EXAMINATION FOR QUALIFYING CERTIFICATES

ADVANCED ALGEBRA

Monday, September 10, 1917—9.15 a. m. to 12.15 p. m., only

Answer eight questions. Each answer should be reduced to its simplest form. Papers entitled to less than 75 credits will not be accepted.

1 *a* How many signals may be made by hanging 15 flags on a staff, if 2 flags are white, 3 black, 5 blue and 5 red?

    b Nine boy scouts were walking and as they approached a fork in the road decided that five should turn to the right and four should turn to the left; in how many ways could this be done?

2 Transform $x^4 + 3x^2 - 5x + 2 = 0$ into an equation whose roots shall be respectively those of the first equation decreased by 6.

3 By using Descartes' rule of signs determine the nature of the roots of the equation $3x^5 - 2x^4 - 45x^3 + 92x - 48 = 0$

4 Find the successive derivatives of $5x^5 - 4x^4 + 3x^3 - 2$

5 Find by Horner's method a real root of the equation $x^3 + 4x^2 + x + 1 = 0$ correct to *two* decimal places.

6 Represent graphically the complex numbers
$$-3 + 2\sqrt{-1} \text{ and } 1 - 4\sqrt{-1}$$
and also their sum and their difference.

7 A starts to walk to a town 21 miles away and one hour afterward B follows him. When B has overtaken A he turns back and reaches the starting point at the same time as A reaches his destination. B walked at the rate of 4 miles an hour. Find A's rate and the distance from the starting point to the place where B overtook him.

8 For what value of k is the sum of the roots of the equation $x^2 + (4 - 6k)x + k^2 = 0$ equal to twice their product?

9 Write *four* equations showing the relation between the roots and the coefficients in the following equation:
$x^4 - 15x^2 + 10x + 24 = 0$

10 Plot the graph of the equation
$$x^4 - 3x^3 + 6x^2 + x - 1 = 0$$
and determine from the graph the value of the real roots to the nearest tenth.

7 Solve the equation $x^3 - 1 = 0$. Show graphically that the sum of the roots is 0.

8 A sum of money, P dollars, is put out at simple interest, at the annual rate r.

    a Find a formula for the amount A at the end of n years.

    b From this formula find P.

9 Given two geometric progressions. Prove that if their corresponding terms are multiplied together, the products will also form a geometric progression.

10 A certain point of a pendulum describes a path 8 inches long during the first swing; if each consecutive swing is $\frac{1}{8}$ less than the preceding one, what is the total length of the path described by the point before the pendulum comes to rest?

11 Examine the following "proof" and point out the mistake in the argument:

Given $x = a$. Then $x^2 = ax$. Hence $x^2 - a^2 = ax - a^2$, and $(x + a)(x - a) = a(x - a)$. Therefore $x + a = a$. But $x = a$. Hence $2a = a$, and $2 = 1$.

12 The capacity of a tank $3 \times 4 \times 5$ feet is to be doubled by increasing each dimension by the same amount; find the new dimensions algebraically.