June 22, 1960

Part I

Answer all questions in this part. Questions 1-10 *count* 1 *credit each. Questions* 11-30 *count* 2 *credits each. No partial credit will be allowed. Write the answer to each question on the line at the right.*

1-3 Questions 1-3 refer to the graph of the equation $2x + 4y + 5 = 0$.

1. Find the slope of the line. 1_____

2. Write an equation of the straight line parallel to the given line and passing through the origin. 2_____

3. Find the x-intercept of the given line. 3_____

4-6 Questions 4-6 refer to the equation $x^3 + 10x + 2 = 0$.

4. Find the sum of the roots of the equation. 4_____

5. Find the product of the roots of the equation. 5_____

6. How many rational roots does the equation have? 6_____

Directions (7-10): Indicate whether *each* of the following statements is true for

(1) all real values of x
(2) one or more, but not all, real values of x
(3) no real value of x

by writing on the line at the right the *number* 1, 2 or 3.

7. $\sqrt{x^2 + 9} = x + 3$ 7_____

8. $x^2 + 1 = 0$ 8_____

9. $(x - 1)^3 = x^3 - 3x^2 + 3x - 1$ 9_____

10. $x^2 - 6x + 9 < 0$ 10_____

11. Express in the form $a + bi$ the reciprocal of $2 + i$. 11_____

12. Find the numerical value of a if $x + a$ is a factor of $x^5 + 32$. 12_____

13. John travels from A to B, a distance of 30 miles, at the rate of 6 miles per hour, and then without stopping returns from B to A. What should his return rate be in miles per hour, in order that the average rate for the entire trip be 5 miles per hour? 13_____

14. If $f(x) = (2x)^0 + x^{-\frac{2}{3}}$, find the value of $f(64)$. 14_____

15. There are ten people at a conference. How many different committees of three members each can be formed from these ten people? 15_____

16. A three-digit number is to be formed using the digits from 1 to 9, inclusive. What is the probability that the number will be odd? [Repetitions of digits are permitted.] 16_____

17. Three cards (ace, king, jack) are face down on a table. If two of these cards are picked at random, what is the probability that one of them is an ace? 17_____

18. Write in *simplest form* the fourth term *only* of $(1 + i)^6$, where $i = \sqrt{-1}$. 18_____

19. The distance that a body falls from rest in t seconds is given by the formula $S = \frac{1}{2} gt^2$, and the final velocity is given by the formula $V = gt$. Express V in terms of g and S. 19_____

20. Express in *simplest form* $\dfrac{1}{1 + \dfrac{1}{1 + x}}$. 20_____

21. If $\frac{1}{2}$, $\frac{1}{x}$ and $\frac{1}{3}$ are three consecutive terms of an arithmetic progression, find the value of x. 21_____

22. Find the coordinates of the minimum point of the graph of the equation $y = x^2 - 6x + 9$. 22_____

23. Ten quarts of a solution containing $x\%$ antifreeze is mixed with twenty quarts of a solution containing $y\%$ antifreeze. The fractional part of antifreeze in the resulting mixture is

(1) $\dfrac{x + 2y}{100}$ (2) $\dfrac{x + 2y}{300}$ (3) $\dfrac{2x + y}{100}$ (4) $\dfrac{2x + y}{300}$ 23_____

24. If $\log_{10} x = 1.5421$, then $10^{3.5421}$ equals (1) $2 + x$ (2) $2x$ (3) $100 + x$ (4) $100x$ 24_____

25. If $f(x)$ is divided by $x - 2$, the remainder is (1) $f(2)$ (2) 2 (3) $f(-2)$ (4) -2 25_____

26. If ${}_nC_x = {}_nC_y$, where n, x and y are positive integers such that $x \neq y$, then (1) $x = \dfrac{n}{2}$ (2) $y = \dfrac{n}{2}$ (3) $x + y = n$ (4) $x - y = n$ 26_____

