June 22, 1967

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Write your answers in the spaces provided.

1. When $x^{25} - 3$ is divided by $x + 2$, there is an integral remainder. If this remainder is a positive integer, write "positive"; if it is a negative integer, write "negative."   1_____

2. Express the product of $3 + 2i$ and $2i$ in the form $a + bi$.   2_____

3. Solve for a in terms of b and c: $\dfrac{1}{a} = \dfrac{1}{b} + \dfrac{1}{c}$ 3_____

4. How many different groups of three students each may a teacher choose from among 8 students in her class?   4_____

5. Write the expression $[4(\cos 30° + i \sin 30°)]^2$ in the form $r(\cos \theta + i \sin \theta)$.   5_____

6. Express the repeating decimal $0.1212\ldots$, in which the digits 1 and 2 are repeated endlessly, as a rational nuumber.   6_____

7. Find the value(s) of x which will satisfy the equation $\sqrt{2x} + 4 = x$.   7_____

8. Find the positive value of b such that the graph of $x^2 + bx + 4 = y$ touches the x-axis in one and only one point.   8_____

9. Find the fourth term of the geometric progression whose first two terms are $\dfrac{1}{\sqrt{a}}$ and 1, respectively.   9_____

10. Find all the values of x which satisfy the inequality
$$\frac{7}{6} + \frac{x}{2} < x - \frac{1}{3}.$$ 10_____

11. How many cars would a dealer have to have on hand in order to show a customer all of the models available from a manufacturer who builds cars in 6 body styles, with a choice of 4 engines, and equipped with either a standard or an automatic transmission?   11_____

12. The graph of $y = f(x)$ crosses the x-axis just once between $x = -0.9$ and $x = -0.8$. If $f(-0.9) > 0$, $f(-0.85) > 0$, and $f(-0.8) < 0$, find to the *nearest tenth* a negative root of $f(x) = 0$.   12_____

13. Write an equation for the axis of symmetry of the graph of $y = 10x - 2x^2$.   13_____

14. Find the rational fractional root of $6x^3 + x^2 + 5x - 2 = 0$.   14_____

15. What is the *least* possible degree of a rational integral equation with rational coefficients, if 3 of its roots are -3, $2 + 3i$, and $4 - \sqrt{7}$?

15._____

16. What is the sum of the roots of the equation $3x^3 - 12x^2 + x = 0$?

16._____

17. If $y = x^4 - 2x^2 - 4$, find the average rate of change of y with respect to x as x varies from $x = -1$ to $x = 0$.

17._____

18. The tens digit of a two-digit number is 4 less than the units digit. If the units digit is represented by u, express the number in terms of u.

18._____

Directions (19-24) : Indicate the correct completion for *each* of the following by writing the number 1, 2, 3, or 4 in the space provided.

19. Which defines a rational integral function of x?
(1) $x^{1/2} - 1$ (2) $x^2 - 1$ (3) $x^{-1/2} - 1$ (4) $x^{-2} - 1$

19._____

20. The graphs of $y = ax + b$ and $y = cx + d$ are distinct parallel lines if   (1) $a \neq c$ and $b \neq d$ (2) $a \neq c$ and $b = d$
(3) $a = c$ and $b \neq d$ (4) $a = c$ and $b = d$

20._____

21. Given $\log_9 3^x = 8$. The value of x that satisfies this equation is   (1) 16   (2) 2   (3) 3   (4) 4

21._____

22. When drawn on the same set of axes, the points of intersection of the graphs of $x^2 + 4y^2 = 16$ and $xy = 1$ are located in quadrants   (1) I and III   (2) I and IV   (3) II and III
(4) II and IV

22._____

23. Given that s varies inversely as the square of t. If a value of t is halved, then the corresponding value of s is multiplied by
(1) ½   (2) 2   (3) ¼   (4) 4

23._____

24. How many real roots does $x^4 + 6x^2 + 3 = 0$ have?
(1) 1   (2) 2   (3) 0   (4) 4

24._____

Part II

*Answer sixteen questions from this part, 25-48. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus Write your answers in the space provided.*

25. When drawn on the same set of axes, the graphs of $y = x^3 - x$ and $y = 2x + k$ are tangent to each other at a point whose abscissa is 1. Find the value of k.

