

June 22, 1961

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

- If $x - 2$ is a factor of $x^3 + hx + 10$, find the value of h . 1 _____
 - Find the value of $x^2 - 4^x$ when $x = -\frac{1}{2}$. 2 _____
 - Find all of the roots of the equation $x^3 - 2x^2 - 5x + 6 = 0$. 3 _____
 - Find the coordinates of a point of intersection of the graphs of $x^2 + y = 10$ and $x + y = 10$. 4 _____
 - Write the repeating decimal $0.2555\dots$ as the quotient of two integers. 5 _____
 - The parabola $y = x^2 - 6x + k$ has its turning point on the x -axis. What is the value of k ? 6 _____
 - Three boys agree to divide a bag of marbles in the following manner: The first boy takes one-half of the marbles. The second boy takes one-third of the remaining marbles, leaving 10 marbles for the third boy. Find the original number of marbles. 7 _____
 - A box contains 5 red, 4 white and 3 black balls. If two balls are drawn at random, without replacement, what is the probability that both are white? 8 _____
 - If $t = x^{1.5}$, find to the nearest integer the value of t when $x = 20$. 9 _____
 - Find a value of x that satisfies the equation:
 $\log_{10}(x^2 - 3x) = 1$ 10 _____
 - Given the triangle whose vertices are at $A(3, 3)$, $B(6, 2)$ and $C(8, -2)$. What is the slope of the altitude to side AC ? 11 _____
 - Write an equation of the line tangent to the curve $y = 2x^2 + 6$ at the point $(1, 8)$. 12 _____
 - If $f(x) = 2 - x$, express $2f(a) + f(2a)$ in simplest form, 13 _____
 - Find in simplest form the 5th term in the expression of
 $\left\{ x^2 + \frac{1}{x} \right\}^8$ 14 _____
 - Write an equation of the circle that passes through the origin and whose center is the point $(-3, 4)$. 15 _____
- Directions (16-20):* Write on the line at the right of each of the following the number preceding the expression that best completes the statement.
- If $x^{-1} - 1$ is divided by $x - 1$, the quotient is
 (1) $\frac{1}{x-1}$ (2) $-\frac{1}{x-1}$ (3) $\frac{1}{x}$ (4) $-\frac{1}{x}$ 16 _____
 - In the equation $x^3 - hx^2 + 2k = 0$, the sum of the roots is 3 and the product of the roots is 4. Then h and k have the values, respectively, (1) -3 and 2 (2) -3 and -2 (3) 3 and -2 (4) 3 and 2 17 3

18. The sum of $\frac{1}{2 + \sqrt{2}}$ and $\frac{1}{2 - \sqrt{2}}$ is (1) $\frac{1}{2}$ (2) 2
 (3) $\sqrt{2}$ (4) $-\sqrt{2}$ 18. 2
19. The simplest form of $1 - \frac{1}{1 + \frac{a}{1-a}}$ is
 (1) 1 (2) $1 - a$ (3) a (4) 0 19. 3
20. The graphs of $3x - 2y = 8$, $2x + 2y = 7$, $x = 3$ and $y = \frac{1}{2}$ are drawn on the same set of axes. The number of distinct intersections of all pairs of these straight lines is (1) 1 (2) 2 (3) 6 (4) 4 20. 1
21. In how many ways may a committee of three students be selected from a group of six students? 21. 20
22. Given that t varies directly as v and inversely as d^3 . If $t = \frac{1}{2}$ when $v = 25$ and $d = 10$, find the value of the constant of variation. 22. 20
23. Solve for $x: 3x - 6 < 5x$ 23. $x > 3$
24. By how much does the arithmetic mean between 1 and 25 exceed the positive geometric mean between 1 and 25? 24. 8
25. If $(2 + i)^2 = c + di$, find the value of d . 25. 4

Part II

Answer ten questions from this part, 26-40. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.

