ADVANCED ALGEBRA

Wednesday, June 22, 1955—9.15 a.m. to 12.15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1. Find the slope of a line parallel to the line whose equation is $7y = 3x + 5$. 1.....

2. Write an equation of the straight line which passes through the origin and through the point (3, 1). 2.....

3. For what positive value of k is the graph of $x = k$ tangent to the graph of $x^2 + y^2 = 25$? 3.....

4. Is the repeating decimal 0.737373 . . . a rational number? [Answer *yes or no.*] 4.....

5. Find the sum of the infinite progression 1, ½, ¼, ⅛, . . . 5.....

6. Express $\frac{1-i}{i}$ in the form $a + bi$. 6.....

7. If y varies inversely as the square of x and if $y = 2$ when $x = 3$, find the positive value of x when $y = 3$. [Answer may be left in radical form.] 7.....

8. If $f(r) = 4\pi r^2$, find $f(\frac{1}{2})$. [Answer may be left in terms of π.] 8.....

9. Find the value of $\log_2 \frac{1}{4}$. 9.....

10. Find to the *nearest thousandth* the cube root of .0731. 10.....

11. If ${}_nP_2 = 12$, find n. 11.....

12. A bag contains 3 white balls and 9 black balls. If one ball is drawn from the bag, what is the probability that it will be white? 12.....

13. Solve the equation $4^{-r} = 2^{5-r}$ for r. 13.....

14. Find the remainder when $y^6 - y^5 - 1$ is divided by $y + 1$. 14.....

15. Two of the roots of an equation with real and rational coefficients are $3i$ and $2 + \sqrt{3}$. What is the lowest possible degree of the equation? 15.....

16. Find the product of the roots of the equation $x^3 - 3x^2 - 4x - 1 = 0$. 16.....

17. Find the rational root of the equation $2x^3 + x^2 + x - 1 = 0$. 17.....

18. How many imaginary roots does the equation $x^3 + 3x - 4 = 0$ have? 18.....

19. Write an equation whose roots are the roots of the equation $y^3 + 2y^2 - 4y + 8 = 0$ each multiplied by ½. 19.....

20. Write an equation whose roots are the roots of the equation $y^3 + 2y^2 - 4y + 8 = 0$ each decreased by 2. 20.....

Part II

Answer five questions from this part. Show all work.

21. Solve the equation $x^4 - 6x^3 + 10x^2 - 6x + 9 = 0$. [10]

22. Find to the *nearest tenth* the positive root of the equation $x^3 + 5x - 10 = 0$. [10]

23. *a.* Using values of x from $x = 0$ to $x = 6$, draw the graph of $y = -x^2 + 6x - 6$. [3]

    b. On the same set of axes used in answer to *a*, draw the graph of $(x - 3)^2 + y^2 = 9$. [3]

    c. From the graphs drawn in answer to parts *a* and *b*, find *all* real roots of the following system of equations. [Express to the *nearest tenth* any values of x and y that are not integers.] [4]

$$y = -x^2 + 6x - 6$$
$$(x - 3)^2 + y^2 = 9$$

24. *a.* How many different committees consisting of 2 seniors and 3 juniors can be formed from a group of 5 seniors and 8 juniors? [4]

    b. How many odd numbers of three digits each can be formed from the digits 1, 2, 4, 6 if repetition of digits is allowed? [3]

    c. A signalman has 5 different-colored flags, one of which is red. If they are arranged at random on a vertical staff, what is the probability that the red one will be the middle flag? [3]

25. The first three terms of a geometric progression are x, 2, and y. If the first of these terms is multiplied by 2, the second is left unchanged and the third is diminished by 5, the resulting numbers taken in that order form an arithmetic progression. Find the two values of x and the corresponding two values of y which satisfy these conditions. [6, 4]

26. *a.* Find to the *nearest hundredth* the value of x in the equation $2^x = 1.5$ [4]

    b. Using logarithms, find to the *nearest thousandth* the value of the third term of the expansion of $(x + y)^7$ if $x = 1/6$ and $y = 5/6$. [6]

27. An airplane whose average speed is r miles per hour travels d miles in a certain length of time. If this speed were decreased by s miles per hour, the plane would take one hour longer to travel the same distance d.

    a. Write an equation showing the relationship between r, s and d. [5]

    b. Solve for r the equation obtained in part *a*. [4]

    c. May each solution for r obtained in part *b* be retained as an admissible value of r? [Answer *yes* or *no*.] [1]

*28. A body moves according to the law $s = t^2 - 3t + 8$, where s is the distance in feet and t is the time in seconds.

    a. Find its average velocity in feet per second for the time interval from 1.5 seconds to 2 seconds. [4]

    b. Find its velocity in feet per second when $t = 2$. [4]

    c. Find its acceleration in feet per second per second. [2]

*29. *a.* Write the number $2 + 3i$ in polar form. [The modulus may be left in radical form and the amplitude (angle) given to the nearest degree.] [4]

    b. If the number $10 (\cos 333^\circ + i \sin 333^\circ)$ is written in the form $a + bi$, find a to the *nearest tenth*. [3]

    c. Write in polar form the *three* roots of the equation $x^3 - 64 = 0$. [3]

* This question is based upon one of the optional topics in the syllabus. Either 28 *or* 29, or *both*, may be used for a total of *five* questions to be answered from part II.

