The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# ALGEBRA 2/TRIGONOMETRY

**Tuesday,** January 28, 2014 — 1:15 to 4:15 p.m., only

**Student Name:** Mr. Sibol

**School Name:** JMAP

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

**Print your name and the name of your school on the lines above.**

**A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.**

**This examination has four parts, with a total of 39 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.**

**The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.**

**Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.**

**When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.**

**Notice...**

**A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.**

**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

ALGEBRA 2/TRIGONOMETRY

## Part I

**Answer all 27 questions in this part. Each correct answer will receive 2 credits. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet.** [54]

**Use this space for computations.**

**1** What is the common difference in the sequence $2a + 1, 4a + 4, 6a + 7, 8a + 10, \ldots$?

(1) $2a + 3$
(2) $-2a - 3$
(3) $2a + 5$
(4) $-2a + 5$

$(4a+4)-(2a+1)$

**2** Which expression is equivalent to $\left(3x^2\right)^{-1}$?

(1) $\frac{1}{3x^2}$
(2) $-3x^2$
(3) $\frac{1}{9x^2}$
(4) $-9x^2$

$h(-8) = \frac{1}{2}(-8) - 2 = -4 - 2 = -6$

**3** If $g(x) = \frac{1}{2}x + 8$ and $h(x) = \frac{1}{2}x - 2$, what is the value of $g(h(-8))$?

(1) 0
(2) 9
(3) 5
(4) 4

$g(-6) = \frac{1}{2}(-6) + 8$

$-3 + 8$

$5$

**4** The expression $\frac{1}{7 - \sqrt{11}}$ is equivalent to

(1) $\frac{7 + \sqrt{11}}{38}$
(2) $\frac{7 - \sqrt{11}}{38}$
(3) $\frac{7 + \sqrt{11}}{60}$
(4) $\frac{7 - \sqrt{11}}{60}$

$\frac{7+\sqrt{11}}{7+\sqrt{11}} = \frac{7+\sqrt{11}}{49-11} = \frac{7+\sqrt{11}}{38}$

**Use this space for computations.**

**5** The expression $\dfrac{a+\dfrac{b}{c}}{d-\dfrac{b}{c}}$ is equivalent to

(1) $\dfrac{c+1}{d-1}$

(3) $\dfrac{ac+b}{cd-b}$

(2) $\dfrac{a+b}{d-b}$

(4) $\dfrac{ac+1}{cd-1}$

**6** A school cafeteria has five different lunch periods. The cafeteria staff wants to find out which items on the menu are most popular, so they give every student in the first lunch period a list of questions to answer in order to collect data to represent the school. Which type of study does this represent?

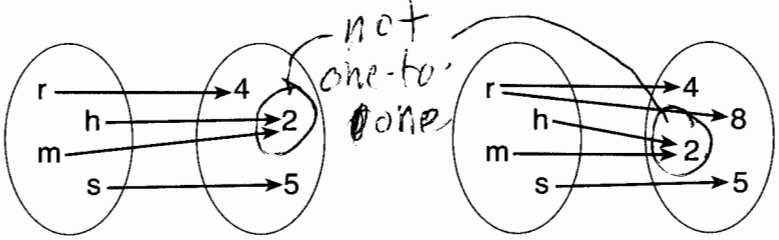
(1) observation

(2) controlled experiment

(3) population survey

(4) sample survey

**7** Which relation is both one-to-one and onto?

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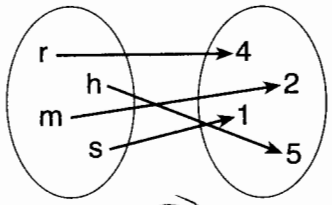


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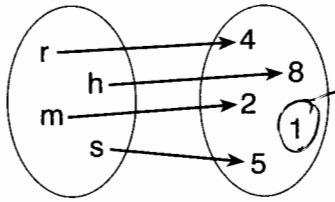


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**Use this space for computations.**

**8** Max solves a quadratic equation by completing the square. He shows a correct step:

$$\sqrt{(x+2)^2} = \sqrt{-9}$$

What are the solutions to his equation?