26. Find to the nearest tenth the root of $x^3 + 4x - 1 = 0$ which lies between 0 and 1. 26. 0.2
27. The points (2, -2), (4, 2) and (5, k) are on the same straight line. What is the value of k ? 27. 4
28. If a certain number is added to each of the numbers 10, 25 and 50, the resulting numbers form a geometric progression. What is this number? 28. $12\frac{1}{2}$
29. Solve to the nearest tenth the equation: $3^x = 20$ 29. 2.7
30. A certain law of motion is given by the equation $S = \frac{1}{2}t^3$ where S is in feet and t is in seconds. Find in feet per second the average velocity over the time interval $t = 2$ to $t = 4$. 30. 14
31. Express in $a + bi$ form the product of $2(\cos 60^\circ + i \sin 60^\circ)$ and $3(\cos 90^\circ + i \sin 90^\circ)$. 31. $-3\sqrt{3} + 3i$
32. Find the fractional root of the equation $3x^3 - 5x^2 + 5x - 2 = 0$. 32. $\frac{2}{3}$
- *33. Transform $r^2 = 4r \cos \theta - r \sin \theta$ from polar to rectangular coordinates. 33. $x^2 + y^2 - 4x + y = 0$
- *34. The value of the determinant $\begin{vmatrix} 1 & 0 & 0 \\ 0 & a & 2 \\ 0 & 0 & 3 \end{vmatrix}$ is 4. Find the value of a . 34. 3

Directions (35-38): Write on the line at the right of each of those chosen the number preceding the expression that best completes the statement or answers the question.

35. Which equality is an example of the distributive law?

(1) $\frac{a+b}{2} = \frac{b+a}{2}$ (2) $a + (b + c) = (a + b) + c$

(3) $\frac{1}{2}(a + b) = \frac{1}{2}a + \frac{1}{2}b$ (4) $a(b + c) = (b + c)a$ 35 3

36. The logarithm of $\sqrt[3]{8}$ to the base 2 is (1) 1 (2) $\frac{3}{4}$
(3) $\frac{1}{2}$ (4) $\frac{1}{4}$ 36 2

37. A rectangular field, twice as long as it is wide, is completely enclosed by x yards of fencing. The area in terms of x is

(1) $\frac{x^2}{2}$ (2) $2x^2$ (3) $\frac{2x^2}{9}$ (4) $\frac{x^2}{18}$ 37 4

38. Given that y varies directly as the square of x . The graph of this relation is (1) a straight line (2) an ellipse
(3) a hyperbola (4) a parabola 38 4

39. Find the coordinates of the point of inflection of the graph of the equation $y = 6 + 3x^2 - x^3$. 39 1, 8

40. How many even numbers consisting of three digits each can be formed from the digits 1, 2, 3, 4 and 5? [Repetition of digits are permitted.] 40 50

Part III

Answer ten questions from this part, 41-55. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.

41. If $s = x - x^2$, express x in terms of s . 41 $\frac{1 \pm \sqrt{1-4s}}{2}$

42. Find the abscissa of the point of intersection of the x -axis and the line joining the points $(-1, 1)$ and $(3, 9)$. 42 $-\frac{3}{2}$

43. Write an equation with real coefficients and of lowest possible degree, two of whose roots are 1 and $1 + i$. 43 $x^3 - 3x^2 + 4x - 2 = 0$

44. Find the value of r when $(\sqrt{3} + i)^5$ is expressed in the form $r(\cos \theta + i \sin \theta)$. 44 32

*45. [Write the number preceding the correct answer in the space provided.] Which set of coordinates does not satisfy the equation $r = 4(1 - \cos \theta)$? (1) $(8, \pi)$ (2) $(0, 0)$

(3) $\left\{ -4, -\frac{\pi}{2} \right\}$ (4) $\left\{ 4, \frac{\pi}{2} \right\}$ 45 3

*46. Write in determinant form the value of y for the system

$$\begin{aligned}x &= 1 \\x + y &= 2 \\2x + y + z &= 1\end{aligned}$$

$$46 \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 1 & 1 \end{vmatrix}$$

47. How many ounces of water must be added to 9 ounces of a 50% salt solution to change it to a 30% salt solution?

47. 6

48. The roots of the equation $x^5 = 1$ are represented in polar form. Find the amplitude of the root that lies in quadrant II.

48. 144°

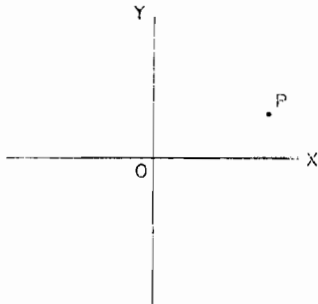
49. A certain law of motion is given by the equation $S = 24t + \frac{1}{2}t^3$ where S is in feet and t is in seconds. Find in feet per second the velocity at $t = 4$.

