June 18, 1959

Part I

Answer all questions in this part. Questions 1-10 count 1 credit each. Questions 11-30 count 2 credits each. No partial credit will be allowed.

Directions (1-10): Indicate the correct completion for *each* of the following by writing the letter *a*, *b*, *c* or *d* on the line at the right.

1 - 5 Questions 1 - 5 refer to the graph of the equation $y = 2x - 2$.

1. The slope of this line is (a) -2 (b) -1 (c) 1 (d) 2 1_____

2. The x-intercept is (a) -2 (b) -1 (c) 1 (d) 2 2_____

3. The y-intercept is (a) -2 (b) -1 (c) 1 (d) 2 3_____

4. The graph of this line does *not* pass through quadrant (a) I (b) II (c) III (d) IV 4_____

5. An equation of the line parallel to the given line and passing through the origin is (a) $y = -2x$ (b) $x = -2y$ (c) $y = 2x$ (d) $x = 2y$ 5_____

6 - 10 Questions 6 - 10 refer to the equation $x^3 - x^2 + 2x - 2 = 0$ whose roots are r, s and t.

6. The value of $r + s + t =$ (a) -2 (b) -1 (c) 1 (d) 2 6_____

7. The value of $rst =$ (a) -2 (b) -1 (c) 1 (d) 2 7_____

8. The value of $rs + rt + st =$ (a) -2 (b) -1 (c) 1 (d) 2 8_____

9. If r represents the only integral root of this equation, then r equals (a) -2 (b) -1 (c) 1 (d) 2 9_____

10. If r represents the only integral root of this equation and $s = i\sqrt{2}$, then t equals (a) $-\sqrt{2}$ (b) $-i\sqrt{2}$ (c) $1-\sqrt{2}$ (d) $\sqrt{2}$ 10_____

11. Solve for the real value of x: $x^{3/2} = 1/64$ 11_____

12. Write in *simplest* form the fifth term in the expansion of $(a + \sqrt{a})^7$. 12_____

13. When the fraction $\frac{1+i}{1-i}$ is expressed in $a+bi$ form, what is the value of a? 13._____

14. The complex number $a+bi$ is the sum of $2+3i$ and $-4-i$. In which quadrant is the graphic representation of $a+bi$ located? 14._____

15. Find the remainder when $x^{15}+5x^8-1$ is divided by $x+1$. 15._____

16. The slope of the line determined by the points $(-2,-5)$ and $(4,k)$ is $1\frac{1}{2}$. Find the value of k. 16._____

17. Write an equation of the axis of symmetry of the graph of $y=-2x^2+6x+5$. 17._____

18. For what value of k will the graph of $y=x^2-10x+k$ be tangent to the x-axis? 18._____

19. Solve for x: $\log 28-\log x=\log 7$ 19._____

20. Factor $3x^2-40x-28$. 20._____

21. If $f(a)=(a-1)^2$, find $f(a+1)$. 21._____

22. If x varies directly as y and inversely as z^2 and if $x=4$ when $y=9$ and $z=6$, find x when $y=0.25$ and $z=3$. 22._____

23. The first, second and third terms of an arithmetic progression are a, b and c, respectively. Express c in terms of a and b. 23._____

24. Solve for x: $8^{2x+3}=4^{4x+2}$ 24._____

25. Find to the *nearest hundredth* the value of $\sqrt[3]{0.587}$. 25._____

26. From a group of five chemists and four physicists a committee of five is to be chosen so as to include three chemists and two physicists. In how many different ways may the committee be chosen? 26._____

27. John, Dick and Paul are members of a club of ten boys from whom three are to be chosen by lot to receive free circus tickets. What is the probability that they will be the three chosen? 27._____

Directions (28-30): Indicate the correct completion for *each* of the following by writing the letter *a*, *b*, *c* or *d* on the line at the right.