(1) $2 \pm 3i$
(3) $3 \pm 2i$
(2) $-2 \pm 3i$
(4) $-3 \pm 2i$

$x + 2 = \pm 3i$

$x = -2 \pm 3i$

**9** Which expression represents the total number of different 11-letter arrangements that can be made using the letters in the word "MATHEMATICS"?

(1) $\frac{11!}{3!}$
(3) $\frac{11!}{8!}$
(2) $\frac{11!}{2! + 2! + 2!}$
(4) $\frac{11!}{2! \cdot 2! \cdot 2!}$

**10** If \$5000 is invested at a rate of 3% interest compounded quarterly, what is the value of the investment in 5 years? (Use the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where $A$ is the amount accrued, $P$ is the principal, $r$ is the interest rate, $n$ is the number of times per year the money is compounded, and $t$ is the length of time, in years.)

(1) \$5190.33
(3) \$5805.92
(2) \$5796.37
(4) \$5808.08

$5000\left(1 + \frac{.03}{4}\right)^{4 \cdot 5}$

$5000(1.0075)^{20}$

$2x^2 - 9x + 4 = 0$

**11** The roots of the equation $2x^2 + 4 = 9x$ are

(1) real, rational, and equal
(2) real, rational, and unequal
(3) real, irrational, and unequal
(4) imaginary

$b^2 - 4ac$

$(-9)^2 - 4(2)(4)$

$81 - 32$

$49$

**Use this space for computations.**

**12** If $d$ varies inversely as $t$, and $d = 20$ when $t = 2$, what is the value of $t$ when $d = -5$?

(1) 8
(3) $-8$
(2) 2
(4) $-2$

$\frac{20 \cdot 2}{-5} = \frac{-5t}{-5}$

$-8 = t$

**13** If $\sin A = -\frac{7}{25}$ and $\angle A$ terminates in Quadrant IV, $\tan A$ equals

(1) $-\frac{7}{25}$
(3) $-\frac{24}{7}$
(2) $-\frac{7}{24}$
(4) $-\frac{24}{25}$

$\sin^2 A + \cos^2 A = 1$

$\left(-\frac{7}{25}\right)^2 + \cos^2 A = 1$

$\frac{49}{625} + \cos^2 A = \frac{625}{625}$

$\sqrt{\cos^2 A} = \sqrt{\frac{576}{625}}$ Quad IV

$\cos A = +\frac{24}{25}$

$\tan A = \frac{\sin A}{\cos A} = \frac{-7/25}{24/25}$

**14** Which expression is equivalent to $\sum_{n=1}^{4} (a - n)^2$?

(1) $2a^2 + 17$
(3) $2a^2 - 10a + 17$
(2) $4a^2 + 30$
(4) $4a^2 - 20a + 30$

$n=1 \quad (a-1)^2 = a^2 - 2a + 1$

$n=2 \quad (a-2)^2 = a^2 - 4a + 4$

$n=3 \quad (a-3)^2 = a^2 - 6a + 9$

$n=4 \quad (a-4)^2 = a^2 - 8a + 16$

**15** What are the coordinates of the center of a circle whose equation is $x^2 + y^2 - 16x + 6y + 53 = 0$?

(1) $(-8,-3)$
(3) $(8,-3)$
(2) $(-8,3)$
(4) $(8,3)$

$x^2 - 16x + 64 + y^2 + 6y + 9 = -53 + 64 + 9$

$(x-8)^2 + (y+3)^2 = 20$

**Use this space for computations.**

**16** For $y = \frac{3}{\sqrt{x-4}}$, what are the domain and range?

(1) $\{x|x > 4\}$ and $\{y|y > 0\}$
(2) $\{x|x \geq 4\}$ and $\{y|y > 0\}$
(3) $\{x|x > 4\}$ and $\{y|y \geq 0\}$
(4) $\{x|x \geq 4\}$ and $\{y|y \geq 0\}$

**17** A math club has 30 boys and 20 girls. Which expression represents the total number of different 5-member teams, consisting of 3 boys and 2 girls, that can be formed?

