ADVANCED ALGEBRA

Wednesday, June 20, 1956—9:15 a.m. to 12:15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive **2½** *credits. No partial credit will be allowed.*

1. Express $\frac{2}{1-i}$ in the form $a + bi$. 1.....

2. Write an equation of the line which passes through the origin and is parallel to the line $2y = 3x - 6$. 2.....

3. What is the value of $\log_b 1$ if b is a finite positive number ***not*** equal to 0 or 1? 3.....

4. Solve the following set of equations for a:

$$\frac{2}{a} + \frac{3}{b} = -1$$

$$\frac{1}{a} - \frac{3}{b} = 4$$

4.....

5. If $x - 1$ is a factor of $x^{10} - kx^7 - 3$, find k. 5.....

6. Write in simplest form the fifth term in the expansion of $(e^x + e^{-x})^8$. 6.....

7. If the graphs of $y^2 = 4x$ and $x^2 = 4y$ are drawn on the same set of axes, in how many points do the graphs intersect? 7.....

8. Find the sum of the infinite geometric progression 4, -2, 1, ... 8.....

9. A bag contains 6 red and 4 white balls. If 2 balls are drawn at random from the bag, what is the probability that both will be white? 9.....

10. In how many ways can 5 men—*A*, *B*, *C*, *D* and *E*—be seated on a bench if *A* and *B* must occupy end positions? 10.....

Directions (11-14): Indicate the correct completion for *each* of the following by writing the letter *a*, *b* or *c* on the line at the right.

11. If the complex number $2x + iy$ is equal to the complex number $6 + 5i$, then the value of x is (*a*) 2 (*b*) 3 (*c*) 6 11.....

12. The graphs of the equations $x - 2y = 4$ and $2x - 4y = 8$ (*a*) are parallel (*b*) intersect (*c*) coincide 12.....

13. As the value of c increases, the graph of the equation $y = x^2 + c$ moves (*a*) in the positive direction of the x-axis (*b*) in the positive direction of the y-axis (*c*) in the negative direction of the x-axis 13.....

14. The equation $f(x) = 0$ has no real roots between $x = 1$ and $x = 2$ if both $f(1)$ and $f(2)$ are positive. This statement is (*a*) always true (*b*) sometimes true (*c*) never true 14.....

15. Solve for x: $2^{x+2} = (\frac{1}{2})^x$ 15.....

16. Find the product of the four fourth roots of 1. 16.....

17. If w varies directly as a and inversely as d, and if $w = 8$ when $a = 2$ and $d = 3$, find w when $a = 1$ and $d = 6$. 17.....

18. What is the rational fractional root of the equation $5x^3 + 6x^2 + 6x + 1 = 0$? 18.....

19. Find the sum of the roots of the equation whose roots are the roots of the equation $x^3 + 6x^2 - 5x + 7 = 0$ each increased by 2. 19.....

20. Write an equation whose roots are the roots of the equation $x^3 + 6x^2 - 5x + 7 = 0$ each multiplied by 2. 20.....

Part II

Answer five questions from this part. Show all work.

21. Find to the *nearest tenth* the positive root of the equation $x^3 - 4x^2 + 14x - 22 = 0$. [10]

22. Solve the equation $x^4 - 3x^3 + x^2 - x - 6 = 0$. [10]

23. Using logarithms, find to the *nearest hundredth*
    a. $\log_5 1.71$ [4]
    b. $\dfrac{28.4}{\sqrt[3]{-0.648}}$ [6]

24. *a.* Draw the graph of $(x - 3)^2 + (y + 4)^2 = 25$. [4]
    b. On the same set of axes used in answer to part *a*, draw the graph of $3x - y - 8 = 0$. [2]
    c. Find the length of the line segment whose end points are the intersections of the graphs drawn in parts *a* and *b*. [Answer may be left in radical form.] [4]