49. 48

50. To solve the inequality $x^2 - x - 6 < 0$, we may consider the values of x for which the graph of $y = x^2 - x - 6$ lies below the x -axis. If the solution has the form $a < x < b$, what is the value of b ?

50. 3

51. In the accompanying diagram, OX is the axis of reals and OY is the axis of imaginaries. Point P represents the complex number $a + bi$. Indicate on the diagram by the letter Q , the point which represents the complex number $c + di$ such that $(a + bi) + (c + di) = 0$.



Directions (52-55): Write on the line at the right of each of those chosen the number preceding the expression that best completes the statement.

52. The equation $\sqrt{2 - x} = x$ has (1) 1 as its only root
(2) -2 as its only root (3) 1 and -2 as its only roots (4)
 -1 and 2 as its only roots

52. 1

53. If $3^x = k + 1$, then 3^{2x} is equal to (1) $2(k + 1)$
(2) $k^2 + 1$ (3) $k^2 + 2k + 1$ (4) $3(k + 1)$

53. 3

54. The equation $2x + 1 = bx + 3$ has a solution for x
(1) for all values of b (2) for no values of b (3) only if b is
equal to 2 (4) only if b is not equal to 2

54. 4

55. If $\log_a 2 = x$ and $\log_b 3 = y$, then $a^x b^y$ equals
(1) $(ab)^{x+y}$ (2) 6 (3) $\log_{ab} 6$ (4) $\log_{ab} 5$

55. 2

FOR TEACHERS ONLY

12A

INSTRUCTIONS FOR RATING TWELFTH YEAR MATHEMATICS 12A (Advanced Algebra)

Thursday, June 22, 1961 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 16–20, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) -9

(2) $-\frac{1}{4}$

(3) $-2, 1$ and 3

(4) $(0, 10)$ or $(1, 9)$

(5) $\frac{23}{90}$

(6) 9

(7) 30

(8) $\frac{1}{11}$

(9) 89

(10) -2 or 5

(11) 1

(12) $4x - y + 4 = 0$

(13) $6 - 4a$

(14) $70x^4$

(15) $(x + 3)^2 + (y - 4)^2 = 25$ or
 $x^2 + y^2 + 6x - 8y = 0$

(16) 4

(17) 3

(18) 2

(19) 3

(20) 1

(21) 20

(22) 20

(23) $x > -3$

(24) 8

(25) 4

Part II

Allow $2\frac{1}{2}$ credits for each of not more than 10 correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. For questions 35–38, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(26) 0.2

(35) 3

(27) 4

(36) 2

(28) $12\frac{1}{2}$

(37) 4

(29) 2.7

(38) 4

(30) 14

(39) (1, 8)

(31) $-3\sqrt{3} + 3i$

(40) 50

(32) $\frac{2}{3}$

(33) $x^2 + y^2 - 4x + y = 0$

(34) $\frac{4}{3}$

Part III

Allow $2\frac{1}{2}$ credits for each of not more than 10 correct answers; allow no partial credit. If more than ten questions have been answered, only the first ten of these should be considered. For questions 52–55, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(41) $\frac{1 \pm \sqrt{1 - 4s}}{2}$

(49) 48

(42) $-\frac{3}{2}$

(50) 3

(43) $x^3 - 3x^2 + 4x - 2 = 0$

(44) 32

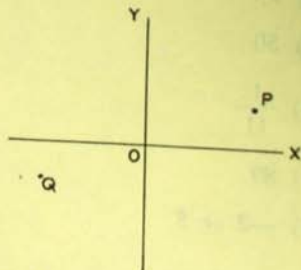
(45) 3

(46)
$$\begin{array}{c} \left| \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 1 & 1 \end{array} \right| \\ \hline \left| \begin{array}{ccc} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{array} \right| \end{array}$$

(47) 6

(48) 144° or $\frac{4\pi}{5}$

(51)



One method of locating Q is to extend PO through O a distance equal to its own length.

(52) 1

(53) 3

(54) 4

(55) 2