(1) ${}_{30}P_3 \cdot {}_{20}P_2$
(2) ${}_{30}C_3 \cdot {}_{20}C_2$
(3) ${}_{30}P_3 + {}_{20}P_2$
(4) ${}_{30}C_3 + {}_{20}C_2$

**18** What is the product of the roots of $x^2 - 4x + k = 0$ if one of the roots is 7?

(1) 21
(2) $-11$
(3) $-21$
(4) $-77$

sum: $\frac{-b}{a}$, $\frac{-(-4)}{1}$, 4

If sum is 4, roots must be 7 and $-3$

**19** In $\triangle DEF$, $d = 5$, $e = 8$, and $m\angle D = 32$. How many distinct triangles can be drawn given these measurements?

(1) 1
(2) 2
(3) 3
(4) 0

$\frac{5}{\sin 32}$, $\frac{8}{\sin E}$

$E \approx 57.98°$

$57.98 + 32 < 180$ $(180 - 57.98) + 32 < 180$

**20** Liz has applied to a college that requires students to score in the top 6.7% on the mathematics portion of an aptitude test. The scores on the test are approximately normally distributed with a mean score of 576 and a standard deviation of 104. What is the minimum score Liz must earn to meet this requirement?

(1) 680
(2) 732
(3) 740
(4) 784

$> 6.7\%$ is $1.5$ s.d. + $\bar{x}$

$1.5(104) + 576$

732

**Use this space for computations.**

**21** The expression $\left(\sqrt[3]{27x^2}\right)\left(\sqrt[3]{16x^4}\right)$ is equivalent to

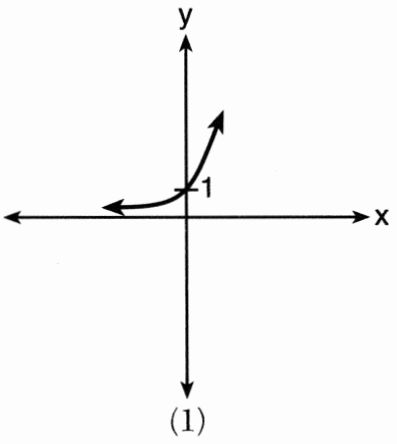
(1) $12x^2\sqrt[3]{2}$

(2) $12x\sqrt[3]{2x}$

(3) $6x\sqrt[3]{2x^2}$

(4) $6x^2\sqrt[3]{2}$

**22** Which sketch shows the inverse of $y = a^x$, where $a > 1$?

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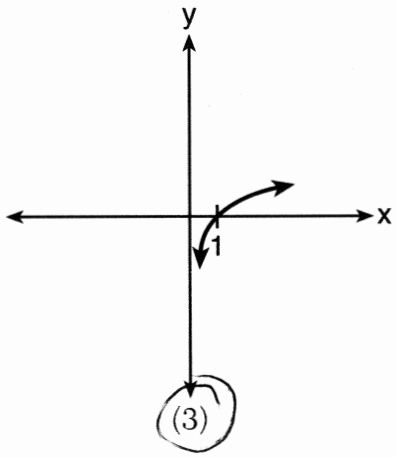


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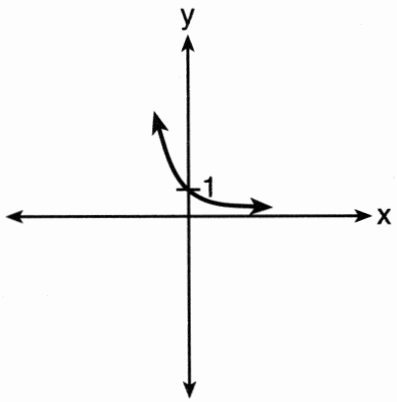