25. *a.* Derive the formula for the roots of the quadratic equation $ax^2 + bx + c = 0$ in terms of the coefficients a, b and c. [7]
    b. Solve for x: $rx^2 - 3rx + r + 2 = 0$. [3]

26. A boat travels in still water at the rate of 5 miles per hour with a load and 15 miles per hour without a load. If the boat travels upstream a distance of 81 miles with a load and makes the return trip without a load, taking 48 hours for the entire trip including 3 hours for unloading, what is the rate of the river current? [10]

27. A man has 20 machines that work automatically when started. Each machine produces 6 yards of material per hour. The man starts the first machine at 8 a.m. and each of the others at 5-minute intervals thereafter. How many yards of material will be completed at 10 a.m.? [6, 4]

*28. *a.* Write equations of the tangents to the graph of $y = x^2 + 2x - 3$ at the points where the graph cuts the x-axis. [6]
    b. Show that these tangents intersect at a point on the axis of symmetry of the graph of $y = x^2 + 2x - 3$. [4]

*29. *a.* Find the difference between the amplitudes (angles) of $-1 + i\sqrt{3}$ and $1 + i$. [4]
    b. Express in polar form $4 - 3i$. [Give the amplitude to the *nearest degree*.] [3]
    c. Find, in polar form, the fifth root of 32 whose amplitude is between 90° and 180°. [Give the amplitude to the *nearest degree*.] [3]

* These questions are based upon the optional topics in the syllabus.

The University of the State of New York

327TH HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Wednesday, June 20, 1956—9:15 a.m. to 12:15 p.m., only

Note to teacher: These questions may be used in conjunction with the regular Regents examination in advanced algebra by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in advanced algebra. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Part I

Directions: Since questions 18, 19 and 20 on the examination in advanced algebra are not based on topics in the twelfth year syllabus, you may replace one or more of those by any of the following questions. Indicate any substitutions by labeling the answers *A, B* or *C*. [Write answers on the regular question paper opposite the questions you are replacing.]

A What is the average rate of change of $y = 2x^2 - x + 1$ with respect to x over the interval from $x = 2$ to $x = 5$?

B Write an equation of the line which is perpendicular to the line $3x - 2y = 5$ and which passes through the point $(0, 2)$.

C Solve for x the inequality $x + \frac{x}{2} > 5$.

Part II

Directions: The following questions, 30 and 31, are based upon optional topics of the twelfth year syllabus. Either 30 or 31, or *both,* may be used toward a total of *five* questions to be answered on part II of the examination in advanced algebra.

30 *a* Transform the equation $xy = 12$ from rectangular coordinates to polar coordinates. [4]

b Find the coordinates of the points of intersection of the circles $r = a$ and $r = 2a \cos \theta$ for values of θ between 0 and 2π. [6]

31 *a* Using a determinant, find the area of a triangle whose vertices are $(2, 3)$, $(-6, 4)$, $(1, -1)$. [5]

b Write an equation of the straight line through the points $(2, 3)$ and $(1, -1)$

(1) in determinant form [3]

(2) in the form $ax + by + c = 0$ [2]

FOR TEACHERS ONLY

AA

INSTRUCTIONS FOR RATING
ADVANCED ALGEBRA
and
TWELFTH YEAR MATHEMATICS
12A (Advanced Algebra)

Wednesday, June 20, 1956 — 9:15 a.m. to 12:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 20 is not expressed as an equation. For questions 11–14, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

(1) $1 + i$
(2) $2y = 3x$
(3) 0
(4) 1
(5) -2
(6) 70
(7) 2
(8) $2\frac{2}{3}$
(9) $\frac{2}{15}$
(10) 12
(11) *b*
(12) *c*
(13) *b*
(14) *b*
(15) -1
(16) 1
(17) 2
(18) $-\frac{1}{5}$
(19) 0
(20) $x^3 + 12x^2 - 20x + 56 = 0$

Twelfth Year Mathematics (Advanced Algebra)

A 13
B $2x + 3y = 6$
C $x > 3\frac{1}{3}$