ is in standard position and $(-4,0)$ is a point on the terminal side of θ. What is the value of $\sec\theta$?

(1) -4 (3) 0

(2) -1 (4) undefined

$\sqrt{(-4)^2 + 0^2} = 4$

$\cos\theta = \dfrac{-4}{4} = -1$

$\sec\theta = -1$

21 The domain of $f(x) = -\dfrac{3}{\sqrt{2-x}}$ is the set of all real numbers

(1) greater than 2

(2) less than 2

(3) except 2

(4) between -2 and 2

22 Which equation could be used to solve $\left(\dfrac{5}{x-3} - \dfrac{2}{x} = 1\right)$

(1) $x^2 - 6x - 3 = 0$

(2) $x^2 - 6x + 3 = 0$

(3) $x^2 - 6x - 6 = 0$

(4) $x^2 - 6x + 6 = 0$

$x(x-3)$

$5x - 2(x-3) = x(x-3)$

$5x - 2x + 6 = x^2 - 3x$

$0 = x^2 - 6x - 6$

23 How many distinct triangles can be constructed if $m\angle A = 30$, side $a = \sqrt{34}$, and side $b = 12$?

(1) one acute triangle

(2) one obtuse triangle

(3) two triangles

(4) none

$\dfrac{\sqrt{34}}{\sin 30} = \dfrac{12}{\sin B}$

$\sin B = \dfrac{12\sin 30}{\sqrt{34}}$

$\sin B = \dfrac{6}{\sqrt{34}} > 1$

24 The expression $\left(\dfrac{3}{2}x + 1\right)\left(\dfrac{3}{2}x - 1\right) - \left(\dfrac{3}{2}x - 1\right)^2$ is equivalent to

(1) 0

(2) $-3x$

(3) $\dfrac{3}{4}x - 2$

(4) $3x - 2$

$\left(\dfrac{3}{2}x - 1\right)\left(\dfrac{3}{2}x + 1 - \left(\dfrac{3}{2}x - 1\right)\right)$

$\left(\dfrac{3}{2}x - 1\right) 2$

$3x - 2$

25 The table below shows five numbers and their frequency of occurrence.

Number	Frequency
5	9
7	5
8	8
12	8
14	8

The interquartile range for these data is

(1) 7

(2) 5

(3) 7 to 12

(4) 6 to 13

26 A wheel has a radius of 18 inches. Which distance, to the *nearest inch*, does the wheel travel when it rotates through an angle of $\frac{2\pi}{5}$ radians?

(1) 45

(2) 23

(3) 13

(4) 11

$s = \theta r$

$= \frac{2\pi}{5} 18$

≈ 22.6

27 If $f(x) = 4x^2 - x + 1$, then $f(a + 1)$ equals

(1) $4a^2 - a + 6$

(2) $4a^2 - a + 4$

(3) $4a^2 + 7a + 6$

(4) $4a^2 + 7a + 4$

$4(a+1)^2 - (a+1) + 1$

$4(a^2 + 2a + 1) - a$

$4a^2 + 8a + 4 - a$

$4a^2 + 7a + 4$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

28 If p and q vary inversely and p is 25 when q is 6, determine q when p is equal to 30.

$$\frac{25 \cdot 6}{30} = \frac{30q}{30}$$

$$5 = q$$

29 Express in simplest form:

$$\frac{\dfrac{36 - x^2}{(x + 6)^2}}{\dfrac{x - 3}{x^2 + 3x - 18}}$$

$$\frac{(6-x)(6+x)}{(x+6)^2} \cdot \frac{(x+6)(x-3)}{x-3}$$

$$6-x$$

30 Solve $e^{4x} = 12$ algebraically for x, rounded to the *nearest hundredth*.

$$\ln e^{4x} = \ln 12$$

$$\frac{4x}{4} = \frac{\ln 12}{4}$$

$$x \approx .62$$

31 Determine, to the *nearest minute*, the degree measure of an angle of $\frac{5}{11}\pi$ radians.

$$\frac{5\pi}{11} \cdot \frac{180}{\pi} \approx 80°\ 49'$$

32 The probability of Ashley being the catcher in a softball game is $\frac{2}{5}$. Calculate the exact probability that she will be the catcher in *exactly* five of the next six games.