The University of the State of New York

324TH HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12A (Advanced Algebra)

Wednesday, June 22, 1955 — 9.15 a.m. to 12.15 p.m., only

Note to teacher: These questions may be used in conjunction with the regular Regents examination in advanced algebra by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in advanced algebra. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Part I

Directions: Since questions 18, 19 and 20 on the examination in advanced algebra are not based on topics in the twelfth year syllabus, you may replace one or more of those by any of the following questions. Indicate any substitutions by labeling the answers *A, B* or *C*. [Write answers on the regular question paper opposite the questions you are replacing.]

A Find the distance from the origin to the midpoint of the line segment whose end points are (2, 7) and (4, 1).

B Write an equation of the circle whose center is the point (0, 2) and which is tangent to the x-axis.

C The inequality $\frac{5x}{2} + \frac{2x}{3} > 19$ is satisfied by *all* values of x greater than a certain number n and by no other values. Find n.

Part II

Directions: The following questions, 30 and 31, are based upon optional topics of the twelfth year syllabus. Either 30 or 31, or *both,* may be used toward a total of *five* questions to be answered on part II of the examination in advanced algebra.

30 *a* Transform the equation $r = \frac{\tan\theta}{\cos\theta}$ from polar coordinates to rectangular coordinates and identify (give the name of) the curve. [4, 2]

b Which *two* of the following points lie on the graph of the equation $r = \frac{\tan\theta}{\cos\theta}$? [4]

(*A*) $(2\sqrt{3}, 60°)$ (*C*) $(-\frac{2}{3}, 210°)$

(*B*) $(-\sqrt{2}, 135°)$ (*D*) $(\sqrt{2}, -45°)$

31 Write the numbers 1–4 on your answer paper and after *each* number indicate the correct completion for each of the following by writing the letter *a*, *b*, or *c*.

(1) The abscissa of the point of intersection of the straight lines whose equations are $3x + 2y = 8$ and $5x + 4y = 11$ is

(*a*) $\dfrac{\begin{vmatrix} 3 & 8 \\ 5 & 11 \end{vmatrix}}{2}$ (*b*) $\dfrac{\begin{vmatrix} 8 & 3 \\ 11 & 5 \end{vmatrix}}{2}$ (*c*) $\dfrac{\begin{vmatrix} 8 & 2 \\ 11 & 4 \end{vmatrix}}{2}$ [3]

(2) The straight line whose equation is $\begin{vmatrix} x & y & 1 \\ 2 & 3 & 1 \\ -2 & 4 & 1 \end{vmatrix} = 0$

passes through the point (*a*) (2, 3) (*b*) (4, −2) (*c*) (3, 4) [3]

(3) The y-intercept of the straight line whose equation is $\begin{vmatrix} x & y \\ -2 & 4 \end{vmatrix} = 6$ is

(*a*) 4 (*b*) 3 (*c*) −3 [2]

(4) The graph of the equation $\begin{vmatrix} x^2 & y^2 \\ 2 & 1 \end{vmatrix} = 4$ is (*a*) a circle (*b*) an ellipse

(*c*) a hyperbola [2]

FOR TEACHERS ONLY

AA

INSTRUCTIONS FOR RATING
ADVANCED ALGEBRA
and
TWELFTH YEAR MATHEMATICS
12A (Advanced Algebra)

Wednesday, June 22, 1955 — 9.15 a.m. to 12.15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit.

(1) $\frac{3}{7}$
(2) $x = 3y$
(3) 5
(4) yes
(5) 2
(6) $-1-i$
(7) $\sqrt{6}$
(8) π
(9) -2
(10) .418
(11) 4
(12) $\frac{1}{4}$
(13) -5
(14) 1
(15) 4th
(16) 1
(17) $\frac{1}{2}$
(18) 2
(19) $x^3 + x^2 - x + 1 = 0$
(20) $x^3 + 8x^2 + 16x + 16 = 0$

Twelfth Year Mathematics (Advanced Algebra)

A 5
B $x^2 + (y-2)^2 = 4$
C 6