# 12 A

# TWELFTH YEAR MATHEMATICS

## 12A (Advanced Algebra)

Wednesday, June 22, 1966 — 9:15 a.m. to 12:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

### Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 Find the rational root of $x^3 - 3x^2 + 5x - 6 = 0$.

2 Between what two consecutive integers does the real root of $x^3 + 2x + 7 = 0$ lie?

3 A particle moves according to the formula
$s = 6t^2 - 3t + 11$ (where $s$ represents distance and $t$, time). Find the instantaneous velocity when $t = 3$.

4 Solve the equation for the positive value of $n$:
$_nC_2 = 78$

5 Express the product of $\dfrac{2}{1+i}$ and $\dfrac{3-i}{1-i}$ in the form $a + bi$.

6 If the point whose coordinates are $(a,-3)$ lies on the line joining the points whose coordinates are $(-5,-8)$ and $(7,2)$, find the numerical value of $a$.

7 Solve for all values of $x$: $\sqrt{3x + 10} = x + 4$

8 Find the remainder when
$x^5 + 4x^4 + 7x^3 + 9x^2 + 2x + 4$ is divided by $x - 10$.

9 Solve for $x$: $\dfrac{x}{2} < \dfrac{3x}{4} + 1$

10 Express in the rectangular form $a + bi$, the product of $2(\cos 30° + i \sin 30°)$ and $4(\cos 60° + i \sin 60°)$.

11 If $a$ varies directly as $b$ and inversely as the positive square root of $c$, and if $a = 6$ when $b = 5$ and $c = 4$, find $a$ when $b = 4$ and $c = 9$.

12 Write an equation of the line which has a slope of $-\frac{2}{3}$ and passes through the point whose coordinates are $(-6,8)$.

13 Write the coordinates of the point of inflection of the graph of $y = 2x^3 + 6x^2 - 7$.

14 A class of 10 students meets in a room which has 5 seats in the front row. Because of a hearing difficulty, one of the students must sit in the middle seat in the front row. If all of the front row seats are always used, how many different seating arrangements are possible for the front row?

15 Find the tenth term of the geometric progression
$1, 2^{\frac{1}{3}}, \sqrt[3]{4}, \ldots$

16 Express in radical form the radius of the circle whose equation is $x^2 + y^2 + 2x - 10y + 8 = 0$.

17 Ed and Fred were each asked independently to write down one of the three numbers 1, 2 or 3. What is the probability that the sum of the two numbers selected is exactly 4?

18 Factor: $x^{n+1} + 2x$

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[1]

19 How many common solutions of the equation $x^2 + 2y^2 = 36$ and $5x - y = 4$ are indicated by their graphs?

  (1) one only         (3) exactly three
  (2) exactly two      (4) none

20 The equation $y = e^x$ is equivalent to

  (1) $x = e^y$          (3) $x = \log_e y$
  (2) $y = \log_e x$     (4) $x = \log_y e$

21 If $a \neq 0$, the graph of the sum of the complex number $a + bi$ and its conjugate lies

  (1) in the first quadrant
  (2) in the second quadrant
  (3) on the axis of reals
  (4) on the axis of imaginaries

22 If $p$ and $q$ are integers, which can *not* be a root of $6x^2 + px^2 + qx + 4 = 0$?

  (1) $\frac{1}{4}$         (3) $\frac{2}{3}$
  (2) $\frac{1}{2}$        (4) $\frac{3}{4}$

23 If $f(x) = x^2 - 2x$, then $f(x + 2) - f(2)$ is equivalent to

  (1) $f(x + 2)$      (3) $x^2 + 2x + 8$
  (2) $f(x)$         (4) $x^2 + 2x - 8$

24 The graph of $(\frac{1}{2})^x = y$ lies only in

  (1) quadrant I
  (2) quadrant II
  (3) quadrants I and II
  (4) quadrants I and IV

[2]

Answer sixteen questions from this part, 25–48. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers on the separate answer sheet.

25 If one of the roots of the equation $x^3 - 3x + 52 = 0$ is $2 + 3i$, find the real root.

26 Express as a single fraction in simplest form:

$$1 + \cfrac{1}{\cfrac{1}{x-1} - 1}$$

27 Find to the *nearest tenth* the value of $\log_2 10$.

28 Find the slope of the straight line which is tangent to the curve $y = x^3 - 2x^2 + 3x - 1$ at the point whose abscissa is $-1$.

29 Solve for $x$: $2^{4x} = (\tfrac{1}{2})^{x+5}$

30 Find the roots of the equation
$(x - 1)(x + 3) = 5(x - 1)$.

31 Write an equation of the straight line whose $x$-intercept is 3 and whose $y$-intercept is $-2$.

32 Solve for the positive value of $x$:
$\log 15 - \log x = \log (2x - 7)$

33 Write in *simplest* form the fourth term only of the expansion of $\left(1 - \dfrac{x}{2}\right)^6$.

34 A boy riding a bicycle at the rate of 10 miles per hour and a car traveling 40 miles per hour are traveling in the same direction on a road. If the car passes a signpost 30 minutes after the boy does, in how many minutes will the car overtake the bicycle?

35 The first three terms of an arithmetic progression are $1, 2\tfrac{1}{3}, 3\tfrac{2}{3}$. The last term is 53. Find the number of terms.

