

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION
TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Friday, June 19, 1964 — 9:15 a.m. to 12:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

- 1 Find the integral root of $x^3 - 2x^2 - x - 6 = 0$.
 - 2 If $x = \sqrt{t}$ and $y = t - 3$, express y in terms of x .
 - 3 If $f(x) = \frac{1-x}{x}$, find, in simplest form, $f\left(\frac{1}{t}\right)$.
 - 4 Find the product of $4(\cos 30^\circ + i \sin 30^\circ)$ and $3(\cos 90^\circ + i \sin 90^\circ)$. [Answer may be left in polar form.]
 - 5 Find the y -intercept of the graph of $y = (2 - x)^3$.
 - 6 Combine into a single term involving only one radical:
 $\frac{\sqrt{3}}{2} + \frac{3}{\sqrt{3}}$
 - 7 Solve for the positive value of x : $x^{\frac{3}{2}} = 64$
 - 8 Given that $f(x) = kx^3 + 2x^2 + 3x + 6$. Find k such that if $f(x)$ is divided by $x - 2$, the remainder will be 24.
 - 9 A committee is to consist of 5 seniors. How many ways may the committee be selected from the group of 10 seniors who volunteer for the task?
 - 10 The real root of $x^3 - 3x^2 + x + 7 = 0$ lies between two consecutive integers. Find these two integers.
 - 11 Express $\frac{2 - i\sqrt{2}}{1 - i\sqrt{2}}$ as an equivalent fraction with a real denominator.
 - 12 Write an equation of the locus of points equidistant from the two lines whose equations are $y = 2x + 4$ and $y = 2x - 4$.
 - 13 If $x - 2$ is a factor of $x^4 - x^3 - mx^2 - 4$, find the value of m .
 - 14 Express in $a + bi$ form the cube of $(1 + i)$.
 - 15 Find the sum of the infinite geometric progression, the first two terms of which are 2 and $-\frac{1}{2}$.
 - 16 Each of a group of n boys plans to pay d dollars in order to purchase a certain boat. If one boy drops out of the group and the remaining boys wish to share equally the cost of the boat, express in terms of n and d the number of dollars each of the remaining boys must pay.
 - 17 Write an equation of the line parallel to $2x - 5y = 6$ that passes through the point $(1, -3)$.
 - 18 How many different code words, using all of the letters at a time, can be made from the letters of the word BANANA? [Any arrangement of letters is considered a code word.]
 - 19 The equation of a circle is $x^2 - 4x + y^2 + 6y - 36 = 0$. Find the coordinates of the center.
- Directions (20-24): Write in the space provided on the separate answer sheet the number preceding the expression that best completes the statement or answers the question.*
- 20 The value of $4 \log_2 2$ equals
(1) $\frac{1}{2}$ (2) $\frac{1}{2}$ (3) $-\frac{1}{2}$ (4) $-\frac{1}{2}$

21 If x and y are real numbers and if $x + 3yi = 6$, then

(1) $x = 0$
(2) $y = 2$

(3) $y = -2$
(4) $x = 6$

22 If the graphs of $x^2 + y^2 = 9$ and $9x^2 + 4y^2 = 36$ are drawn on the same set of axes, how many distinct points will they have in common?

(1) 1
(2) 2

(3) 3
(4) 4

23 If $a^x = b$, then a^{x+2} is equal to

(1) $3 + b$
(2) $a^2 b$

(3) b^a
(4) $3b$

24 If a , b and c are real numbers and $c < 0$ and $a < b$, then

(1) $a + c > b + c$
(2) $a - c > b - c$

(3) $ac > bc$
(4) $ac < bc$

Part II

Answer sixteen questions from this part, 25-48. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answers on the separate answer sheet.

25 Find $\log_3 4$ to the nearest tenth.

26 Write an equation of the straight line whose x -intercept is 3 and whose y -intercept is -2 .

27 If $(n-1)$, $(n+2)$ and $(2n+4)$ are positive numbers which form a geometric progression in that order, find the value of the first term.

28 The point $(h, -1)$ lies on the straight line through the two points $(1, 5)$ and $(-3, -3)$. Determine the value of h .

29 Two roots of the equation $x^3 + 3x^2 + kx + q = 0$ are 3 and 1. Find the third root.

30 Solve for x the inequality $4 - 3x \leq 12 + x$.

31 If a buyer samples two items from a lot of 10 items that contains 3 defective items, what is the probability that both will be defective?

*32 Transform the equation $x^2 + y^2 = 2x + 20$ from rectangular to polar coordinates.

33 If the sum of two consecutive integers is k , express the smaller integer in terms of k .

34 Solve for x in terms of y and a : $y = \frac{a-x}{2a-x}$

*35 If $\begin{vmatrix} a & 1 & 0 \\ 2 & 1 & 1 \\ 1 & 3 & 1 \end{vmatrix} = 0$, find the value of a .

36 Write in simplest form the fifth term only in the expansion of $\left(x + \frac{1}{x^3}\right)^{10}$.

37 Write in the form $r(\cos \theta + i \sin \theta)$ the cube root of $8(\cos 72^\circ + i \sin 72^\circ)$, the amplitude of which lies in the first quadrant.