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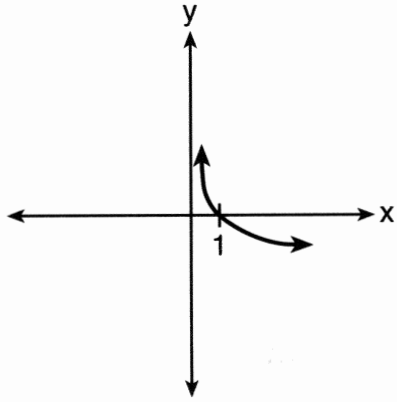


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**Use this space for computations.**

**23** The expression $\frac{x^2 + 9x - 22}{x^2 - 121} \div (2 - x)$ is equivalent to

(1) $x - 11$

(2) $\frac{1}{x - 11}$

(3) $11 - x$

(4) $\frac{1}{11 - x}$

$\frac{(x+11)(x-2)}{(x+11)(x-11)} \cdot \frac{-1}{2-x}$

$\frac{-1}{x-11}$

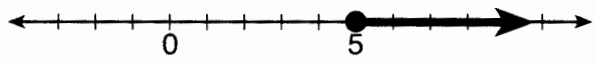
**24** Which graph represents the solution set of $\frac{x + 16}{x - 2} \leq 7$?

$\frac{x+16}{x-2} - \frac{7(x-2)}{x-2} \leq 0$

$\frac{x+16-7x+14}{x-2} \leq 0$

$\frac{-6x+30}{x-2} \leq 0$

(1)

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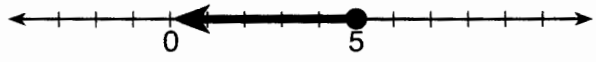


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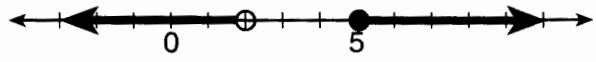


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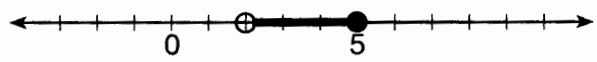



$-6x + 30 = 0$

$-6x = -30$

$x = 5$

$x = 2$ undefined

Check

1 $\frac{-6(1)+30}{1-2} = \frac{24}{-1} \leq 0$ yes

3 $\frac{-6(3)+30}{3-2} = \frac{12}{1} \leq 0$ no

6 $\frac{-6(6)+30}{6-2} = \frac{-6}{4} \leq 0$ yes

**Use this space for computations.**

**25** Which equation represents a graph that has a period of $4\pi$?

(1) $y = 3 \sin \frac{1}{2}x$ $\frac{2\pi}{\frac{1}{2}} = 4\pi$ (3) $y = 3 \sin \frac{1}{4}x$

(2) $y = 3 \sin 2x$ (4) $y = 3 \sin 4x$

**26** The expression $x^2(x + 2) - (x + 2)$ is equivalent to $(x^2-1)(x+2)$

(1) $x^2$ (3) $x^3 + 2x^2 - x + 2$

(2) $x^2 - 1$ (4) $(x + 1)(x - 1)(x + 2)$ $(x+1)(x-1)(x+2)$

**27** Approximately how many degrees does five radians equal?

(1) 286 (3) $\frac{\pi}{36}$ $5 \text{ rad} \cdot \frac{180}{\pi} \approx 286$

(2) 900 (4) $5\pi$

## Part II

**Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [16]

**28** Show that $\sec\theta \sin\theta \cot\theta = 1$ is an identity.

$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} \cdot \frac{\cos\theta}{\sin\theta} = 1$

$1 = 1$

**29** Find, to the *nearest tenth of a square foot*, the area of a rhombus that has a side of 6 feet and an angle of 50°.

$6 \cdot 6 \sin 50 \approx 27.6$

**30** The following is a list of the individual points scored by all twelve members of the Webster High School basketball team at a recent game:

2 2 3 4 6 7 | 9 10 10 11 12 14

Find the interquartile range for this set of data.