27. The area of a circle varies directly as the square of its diameter. The constant of variation is (1) 1 (2) π (3) $\dfrac{\pi}{4}$ (4) $\dfrac{1}{4}$ 27_____

28. If r_1 and r_2 are real roots of the quadratic equation $x^2 + px + q = 0$ such that $r_1 > 0$, $r_2 < 0$ and p and q are integers, it is always true that (1) $q > 0$ (2) $q < 0$ (3) $p > 0$ (4) $p < 0$ 28_____

29. If $4^x = 8^y$, then x equals (1) $\frac{1}{2}y$ (2) $2y$ (3) $\frac{3}{2}y$ (4) $\frac{2}{3}y$ 29_____

30. In the equation $px^2 + qx + s = 0$, p, q and s are real numbers with $p \neq 0$. If the two roots of the equation are equal, then (1) $q^2 = 4ps$ (2) $q^2 = -4ps$ (3) $q^2 = ps$ (4) $q^2 = -ps$ 30_____

Part II

*Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. The question marked * is based upon an optional topic in the syllabus. Write your answer on the line at the right.*

31. If two of the roots of $x^3 + px + q = 0$ are 3 and —1, find the third root. 31_____

32. If one of the roots of $x^3 - 2x^2 + x - 2 = 0$ is 2, find the other two roots. 32_____

33. The x-intercepts of the graph of the equation $y = x^2 + bx + c$ are 2 and 3. Find the value of c. 33_____

34. The points P_1 (2, 3), P_2 (4, 9), P_3 (6, k) are collinear. Find the value of k. 34_____

Directions (35-37) : Indicate the correct completion for *each* of the following by writing on the line at the right the *number* 1, 2, 3 or 4.

35. A possible root of the equation $6x^4 + px^3 + qx^2 + rx + 4 = 0$ where p, q and r are integers is (1) $\frac{3}{2}$ (2) $-\frac{3}{2}$ (3) —3 (4) $\frac{4}{3}$ 35_____

36. If a and b are real numbers, then the product of $a + bi$ and $a - bi$ is (1) always a real number (2) sometimes, but not always, a real number (3) always imaginary (4) sometimes, but not always, imaginary 36_____

37. A rational integral function of x is (1) $x + \frac{1}{x}$ (2) $\sqrt{x} + 2$ (3) $x^2 + x^{3/2}$ (4) $x + \sqrt{2}$ 37_____

38. The circle whose center is (3, —2) passes through the point (5, 1). Find the length of the radius of the circle. 38_____

39. The first term of an arithmetic progression is x and the common difference is 2. The first, third and seventh terms form a geometric progression. Write an equation that could be used to find the value of the first term. 39_____

40. In how many ways may three pupils be seated in a row containing 5 seats? 40_____

41. Find the slope of the line tangent to the curve whose equation is $y = x^3 - 5x + 2$, at the point where the graph crosses the y-axis. 41_____

42. Find the coordinates of the point of inflection of the curve whose equation is $y = x^3 - 5x + 2$. 42_____

43. The area of a rectangle is represented by $12x - x^2$ where x is a side of the rectangle. For what value of x will the area be a maximum? 43_____

44. Find the set of values of x that satisfies the inequality $4 - 2x < 10$. 44_____

*45. Write in determinant form an equation of the straight line through the points (3, 2) and (—1, 0). 45_____

Part III

*Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

46. Find to the *nearest tenth* the value of $\log_2 5$. 46_____

47. Given $A = Pe^r$. Express r in terms of $\log A$, $\log P$ and $\log e$. 47_____

Directions (48-53): Indicate the correct completion for each of the following by writing on the line at the right the *number* 1, 2, 3 or 4.