36 If $x$ men are each paid $p$ dollars a week and $y$ men are each paid $q$ dollars a week, express the average weekly salary of these men.

37 Express the complex number $-\tfrac{1}{2} + \tfrac{1}{2}\sqrt{3}i$ in polar form.

38 Find the average rate of change with respect to $x$ of the function defined by $y = x^3 - 2x + 3$ as $x$ varies from $x = 1$ to $x = 3$.

39 The product of the roots of the equation $(k - 1)x^3 + kx^2 + (k + 2)x - 12 = 0$ is 3. Find the value of $k$.

40 Find the maximum value of $k$ for which the roots of $x^2 - 8x + k = 0$ are real.

41 The parabola defined by $y = x^2 + bx + c$ crosses the $x$-axis at $(1,0)$. The equation of the axis of symmetry is $x = -2$. Find the value of $c$.

*42 Solve the equation $\begin{vmatrix} 2 & 0 & x \\ 3 & 5 & 2 \\ 1 & 4 & x \end{vmatrix} = 1$.

Directions (43–48): For *each of those chosen*, write in the space provided on the separate answer sheet the *number* preceding the expression that best completes the statement.

43 If the coordinates of two points are $A$ $(2,5)$ and $B$ $(-4,3)$, an equation of the perpendicular bisector of $AB$ is
(1) $3x - y + 7 = 0$
(2) $3x + y - 1 = 0$
(3) $x - 3y + 13 = 0$
(4) $x + 3y - 11 = 0$

44 The values of $x$ which satisfy the inequality $3x^2 + 5x - 2 > 0$ are those that satisfy
(1) $-2 < x < \tfrac{1}{3}$     (3) $x > \tfrac{1}{3}$ or $x < -2$
(2) $-\tfrac{1}{3} < x < 2$     (4) $x > 2$ or $x < -\tfrac{1}{3}$

45 The *smallest* positive value of $n$ for which $[5(\cos 30° + i \sin 30°)]^n$ is a real negative number is
(1) 12     (3) 3
(2) 2     (4) 6

46 If $x \neq 1$, the ratio $\dfrac{\log_a x^3}{\log_a x^2}$ is
(1) $x$     (3) $\tfrac{2}{3}$
(2) $\log_a x$     (4) $\tfrac{3}{2}$

*47 The graph of $r = 2 \sin \theta$ in polar coordinates is a
(1) straight line whose $y$-intercept in rectangular coordinates is 2
(2) spiral passing through the origin
(3) circle passing through the origin
(4) circle having its center at the origin

48 The distance between the points whose coordinates are $A$ $(\sqrt{5},\sqrt{3})$ and $B$ $(\sqrt{3},-\sqrt{5})$ is
(1) $\sqrt{15}$     (3) $\sqrt{68}$
(2) 16     (4) 4

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# TWELFTH YEAR MATHEMATICS

## 12A (Advanced Algebra)

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### ANSWER SHEET

Pupil.............................................................Teacher...............................................................

School..............................................................................................................................

All of your answers should be recorded on this answer sheet.

## Part I

Answer all questions in this part.

1..................................    9..................................    17..................................

2..................................    10..................................    18..................................

3..................................    11..................................    19..................................

4..................................    12..................................    20..................................

5..................................    13..................................    21..................................

6..................................    14..................................    22..................................

7..................................    15..................................    23..................................

8..................................    16..................................    24..................................

Your answers for part II should be placed in the proper spaces on the back of this sheet.

**Answer only sixteen questions from this part.** Be sure to write in the properly numbered spaces the answers to the questions you have chosen. Leave blank the spaces for the questions you do not choose to answer.

25.............................................

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I do so declare.................................................
                                                    *(Signature)*

# FOR TEACHERS ONLY

## 12A

Wednesday, June 22, 1966 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow 2½ credits for each correct answer; allow no partial credit. For questions 19–24, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

| | | |
|---|---|---|
| (1) 2 | (9) $x > -4$ | (17) ½ |
| (2) —1 and —2 | (10) $8i$ | (18) $x(x^n + 2)$ |
| (3) 33 | (11) $3\frac{1}{3}$ | (19) 2 |
| (4) 13 | (12) $y = -\frac{2}{3}x + 4$ | (20) 3 |
| (5) $3 - i$ | (13) $(-1, -3)$ | (21) 3 |
| (6) 1 | (14) 3024 | (22) 4 |
| (7) —3, —2 | (15) 8 | (23) 1 |
| (8) 147,924 | (16) $3\sqrt{2}$ | (24) 3 |

[OVER]

## Part II

Allow 2½ credits for each of not more than 16 correct answers; allow no partial credit. If more than sixteen questions have been answered, only the first sixteen of these should be considered. For questions 43–48, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(25) —4

(26) $\dfrac{1}{2 - x}$

(27) 3.3

(28) 10

(29) —1

(30) 1 and 2

(31) $y = \frac{3}{2}x - 2$

(32) 5

(33) $-\dfrac{5x^3}{2}$

(34) 10

(35) 40

(36) $\dfrac{px + qy}{x + y}$

(37) $\cos 120° + i \sin 120°$

(38) 11

(39) 5

(40) 16

(41) —5

(42) 1

(43) 2

(44) 3

(45) 4

(46) 4

(47) 3

(48) 4