The University of the State of New York

Centennial of Regents Examinations

1865–1965

12A

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Wednesday, June 23, 1965 — 9:15 a.m. to 12:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 Express $\frac{1}{1+i}$ as an equivalent fraction with a real denominator.

2 Find the value of the remainder obtained when $6x^4 + 5x^3 - 2x + 8$ is divided by $x - \frac{1}{2}$.

3 Given that T varies inversely as the square root of D and directly as P. If $T = 12$ when $D = 36$ and $P = 8$, find the value of the constant of variation.

4 Solve the equation: $2x + \sqrt{x} - 1 = 0$

5 Given the quadratic function $y = 4x^2 + 2x + 1$. Find the average rate of change of the function as x varies from $x = 1$ to $x = 3$.

6 If $f(x) = 2x^2$, write $f(x - 2)$ as a polynomial without parentheses.

7 Solve for t: $27^{5-t} = 9^{t-1}$

8 Determine all the values of x which satisfy the inequality $3 + 2(x - 1) > -4$.

9 For what value of k is the polynomial $kx^2 + kx + 20$ exactly divisible by $x - 2$?

10 Between what two successive positive integers does a real root of the equation $x^3 - x - 1 = 0$ lie?

11 A root of the equation $x^3 + 2x^2 - x - 1 = 0$ lies between 0.8 and 0.9. Find this root to the *nearest tenth*.

12 How many integers greater than 1,000 can be formed from the digits 0, 2, 3, 5 if no digit is repeated in any number?

13 Express the repeating decimal 0.191919..., in which the digits 1 and 9 are repeated endlessly, in the form $\frac{a}{b}$ where a and b are integers.

14 Find the radius of the circle whose equation is $x^2 + y^2 - 6x + 8y = 0$.

15 The slope of the straight line joining the points (5,7) and (2,y) is 1. Find the value of y.

[OVER]

Directions (16–24): Indicate the correct completion for *each* of the following by writing the number 1, 2, 3 or 4 in the space provided on the separate answer sheet.

16 The members of the family of lines having the equation $y = -2(x - a)$, where x, y and a are real, have
(1) the same x-intercept
(2) the same y-intercept
(3) the same slope
(4) a common point not on either coordinate axis

17 If the sum of two consecutive integers is k, then the smaller integer is
(1) $k-1$
(2) $\frac{k}{2}$
(3) $\frac{k-1}{2}$
(4) $\frac{k+1}{2}$

18 The sum of $\frac{\sqrt{3}}{2}$ and $\frac{2}{\sqrt{3}}$ is
(1) 1
(2) $\frac{5}{2\sqrt{3}}$
(3) $\frac{7\sqrt{3}}{6}$
(4) $\frac{\sqrt{3}+2}{2\sqrt{3}}$

19 When drawn on the same axes, the graphs of $x^2 - 3y^2 = 9$ and $(x-2)^2 + y^2 = 9$ have in common exactly
(1) 1 point
(2) 2 points
(3) 3 points
(4) 4 points

20 The eighth term of the geometric progression $3\sqrt{2}$, -6, $6\sqrt{2}$, ... is
(1) 144
(2) -144
(3) 48
(4) -48

21 The fifth term in the expansion of $(a + bi)^7$, where $i = \sqrt{-1}$, is
(1) $35a^3b^4$
(2) $-35a^3b^4$
(3) $21a^3b^4i$
(4) $-21a^3b^4i$

22 In order that $\frac{x+y}{z} - 1 = \frac{x-z}{y} + 1$, it is sufficient that
(1) $x = y$
(2) $y = z$
(3) $x = z$
(4) $x = 0$

23 If the equation $x^3 - 6x^2 + px + q = 0$ has 3 equal roots, then
(1) $q = 0$
(2) $p = 0$
(3) each root $= 2$
(4) each root $= -2$

24 An expression which is a rational, integral function of x is
(1) $x^5 - \sqrt{2}x^2 + 4$
(2) $x^5 - 2x^{\frac{1}{2}} + 6$
(3) $x^5 - 2x^{-\frac{1}{2}} + \frac{1}{2}$
(4) $x + \frac{1}{x} + 1$

Part II

Answer sixteen questions from this part, 25–48. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers on the separate answer sheet.

25 Find the value of $\frac{(x-3)^n + x^{-n}}{n^{x-3}}$ when $x = 3$ and $n = 2$.

26 If $\log (9!) = 5.5598$, determine the value of $\log (10!)$.

27 Find the abscissa of the point of inflection of the graph of $y = 2x^3 + 3x^2 + 6x - 4$.

*28 The straight line whose equation is $\begin{vmatrix} x & y & 1 \\ 2 & 3 & 1 \\ 4 & 7 & 1 \end{vmatrix} = 0$ passes through the point $(a,7)$. Find the value of a.

29 If $x = 7$ is the equation of the axis of symmetry of the graph of $y = ax^2 + 7x - 14$, find the value of a.

30 The equation $x^3 - 5x^2 + 8x - C = 0$ has one real root. If $(1 + i)$ is a root, find the value of the real root.

31 The longer side of a rectangle is represented by s, the perimeter by p and the area by A. Express A in terms of p and s.

32 How many different chords can be drawn between 10 different points on the circumference of a circle?

33 From a standard deck of 52 playing cards, one card is drawn at random, and then returned to the deck. A second card is then drawn. What is the probability of drawing a red card followed by a black card? (*Note:* A standard playing deck consists of two black suits and two red suits, each suit containing 13 cards.)

