June 21, 1962

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. If $f(x)$ is $x^{-3/2} - 2x^0$, find the value of $f(4)$.   1_____

2. Find the radius of the circle $x^2 - 2x + y^2 + 8y = 4$.   2_____

3. The repeating decimal .7666...., in which the digit 6 repeats endlessly, is rational. Show that it is rational by writing it in the form $\dfrac{p}{q}$ where p and q are integers.   3_____

4. Find the coordinates of the point at which the graph of $y = 2^x$ intersects the y-axis.   4_____

5. In the equation $x^3 + px^2 - qx = 0$, p and q are real numbers. If one root is $2 - i$, find the value of p.   5_____

6. Write in simplest form the tenth term *only* of the expansion of $\left\{ 2 - \dfrac{1}{x} \right\}^{12}$ 6_____

7. Write in the form $r(\cos \theta + i \sin \theta)$ the root of $x^3 + 8 = 0$ which, when represented graphically, lies in quadrant I.   7_____

8. If $\dfrac{x^{13} + 17}{x + 1}$ is expressed in the form $Q(x) + \dfrac{R}{x + 1}$, where R is a constant, find the value of R.   8_____

9. The graph of $y = x^2 - 8x + k$ is tangent to the x-axis. Find the value of k.   9_____

10. Find the slope of a straight line which is tangent to the graph of $y = 2x^2 - 7x + 5$ at $(-1, 14)$.   10_____

11. If $y = 3x^3 - 7x$, find the average rate of change of y with respect to x for the interval from $x = -1$ to $x = 1$.   11_____

12. What is the nonintegral rational root of the equation $2x^3 - 11x^2 + 8x + 7 = 0$?   12_____

13. Between what two consecutive integers does the smaller positive irrational root of $2x^3 - 11x^2 + 8x + 7 = 0$ lie?   13_____

14. The fourth term of an arithmetic progression is $3c$ and the ninth term is $-5c$. Find the first term.   14_____

15. If each signal is to consist of five flags arranged in sequence on a pole, how many different signals may be constructed from three identical red flags and two identical blue flags?   15_____

16. Four faces of a die are painted red and the remaining two faces are painted green. If the die is tossed once, what is the probability that it lands with a green face up?   16_____

17. If the distance from $(2, b)$ to $(-3, 5)$ is $\sqrt{74}$, find a value of b.   17_____

Directions (18-23): Indicate the correct completion for *each* of the following by writing the *number* 1, 2, 3 or 4 in the space provided.

18. The sum of $-2 + 3i$ and $3 - 4i$ is a complex number which, when represented graphically, lies in quadrant   (1) I   (2) II (3) III   (4) IV   18_____

19. The equation of the axis of symmetry of the parabola $y = 2x^2 - kx + c$ is $x = 1$. The value of k is   (1) -2 (2) 2   (3) -4 (4) 4   19_____

20. A general equation of the family of lines perpendicular to $2x - 5y = 7$ is   (1) $5x + 2y = k$ (2) $5x - 2y = k$ (3) $2x + 5y = k$ (4) $2x - 5y = k$ 20_____

21. The expression $xy = z$ represents   (1) direct variation of x with respect to y when z is constant   (2) inverse variation of x with respect to y when z is constant   (3) inverse variation of x with respect to z when y is constant   (4) inverse variation of z with respect to the product of x and y.   21_____

22. A line $kx + y = 8$ is drawn parallel to the line $4x + 5y = 7$. The value of k is   (1) $-\dfrac{4}{5}$ (2) $-\dfrac{5}{4}$ (3) $\dfrac{4}{5}$ (4) $\dfrac{5}{4}$ 22_____

23. If $y = -x^2 + 1$ and $9x^2 + 4y^2 = 36$ are graphed on the same axes, the total number of points of intersection is   (1) 1 (2) 2   (3) 3   (4) 4   23_____

24. On the diagram darken the portion of the real axis which is the graph of the non-negative solution of the inequality $3x - 4 \leq 2$.
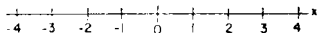
Part II

Answer sixteen questions from this part, 25-48. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus.

