The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Wednesday, June 19, 1957 — 9:15 a.m. to 12:15 p.m., only

Note to teacher: These questions may be used in conjunction with the regular Regents examination in advanced algebra by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in advanced algebra. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Part I

Directions: Since questions 18, 19 and 20 on the examination in advanced algebra are not based on topics in the twelfth year syllabus, you may replace one or more of those by any of the following questions. Indicate any substitutions by labeling the answers *A*, *B* or *C*. [Write answers on the regular question paper opposite the questions you are replacing.]

A Write an equation of the circle one of whose diameters is the line segment joining the points (1, 2) and (7, 2).

B Express in polar form the product of $3(\cos 20° + i \sin 20°)$ and $5(\cos 40° + i \sin 40°)$.

C Solve for x the inequality $\frac{1}{2}x > 5 - 2x$.

Part II

Directions: The following questions, 30 and 31, are based upon optional topics of the twelfth year syllabus. Either 30 or 31, or *both*, may be used toward a total of *five* questions to be answered on part II of the examination in advanced algebra.

30 Each item in column I is represented by one and *only one* of the expressions in column II. On your answer paper list the numbers 1–4, and after *each* number write the *letter* from column II that indicates the corresponding correct polar form of that item. [10]

Column I	*Column II*
(1) $x^2 + y^2 = 8$	(*a*) $r(\cos\theta + \sin\theta) = 2$
(2) $x^2 + y^2 - 2x = 0$	(*b*) $r = 2\sqrt{2}$
(3) $2x = y$	(*c*) $r = 2\cos\theta$
(4) $x + y = 2$	(*d*) $r = 2\sin\theta$
	(*e*) $\tan\theta = 2$
	(*f*) $r\sin\theta = 2$

31 *a* Find by ***determinants*** the coordinates of P, the point of intersection of the straight lines whose equations are $2x - y = 0$ and $x + 2y = 10$. [4]

b Write in determinant form the area of the triangle whose vertices are $(-4, 5)$, $(0, 1)$ and point P found in answer to part *a*. [3]

c Find the area of the triangle by evaluating your answer to part *b*. [3]

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ADVANCED ALGEBRA

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Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Fill in the following lines:

Name of pupil..... Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Write an equation of the line passing through the point $(0, -4)$ and parallel to the line whose equation is $3x - 2y = 7$. 1.....

2 If $f(x) = x^{\frac{3}{2}} - 4x^0 + x^{-\frac{1}{2}}$, find $f(4)$. 2.....

3 Express $\frac{3}{3 - 2i}$ as a fraction with a real denominator. 3.....

4 Find n if ${}_nC_2 = 66$. 4.....

5 Write in *simplest form* the fifth term *only* in the expansion of $(a + \sqrt{a})^8$. 5.....

6 Find the arithmetic mean between $\frac{x+1}{x}$ and $\frac{x-1}{x}$. 6.....

7 Solve for x: $x^{\frac{2}{3}} = 0.25$ 7.....

8 Solve for x: $\log_3 5x = 2$ 8.....

9 Express the repeating decimal 0.171717...as a common fraction. 9.....

10 If $y = \frac{x}{x+1}$, express x as a function of y. 10.....

11 The product of two of the roots of the equation $x^3 - 3x^2 - 2x + 6 = 0$ is 3. Find the third root. 11.....

12 Ann has 6 different blouses and 3 different skirts. In how many ways may she select 3 blouses and 2 skirts to take on a weekend trip? 12.....

13 One letter is selected at random from the word *trip* and one letter is selected at random from the word *chop*. What is the probability that the same letter is selected from each word? 13.....

Directions (14–18): Indicate the correct completion for *each* of the following by writing the letter *a*, *b*, *c* or *d* on the line at the right.

14 If the graphs of the equations, $xy = 6$ and $y = x^2$, are drawn on the same set of axes, the number of points common to the two graphs is (*a*) one (*b*) two (*c*) three (*d*) four 14.....

15 If y varies inversely as the square of x and if x is doubled, y is multiplied by (*a*) $\frac{1}{4}$ (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) 4 15.....

16 An equation whose graph is tangent to the x-axis is (*a*) $x^2 + y^2 = 9$ (*b*) $y = x^2 - 6x$ (*c*) $y = x^2 - 6x + 9$ (*d*) $y = x^2 + 9$ 16.....