$Q_1$ $Q_3$

3.5 10.5

$Q_3 - Q_1$

$10.5 - 3.5$

7

**31** Determine algebraically the $x$-coordinate of all points where the graphs of $xy = 10$ and $y = x + 3$ intersect.

$x(x+3) = 10$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5, 2$

**32** Solve $|-4x + 5| < 13$ algebraically for $x$.

$-4x+5<13$ $\quad$ $-4x+5>-13$

$-4x<8$ $\quad$ $-4x>-18$

$x>-2$ $\quad$ $x<4.5$

$-2<x<4.5$

**33** Express $4xi + 5yi^8 + 6xi^3 + 2yi^4$ in simplest $a + bi$ form.

$4xi+5y-6xi+2y$

$7y-2xi$

**34** In an arithmetic sequence, $a_4 = 19$ and $a_7 = 31$. Determine a formula for $a_n$, the $n^{th}$ term of this sequence.

$\frac{31-19}{7-4} = \frac{12}{3} = 4$

$x + (n-1)4$

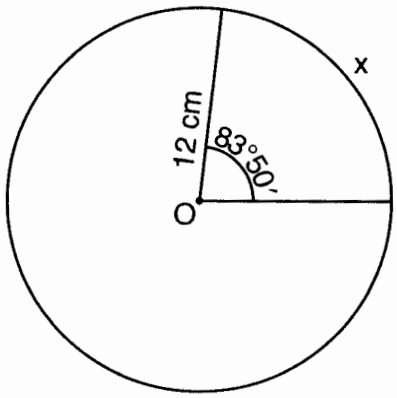
$x + (4-1)4 = 19$

$x + 12 = 19$

$x = 7$

$a_n = 7 + (n-1)4$

**35** Circle $O$ shown below has a radius of 12 centimeters. To the *nearest tenth of a centimeter*, determine the length of the arc, $x$, subtended by an angle of 83°50′.




$83°50 \cdot \frac{\pi}{180} \approx 1.463 \text{ rad}$

$s = \theta r$

$= 1.463 \cdot 12$

$\approx 17.6$

## Part III

**Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [12]

**36** Solve algebraically for all exact values of $x$ in the interval $0 \leq x < 2\pi$:

$$2 \sin^2 x + 5 \sin x = 3$$

$(2\sin x - 1)(\sin x + 3) = 0$

$2\sin x - 1 = 0$ $\quad$ $\sin x + 3 = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -3$ ~~$-3$~~

$x = 30^\circ, 150^\circ$

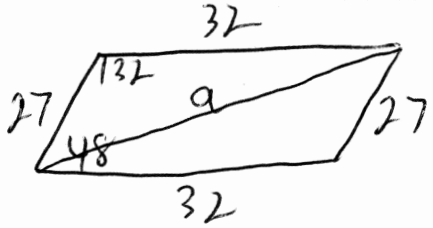
$= \frac{\pi}{6}, \frac{5\pi}{6}$

**37** Because Sam's backyard gets very little sunlight, the probability that a geranium planted there will flower is 0.28. Sam planted five geraniums. Determine the probability, to the *nearest thousandth*, that *at least* four geraniums will flower.

$${}_5C_4(.28)^4(.72)^1 + {}_5C_5(.28)^5(.72)^0 \approx .024$$

**38** Two sides of a parallelogram measure 27 cm and 32 cm. The included angle measures 48°. Find the length of the longer diagonal of the parallelogram, to the *nearest centimeter*.

$a = \sqrt{27^2 + 32^2 - 2(27)(32)(\cos 132)} \approx 54$



## Part IV

**Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen.** [6]

**39** Solve algebraically for all values of $x$:

$$\log_{(x+3)}(2x + 3) + \log_{(x+3)}(x + 5) = 2$$

$\log_{(x+3)}(2x+3)(x+5) = 2$

$x^2 + 6x + 9 = 2x^2 + 13x + 15$

$0 = x^2 + 7x + 6$

$0 = (x+6)(x+1)$

$x = -6$ $-1$