The University of the State of New York

EXAMINATION FOR QUALIFYING CERTIFICATES

ADVANCED ALGEBRA

Monday, September 14, 1925 — 1.15 to 4.15 p. m., only

Answer eight questions. Each answer should be reduced to its simplest form. Papers entitled to less than 75 credits will not be accepted.

1 Determine n so that one root of the equation $3x^3 + 14x^2 - 23x + n = 0$ shall be three times one of the other roots. [12½]

2 *a* Express in the form $a + bi$ the fraction

$$\frac{\sqrt{3} + \sqrt{-2}}{\sqrt{3} - i\sqrt{2}}$$ [7½]

b Represent *each* of the following complex numbers graphically and find their sum graphically:

$4 - 3i$, $5i - 2$ [1, 1, 3]

3 Transform the equation $6x^3 - 8x^2 + 4x - 9 = 0$ so that the coefficient of the highest degree term shall be unity and all other coefficients integers. [12½]

4 The sum of the first two terms of an infinite geometric progression is 1 and every term is twice the sum of all the terms that follow; find the progression. [12½]

5 *a* How many different numbers between 4000 and 7000 can be made from the digits 4, 5, 0, 6, 9? [6]

b In how many different ways can a guard of 4 be selected from a detachment of 15 soldiers, if 2 particular men are always excluded? [6½]

6 What sum of money, invested now at 4½%, interest compounded annually, will amount to $6000 in 20 years? [12½]

7 Find to the *nearest hundredth* the real root of the equation $x^3 + 2x - 4 = 0$ [12½]

8 *a* Expand to *three* terms by the binomial theorem the expression $\sqrt[3]{(1-x)^2}$. [7]

b By using the formula for the general term find the 6th term of this expansion. [5½]

9 Determine graphically the roots of the equation $3x^3 - 16x^2 + 9x + 14 = 0$, plotting the curve from $x = -1$ to $x = +5$. [12½]

10 Show that the equation $x^7 + 3x^5 - 3x^2 - 4 = 0$ has 6 complex roots. Is the 7th root positive or negative? [12½]

11 Coffee is bought at 36 cents a pound and chicory at 9 cents a pound; in what proportion must they be mixed so that 10% may be gained by selling the mixture at 33 cents a pound? [8, 4½]

12 A train A starts to go from P to Q, two stations 240 miles apart, and travels at a uniform rate. An hour later another train B starts from P and, after traveling for 2 hours, comes to the point that A has passed 45 minutes previously. The rate of B is now increased by 5 miles an hour and it overtakes A on entering Q. Find the rates at which the trains started. [9, 3½]