17 The equation $\sqrt{3x + 4} - x = 0$ has (*a*) 4 and -1 as its roots (*b*) 4 as its only root (*c*) -1 as its only root (*d*) neither 4 nor -1 as a root 17.....

18 The equation $x^3 + 3x - 2 = 0$ has (*a*) one positive and two negative roots (*b*) one positive and two imaginary roots (*c*) one negative and two positive roots (*d*) one negative and two imaginary roots 18.....

19 Write an equation whose roots are 2 less than the roots of the equation $x^3 + 2x - 5 = 0$. 19.....

20 Write an equation whose roots are one-half the roots of the equation $x^3 - 4x^2 + 4x + 8 = 0$. 20.....

Part II

Answer five questions from this part. Show all work.

21 Find, to the *nearest tenth*, the positive root of the equation $2x^3 + x^2 - 10x - 4 = 0$. [10]

22 Solve completely the equation $2x^4 - 5x^3 + 11x^2 - 3x - 5 = 0$. [10]

23 (1) Draw the graph of $y = -x^2 + 8$. [4]
(2) On the set of axes used in part (1) draw the graph of $x^2 + (y-4)^2 = 16$. [3]
(3) If the graphs of $x^2 + (y-4)^2 = 16$ and $y = -x^2 + K$ are drawn on the same set of axes, the graphs will have *no* point in common if $K =$ (*a*) 8 (*b*) 4 (*c*) 0 (*d*) -2. [3]

24 *a* If \$3,750, invested at interest which is compounded annually, amounts to \$5,000 in 10 years, what is the annual rate of interest to the *nearest tenth of a percent?* [Use the formula $A = P(1+r)^n$.] [7]
b Find x to the *nearest tenth*: $1.91^x = 54.2$ [3]

25 *a* State and prove the Remainder Theorem. [1, 5]
b If n is an integer, find the remainder when $x^{2n+1} - 3x^{2n} + 6$ is divided by $x + 1$. [2]
c For what value of K is $(x - 2)$ a factor of $x^3 + Kx^2 - 5x - 14$? [2]

26 In the equation $x^3 + 4x^2 + x + K = 0$, the sum of two of the roots equals the third root. Find the roots and the value of K. [10]

27 City A is 40 miles south of city B. Two cars start at the same time, one from A and the other from B, and travel toward each other at uniform rates. At the end of one-half hour they meet. If both cars had traveled north, the car from A would have overtaken the car from B at a point 140 miles north of B. Find the rate of the car from A. [7, 3]

*28 *a* Find the modulus of $5 - 12i$. [2]
b Find the amplitude of $-\sqrt{2} + i\sqrt{6}$. [2]
c Express $2(\cos 210° + i \sin 210°)$ in the form $a + bi$. [3]
d Write in polar form one of the imaginary roots of $x^3 + 8 = 0$. [3]

*29 Given $y = x^3 - 3x^2 + 5$.
a Find the derivative of y with respect to x. [2]
b Find the coordinates of the maximum point. [2]
c Find the coordinates of the minimum point. [2]
d Find the coordinates of the point of inflection. [2]
e *Sketch* the graph of $y = x^3 - 3x^2 + 5$. [2]

* These questions are based upon the optional topics in the syllabus.

FOR TEACHERS ONLY

AA

INSTRUCTIONS FOR RATING
ADVANCED ALGEBRA
and
TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. Do not allow credit if answers to questions 19 and 20 are not expressed as equations. For questions 14–18, allow credit if the pupil has written the correct answer instead of the letter *a, b, c* or *d*.

(1) $3x - 2y = 8$

(2) $4\frac{1}{2}$

(3) $\frac{3(3+2i)}{13}$

(4) 12

(5) $70a^6$

(6) 1

(7) $\frac{1}{8}$ *or* 0.125

(8) $1\frac{4}{5}$

(9) $\frac{17}{99}$

(10) $\frac{y}{1-y}$

(11) -2

(12) 60

(13) $\frac{1}{16}$

(14) a

(15) a

(16) c

(17) b

(18) b

(19) $x^3 + 6x^2 + 14x + 7 = 0$

(20) $x^3 - 2x^2 + x + 1 = 0$

Twelfth Year Mathematics (Advanced Algebra)

A $(x-4)^2 + (y-2)^2 = 9$
B $15(\cos 60° + i \sin 60°)$
C $x > 2$