$$_6C_5\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)$$

$$6 \cdot \frac{32}{3125} \cdot \frac{3}{5}$$

$$\frac{576}{15,625}$$

33 If x is a real number, express $2xi(i - 4i^2)$ in simplest $a + bi$ form.

$$2xi^2 - 8xi^3$$
$$-2x + 8xi$$

34 On a test that has a normal distribution of scores, a score of 57 falls one standard deviation below the mean, and a score of 81 is two standard deviations above the mean. Determine the mean score of this test.

$$\frac{81-57}{3} = \frac{24}{3} = 8$$

$$57 + 8 = 65$$

35 The area of a parallelogram is 594, and the lengths of its sides are 32 and 46. Determine, to the *nearest tenth of a degree*, the measure of the acute angle of the parallelogram.

$$594 = 32 \cdot 46 \sin C$$

$$\frac{594}{1472} = \sin C$$

$$23.8° \approx C$$

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

36 The table below shows the amount of a decaying radioactive substance that remained for selected years after 1990.

Years After 1990 (x)	0	2	5	9	14	17	19
Amount (y)	750	451	219	84	25	12	8

Write an exponential regression equation for this set of data, rounding all values to the *nearest thousandth*.

$$y = 733.646(0.786)^x$$

Using this equation, determine the amount of the substance that remained in 2002, to the *nearest integer*.

$$733.646(0.786)^{12} \approx 41$$

37 Use the recursive sequence defined below to express the next three terms as fractions reduced to lowest terms.
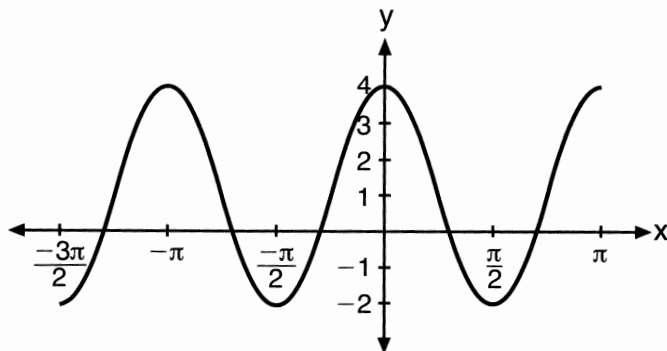
$$a_1 = 2$$

$$a_n = 3\left(a_{n-1}\right)^{-2}$$

$$a_2 = 3(2)^{-2} = \frac{3}{4}$$

$$a_3 = 3\left(\frac{3}{4}\right)^{-2} = \frac{16}{3}$$

$$a_4 = 3\left(\frac{16}{3}\right)^{-2} = \frac{27}{256}$$

38 The periodic graph below can be represented by the trigonometric equation $y = a \cos bx + c$ where $a, b,$ and c are real numbers.

State the values of $a, b,$ and c, and write an equation for the graph.

3 2 1

$$y = 3\cos 2x + 1$$

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

39 A homeowner wants to increase the size of a rectangular deck that now measures 14 feet by 22 feet. The building code allows for a deck to have a maximum area of 800 square feet. If the length and width are increased by the same number of feet, find the maximum number of whole feet each dimension can be increased and *not* exceed the building code.

[Only an algebraic solution can receive full credit.]

$$(x+14)(x+22) \le 800$$

$$x^2 + 36x + 308 \le 800$$

$$x^2 + 36x - 492 \le 0$$

$$\frac{-36 \pm \sqrt{(36)^2 - 4(1)(-492)}}{2(1)}$$

$$\frac{-36 \pm \sqrt{3264}}{2}$$

$$\frac{-36 + \sqrt{3264}}{2} \approx 10.6$$

10