25. Express the number $\dfrac{6 + i\sqrt{3}}{2 - i\sqrt{3}}$ as an equivalent fraction with a real denominator.   25_____

26. From a mixture of vinegar and water which is 20% vinegar, 2 quarts are removed and replaced by pure vinegar. The resulting mixture is 30% vinegar. Find the number of quarts in the original mixture.   26_____

27. A point moves on a straight line through a fixed point P so that the equation $s = 2t^2 - 7t + 4$ represents its distance s from P as a function of the time t. Find the velocity of the point when $t = 3$.   27_____

28. A root of $f(x) = x^3 - 7x^2 + 2x - 3 = 0$ is between 6.7 and 6.8; $f(6.7)$ is -3.067 and $f(6.8)$ is 1.352. Find this root to the *nearest tenth.*   28_____

29. Find to the *nearest tenth* $\log_3 8$.   29_____

30. Write $(\sqrt{3} + i)^3$ in the form $r(\cos \theta + i \sin \theta)$.   30_____

31. Six men are candidates in an election in which two are to be chosen. If a voter is instructed to mark his ballot either for one candidate or for two candidates, in how many ways can he carry out these instructions?   31_____

32. Express as a single fraction in simplest form: $\dfrac{2 - \dfrac{2-a}{2-\dfrac{a}{2}}}{}$ 32_____

*33. The area of a certain triangle is 12. If the area expressed in determinant form is $\frac{1}{2} \begin{vmatrix} a & 3 & 1 \\ -1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$, find the value of a.   33_____

Directions (34-36): For each *of those chosen*, write the *number* preceding the expression that best completes the statement.

34. If $f(x) = -x^2 + 7x$, then $f\left\{\dfrac{1}{x}\right\}$ is

(1) $\dfrac{7x + 1}{x^2}$ (2) $\dfrac{7x - 1}{x^2}$ (3) $\dfrac{7 - x^3}{x}$ (4) $\dfrac{7 + x^3}{x}$ 34_____

35. If $\log x^2 - \log 2a = \log 5a$, then the value of $\log x$ in terms of a is   (1) $\dfrac{2\log a + 1}{2}$ (2) $\dfrac{2\log a + 7}{2}$ (3) $\frac{1}{2}$

(4) $\dfrac{7\log a}{2}$ 35_____

36. The solution set of $2x^2 - x - 3 < 0$ is the set of all real numbers x such that $x > \dfrac{3}{2}$ or $x < -1$ (2) $x < \dfrac{3}{2}$ or $x < -1$

(3) $-\dfrac{3}{2} < x < 1$ (4) $-1 < x < \dfrac{3}{2}$ 36_____

37. A line whose equation is $\dfrac{x}{3} - \dfrac{y}{7} = 1$ intersects the axes at $(a, 0)$ and $(0, b)$. Find the value of b.   37_____

38. Solve $8^{2x} = (\frac{1}{16})^{x-1}$ for x.   38_____

39. The sum of an infinite geometric progression is $2\sqrt{2} + 2$. If the common ratio is $\dfrac{1}{\sqrt{2}}$, find the first term.   39_____

40. If the only roots of $x^n + a_1 x^{n-1} + a_2x^{n-2} + ... + a_n = 0$ are $3 - i$, $3 + i$ and -4, find the value of a_2 40_____

41. If $x^{2.5} = 94.3$, find x to the *nearest tenth.*   41_____

42. The sum of an arithmetic progression of 8 terms is S, the first term is a and the eighth term is $\dfrac{a}{2}$. Express a as a function of S.   42_____

43. Ten balls numbered 1 to 10 are placed in a bag, and two of the balls are drawn at random. What is the probability that balls numbered 3 and 7 are drawn?   43_____

*44. Transform $2x^2 + 5y^2 = 1$ into an equation in polar coordinates.   44_____

45. Using k as the constant of variation, write an equation which represents the following relationship: x varies directly as the square of y and inversely as z·   45_____

Directions (46-48): For each *of those chosen*, write in the space provided the *number* preceding the expression that best completes the statement·

*46. The graph of $r = 2 \cos \theta$ is a   (1) straight line parallel to the y-axis   (2) circle passing through the pole   (3) straight line passing through the pole   (4) circle with center at the pole   46_____

47. The roots of $x^4 + 3x^2 - 28 = 0$ are   (1) $\sqrt{7}, -\sqrt{7}, 2, -2$ (2) $7, 7, 2, 2$ (3) $+7i, -7i, 2, -2$ (4) $+i\sqrt{7}, -i\sqrt{7}, 2, -2$ 47_____

48. The equation $\sqrt{2 + x} - x = 0$ has   (1) no root   (2) 2 as its only root   (3) -1 as its only root   (4) the two roots 2 and -1 48_____