25._____

26. Express the complex number $\sqrt{2} + i\sqrt{2}$ in the polar form $r(\cos \theta + i \sin \theta)$.

26._____

27. A root of $x^3 + x^2 - 24 = 0$ lies between 2 and 3. Find this root to the *nearest integer*.

27._____

28. In the equation $x^3 - kx^2 - 5x + 10 = 0$, k is a rational number. If two roots of this equation are $\sqrt{5}$ and 2, find the numerical value of k.

28._____

29. The illumination I, received by a body from a source of light of strength S, varies directly as S and inversely as the square of the distance d between the light source and the body. If k represents the constant of variation, express I in terms of S, d and k.   29_____

30. If $f(x) = 3x^3 - 2x^2 + 1$, find $f(2) - f(3^{-1})$.   30_____

31. In the complex number, plane points P and Q represent the complex numbers i and $-1 + i$, respectively, and O represents the origin. How many degrees are there in angle POQ?   31_____

*32. For what value of k will the value of the determinant
$$\begin{vmatrix} 4 & k & 3 \\ 2 & 0 & 5 \\ -1 & 0 & 6 \end{vmatrix}$$ be 17?   *32_____

33. Write the term of the expansion of $(a + b)^6$ in which the exponent of a is twice the exponent of b.   33_____

34. An object is moving along a straight line. Its distance s, in feet from a fixed point after t seconds, is given by the equation $s = 5t^2 + t^3$. Find the value of t when the acceleration is 19 feet per second per second.   34_____

35. A school committee of 2 members is to be formed by drawing 2 names from a box containing the names of 5 seniors, 4 juniors, and 3 sophomores. What is the probability that the committee will consist of 2 seniors?   35_____

36. Write an equation of the straight line having an x-intercept of 2 and a y-intercept of -4.   36_____

37. If the number $\dfrac{2 + 3i}{1 - 3i}$ is expressed in the form $a + bi$ where a and b are real numbers, what is the value of a?   37_____

38. Find the coordinates of the point of inflection of the curve whose equation is $y = x^3 - 12x + 20$.   38_____

39. The lengths of the sides of a right triangle form an arithmetic progression whose common difference is 3. Find the length of the shorter leg.   39_____

40. If $y = \log_{10}3$, find the value of 10^{2y}.   40_____

41. Point P in the first quadrant has the coordinates $(2, k)$. The line segment between P and the origin and the line segment between P and the point $(10, 0)$ are perpendicular. Find the value of k.   41_____

*42. Transform the equation $r^2 = 2r \sin \theta - 4r \cos \theta$ from polar to rectangular coordinates.   *42_____

Directions (43-48)   For *each of those chosen,* write in the space provided the *number* preceding the expression that best completes the statement.

43. If $x < 0$, then $\sqrt{x^2}$ is equal to   (1) an imaginary number
(2) x (3) a negative number   (4) $-x$ 43_____

44. If Al is a years old and Bob is b years old, how old was Al when Bob was c years old?   (1) $a - b + c$ (2) $a - b - c$
(3) $a + b - c$ (4) $b - a + c$ 44_____

45. If $\log x = 3 + \log 3$, then x equals   (1) 3^3 (2) 6
(3) 3,000   (4) 1,003   45_____

46. The circle whose equation is $(x + 1)^2 + (y + 3)^2 = 9$ is tangent to   (1) the y-axis only   (2) the x-axis only
(3) both axes   (4) neither axis   46_____

47. If the graphs of $y = 3^x$ and $y = 3^{-x}$ are drawn on the same set of axes, the graphs will intersect at a point which is on the line whose equation is   (1) $x = 0$ (2) $y = 0$ (3) $y = x$
(4) $y = -x$ 47_____

48. The solution of the inequality $x^2 - x - 6 > 0$ is
(1) $-2 < x < 3$ (2) $-3 < x < 2$
(3) $x < -2$ or $x > 3$ (4) $x < -3$ or $x > 2$ 48_____