48. The positive root of the equation $x^3 + 5x - 7 = 0$ lies between (1) 1.0 and 1.2 (2) 1.2 and 1.4 (3) 1.4 and 1.6 (4) 1.6 and 1.8 48_____

49. The graph of $y = 3^x$ (1) intersects the x-axis only (2) intersects the y-axis only (3) intersects both coordinate axes (4) does not intersect either axis 49_____

50. If r is a positive real number and n is a positive integer, then $r^{-\frac{1}{n}}$ is equal to (1) $\frac{1}{r^{-n}}$ (2) $\frac{1}{\sqrt[n]{r}}$ (3) $\sqrt[n]{r}$ (4) r^n 50_____

51. If the relation, x varies inversely as y, is represented graphically, the graph will be (1) a straight line (2) an ellipse (3) a hyperbola (4) a parabola 51_____

52. If in the equation $y = 3^x$, the variable x is increased by 2, then y is (1) increased by 2 (2) multiplied by 2 (3) increased by 9 (4) multiplied by 9 52_____

53. If the roots of the equation $x^2 + x + 1 = 0$ are expressed in the form $a + bi$, then b is equal to (1) $\pm \frac{1}{2}$ (2) $\pm \frac{3}{2}$ (3) $\pm \frac{\sqrt{3}}{2}$ (4) $\pm \frac{\sqrt{3}}{4}$ 53_____

54. The area of a rectangle is represented by A, the diagonal by d and one side by s. Express d in terms of A and s. 54_____

55. In the equation $x^2 + ax + b = 0$, one root is twice the other. Express b in terms of a. 55_____

56. Express in the form $a + bi$: $2(\cos 120° + i \sin 120°)$ 56_____

57. Express in polar form: $-3i$ 57_____

58. Find the amplitude of the complex number $[1(\cos 40° + i \sin 40°)]^{\frac{1}{2}}$ which, when represented graphically, lies in the third quadrant. 58_____

*59. The polar coordinates of a point P are $\left\{2, \frac{\pi}{3}\right\}$. If $\left\{x, \frac{4\pi}{3}\right\}$ are the coordinates of the same point, find the value of x. 59_____

*60. The equation of a circle in polar form is $r = 6 \sin \theta$. Write an equation of this circle in rectangular form. 60_____

ANSWER KEY

Regents Examinations in Advanced Algebra (12A)

TOPICAL REVIEW BOOK CO.

131 North Street

Auburn, New York

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1. $-\frac{1}{2}$
2. $x + 2y = 0$
3. $-2\frac{1}{2}$
4. 0
5. -2
6. none
7. 2
8. 3
9. 1
10. 3
11. $\frac{3}{5} - \frac{1}{5}i$
12. 2
13. 4 2/7
14. 1 1/16
15. 120
16. 5/9
17. $\frac{2}{3}$
18. $-20\sqrt{-1}$
19. $g\sqrt{\frac{2S}{g}}$
20. $\frac{1+x}{2+x}$
21. 2 2/5
22. 3, 0
23. 2
24. 4
25. 1
26. 3
27. 3
28. 2
29. 3
30. 1
31. -2
32. $i, -i$
33. 6
34. 15
35. 4
36. 1
37. 4
38. $\sqrt{13}$
39. $\frac{x}{x+4} = \frac{x+4}{x+12}$
40. 60
41. -5
42. 0, 2
43. 6
44. $x > -3$
45. $\begin{vmatrix} x & y & 1 \\ 3 & 2 & 1 \\ -1 & 0 & 1 \end{vmatrix} = 0$
46. 2.3
47. $\frac{\log A - \log P}{\log e}$
48. 1
49. 2
50. 2
51. 3
52. 4
53. 3
54. $\frac{\sqrt{A^2 + s^4}}{s}$
55. $\frac{2a^2}{9}$
56. $-1 + \sqrt{3}i$
57. $3(\cos 270° + i \sin 270°)$ or $x^2 + (y-3)^2 = 9$
58. 200°
59. -2
60. $x^2 + y^2 - 6y = 0$