34 The positive rational root of the equation
$$4x^3 - x^2 + 16x - 4 = 0$$
is a common fraction. Find this root.

35 Find the value of x, given the simultaneous equations
$$(2^x)(2^y) = 8$$
$$\frac{2^x}{2^y} = 32$$

36 An arrow is shot vertically upward. Its height h in feet after t seconds is given by the formula
$$h = 128t - 16t^2.$$
Find in feet the maximum height to which the arrow will rise.

37 A root of $x^3 - 32 = 0$ lies in quadrant II. Write this root in the form $r(\cos\theta + i\sin\theta)$.

38 Indicate by means of an expression of the form $a < x < b$ all those values of x which satisfy $x^2 < 3x + 10$.

39 If $x^2 - dx - 2d = 0$, express x in terms of d.

40 Write the complex number $-2 - 2i$ in the polar form $r(\cos\theta + i\sin\theta)$.

41 Write a general equation of the family of lines perpendicular to the line $x + 2y + 3 = 0$.

Directions (42–48): For each *of those chosen*, write in the space provided on the separate answer sheet the *number* preceding the expression that best completes the statement.

42 If $\frac{1}{x} + y = 2$ and $x + \frac{1}{y} = 3$, then the ratio of x to y is
(1) 1:2
(2) 2:3
(3) 3:1
(4) 3:2

*43 The point whose polar coordinates are $(5,-30°)$ is the same as the point whose polar coordinates are
(1) $(-5,30°)$
(2) $(-5,150°)$
(3) $(5,-150°)$
(4) $(-5,-30°)$

44 If $0 < x < 1$, then
(1) $0 < \log_{10} x < 1$
(2) $\log_{10} x > 1$
(3) $\log_{10} x < 0$
(4) $\log_{10} x = 0$

45 A formula that describes the growth of a bacterial colony is given by $N = N_o e^{rt}$, where N is the number of bacteria at the end of time t, N_o is the original number, r is the rate of growth and e is a constant. If the formula is solved for t, then t equals
(1) $\frac{\log(N - N_o)}{r \log e}$
(2) $\frac{\log N - \log N_o}{r \log e}$
(3) $\frac{\log(N - N_o)}{(\log e)^r}$
(4) $\frac{\log N - \log N_o}{(\log e)^r}$

46 If a is the arithmetic mean between two different positive real numbers and g is their positive geometric mean, then a true statement is
(1) $a > g$
(2) $g > a$
(3) $a = g$
(4) $a = \frac{1}{2}g$

47 If $7^{-x} = 10$, then 7^{2x} is equal to
(1) $\frac{1}{100}$
(2) $\frac{1}{20}$
(3) 20
(4) 100

48 If the product of the roots of the equation
$$px^3 + qx^2 + rx + s = 0$$
is added to the sum of the roots of the same equation, the result may be expressed as
(1) $\frac{q+s}{p}$
(2) $\frac{q-s}{p}$
(3) $-\frac{s+q}{p}$
(4) $\frac{s-q}{p}$

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Rater's Initials:

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ANSWER SHEET

Pupil..... Teacher.....

School.....

All of your answers should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1.....	9.....	17.....
2.....	10.....	18.....
3.....	11.....	19.....
4.....	12.....	20.....
5.....	13.....	21.....
6.....	14.....	22.....
7.....	15.....	23.....
8.....	16.....	24.....

Your answers for part II should be placed in the proper spaces on the back of this sheet.

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[OVER]

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Part II

Answer only sixteen questions from this part. Be sure to write in the properly numbered spaces the answers to the questions you have chosen. Leave blank the spaces for the questions you do not choose to answer.

5.....	33.....	41.....
6.....	34.....	42.....
7.....	35.....	43.....
8.....	36.....	44.....
9.....	37.....	45.....
30.....	38.....	46.....
31.....	39.....	47.....
32.....	40.....	48.....

I do so declare.....

(Signature)

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FOR TEACHERS ONLY

12A

SCORING KEY

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Wednesday, June 23, 1965 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 16–24, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\frac{1-i}{2}$

(2) 8

(3) 9

(4) $\frac{1}{4}$

(5) 18

(6) $2x^2 - 8x + 8$

(7) 4

(8) $x > -2\frac{1}{2}$

(9) $-\frac{10}{3}$

(10) 1 and 2

(11) 0.8

(12) 18

(13) $\frac{19}{99}$

(14) 5

(15) 4

(16) 3

(17) 3

(18) 3

(19) 2

(20) 4

(21) 1

(22) 2

(23) 3

(24) 1

[OVER]

Part II

Allow $2\frac{1}{2}$ credits for each of not more than 16 correct answers; allow no partial credit. If more than sixteen questions have been answered, only the first sixteen of these should be considered. For questions 42–48, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(25) $\frac{1}{8}$

(26) 6.5598

(27) $-\frac{1}{2}$

(28) 4

(29) $-\frac{1}{2}$

(30) 3

(31) $s\left(\frac{p}{2} - s\right)$ *or* $\frac{sp - 2s^2}{2}$

(32) 45

(33) $\frac{1}{4}$

(34) $\frac{1}{4}$

(35) 4

(36) 256

(37) $2(\cos 144° + i \sin 144°)$

(38) $-2 < x < 5$

(39) $\frac{d \pm \sqrt{d^2 + 8d}}{2}$

(40) $2\sqrt{2}\,(\cos 225° + i \sin 225°)$

(41) $2x - y + k = 0$

(42) 4

(43) 2

(44) 3

(45) 2

(46) 1

(47) 1

(48) 3