38 Find the point of inflection for the following curve: $y = 2x^3 - 6x^2 - 5x + 11$

39 A body moves along a line in such a way that its distance S in feet from a fixed point on the line at the end of t seconds is given by the equation $S = t^3 - 5t^2$. Find the number of feet per second in its velocity at the end of 5 seconds.

40 Solve the following equation for the positive real value of x : $\log \frac{x^4}{2} = \log 8$

41 Express the repeating decimal $0.5444\dots$ where the digit 4 repeats endlessly in the form $\frac{a}{b}$ where a and b are integers.

42 Write a general equation of the family of lines perpendicular to $3x + 2y = 6$.

43 Given that $f(x) = x^3 + x^2 + 2x + 6$. Find $f(i)$ in $a + bi$ form where $i = \sqrt{-1}$ and a and b are real.

44 If $s = 2 - 3t + 2t^2$, find the average rate of change of s with respect to t as t increases from $t = 0$ to $t = 2$.

Directions (45-48): For each of those chosen, write in the space provided on the separate answer sheet the number preceding the expression that best completes the statement or answers the question.

45 The graph of the solution of the inequality $x^2 - 4x \geq 0$ is to be shown by a heavy line or lines on the real number axis. Which graph is correct?



46 Which value of x satisfies the equation $x^2 - 2x + 1 = k$?

- (1) $-1 + \sqrt{k}$ (3) $1 + i\sqrt{k}$
(2) $1 + \sqrt{k}$ (4) $1 - i\sqrt{k}$

47 Given that y varies inversely as the square of x , x is doubled, then y is

- (1) multiplied by 2 (3) multiplied by 4
(2) divided by 2 (4) divided by 4

48 The graph of which of the following functions contains points in quadrants I and IV?

- (1) $y = 2^x$ (3) $y = 2x$
(2) $y = 2^{-x}$ (4) $y = \log_2 x$

Percent:
Rater's Initials:

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

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ANSWER SHEET

Pupil.....Teacher.....

School.....

All of your answers should be recorded on this answer sheet.

Part I

Answer all questions in this part.

- | | | |
|--------|---------|---------|
| 1..... | 9..... | 17..... |
| 2..... | 10..... | 18..... |
| 3..... | 11..... | 19..... |
| 4..... | 12..... | 20..... |
| 5..... | 13..... | 21..... |
| 6..... | 14..... | 22..... |
| 7..... | 15..... | 23..... |
| 8..... | 16..... | 24..... |

Your answers for part II should be placed in the proper spaces on the back of this sheet.

Answer only sixteen questions from this part. Be sure to write in the properly numbered spaces the answers to the questions you have chosen. Leave blank the spaces for the questions you do not choose to answer.

- | | | |
|-------|---------|---------|
| | 33..... | 41..... |
| | 34..... | 42..... |
| | 35..... | 43..... |
| | 36..... | 44..... |
| | 37..... | 45..... |
| | 38..... | 46..... |
| | 39..... | 47..... |
| | 40..... | 48..... |

I have regularly studied twelfth year mathematics 12A (advanced algebra) for weeks and have had sessions per week.

I do so declare.....
(Signature)

FOR TEACHERS ONLY

12A

SCORING KEY TWELFTH YEAR MATHEMATICS 12A (Advanced Algebra)

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 20–24, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 3

(9) 252

(17) $\frac{y+3}{x-1} = \frac{2}{3}$ or $2x - 5y = 17$

(2) $y = x^2 - 3$

(10) -1, -2

(18) 60

(3) $t - 1$

(11) $\frac{4 + i\sqrt{2}}{3}$

(19) (2, -3)

(4) $12(\cos 120^\circ + i \sin 120^\circ)$
or $-6 + 6i\sqrt{3}$

(12) $y = 2x$

(20) 2

(5) 8 or (0,8)

(13) 1

(21) 4

(6) $\frac{3\sqrt{3}}{2}$

(14) $-2 + 2i$

(22) 2

(7) 512

(15) $\frac{8}{5}$

(23) 2

(8) $\frac{1}{2}$

(16) $\frac{nd}{n-1}$

(24) 3

Part II

Allow $2\frac{1}{2}$ credits for each of not more than 16 correct answers; allow no partial credit. If more than sixteen questions have been answered, only the first sixteen of these should be considered. For questions 45–48, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(25) 1.3

(33) $\frac{k-1}{2}$

(41) $\frac{49}{90}$

(26) $\frac{x}{3} + \frac{y}{-2} = 1$

or $2x - 3y = 6$

(34) $x = \frac{a-2ay}{1-y}$

(42) $y = \frac{2x}{3} + b$

or $2x - 3y = k$

or $2x - 3y = -3b$

(27) 3

(35) $-\frac{1}{2}$

(28) -2

(36) $\frac{210}{x^6}$

(43) $5+i$

(29) -7

(44) 1

(37) $2(\cos 24^\circ + i \sin 24^\circ)$

(30) $x \geq -2$

(45) 4

(38) (1, 2)

(31) $\frac{1}{15}$

(46) 2

(39) 25

(32) $r^2 - 2r \cos \theta = 20$

(40) 2

(47) 4

(48) 4