28. The equation $x + 1 = \sqrt{x + 1}$ has (a) no roots (b) one and only one root (c) the roots 0 and -1 (d) the roots 0 and 1 28_____

29. If $K = p^N$, then $N =$ (a) $\dfrac{\log p}{\log K}$ (b) $\dfrac{\log K}{\log p}$ (c) $\log K - \log p$ (d) $\log p - \log K$ 29_____

30. If $\frac{2}{3}$ is a root of the equation $mx^3 + nx^2 + px + s = 0$ in which m, n, p and s are integers, then 3 must be a factor of (a) m (b) n (c) p (d) s 30_____

Part II

*Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

31. Between what two successive positive integers does a root of the equation $x^3 - 10x - 8 = 0$ lie? 31_____

32. A root of the equation $x^3 + 7x^2 - 5x - 15 = 0$ lies between 1 and 2. Find the root to the *nearest integer*. 32_____

33. Find a rational root of the equation $x^4 + x^3 - 5x^2 + x - 6 = 0$. 33_____

34. Given $x^2 + xy - 3y = 0$, express x in terms of y. 34_____

35. In how many different ways may the six letters of the word *summer* be arranged? 35_____

36. If ${}_nC_{n-2} = 28$, find the positive value of n. 36_____

37. Find to the *nearest tenth* $\log_2 14$. 37_____

38. The velocity of a moving body is given by the equation $v = 12t - 6t^2$. For what value of t is the velocity greatest? 38_____

39. Find the slope of the tangent to the graph of $y = 2x^3 + 3x^2 + 4$ at the point $(-2, 0)$. 39_____

*40. Write in determinant form an expression for the area of the triangle whose vertices are the points (2, 3), (—1, 6) and (1, —2). 40

*41. Evaluate the determinant:

$$\begin{vmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{vmatrix}$$ 41

42. Express in polar form the fourth root of 16 ($\cos 240° + i \sin 240°$) whose amplitude is smallest. 42

43. Solve the inequality: $2 - 4x > x + 7$ 43

44. Simplify completely: $1 - \dfrac{1}{1 + \dfrac{a}{1 - a}}$ 44

45. Write an equation of the line through (—1, 4) and perpendicular to the line whose equation is $2x - 3y = 8$. 45

Part III

*Answer ten questions from this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Questions marked * are based upon optional topics in the syllabus. Write your answer on the line at the right.*

46. The difference of the squares of two consecutive integers is d. Express the smaller of these integers in terms of d. 46

47. A cement walk x feet wide surrounds a rectangular plot whose dimensions are a and b. The area of the plot equals the area of the walk. Write, in terms of a, b and x, an equation which may be solved to find x in terms of a and b. 47

48. The units digit of a two-digit number is 3 and the tens digit is t. If the number with its digits reversed is divided by the sum of the digits, the quotient is t and the remainder is 6. Write an equation which may be used to solve for t. 48

49. Given the set of equations $x^2 + y^2 = 10$
$x + 3y = 0$
Find *one* set of values for x and y that satisfies the above equations. 49_____

50. One root of the equation $x^3 + x^2 - 10x - 12 = 0$ is -3. Write the quadratic equation which can be solved to find the other two roots. 50_____

51. Log $p^3 = 0.4290$. Find log $p\sqrt{p}$. 51_____

*52. Transform the equation $x^2 + y^2 = 2x$ from rectangular to polar coordinates. 52_____

53. Express $2(\cos 210° + i \sin 210°)$ in $a + bi$ form. 53_____

54. Express in polar form *one* of the roots of $x^5 + 32 = 0$. 54_____

Directions (55-60): Indicate the correct completion for each of the following by writing the letter *a*, *b*, *c* or *d* on the line at the right.

55. If $f(x) = Q(x)(x - a) + R$, then $f(a)$ is equal to (a) a (b) 0 (c) R (d) $Q(a) + R$ 55_____

56. If $K^x = y$, then K^{x+2} equals (a) $y + 2$ (b) $2y$ (c) y^2 (d) K^2y 56_____

57. If $R = 36P^{1.2t}$, then log R equals (a) log $36 + 1.2t \log P$ (b) $1.2t$ (log $36 +$ log P) (c) $36(1.2t \log P)$ (d) log $36 +$ (log $1.2 +$ log t) log P 57_____

58. If drawn on the same set of axes, the graphs of $y = 2$ and $y = x^2$ have (a) no points in common (b) one and only one common point which lies in the first quadrant (c) two common points, one in the first and the other in the fourth quadrant (d) two common points, one in the first and the other in the second quadrant 58_____

59. The radius of the circle $x^2 + 2x + y^2 - 10y = 10$ is (a) $\sqrt{10}$ (b) 5 (c) 6 (d) 10 59_____

*60. Transform the equation $r \sin \theta = \cot \theta$ from polar to rectangular coordinates. The graph of this equation is (a) a straight line (b) an ellipse (c) a parabola (d) a hyperbola 60_____