The University of the State of New York

318TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, June 17, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find to the *nearest tenth* the real root of the equation $x^3 - 2x^2 + x - 7 = 0$ [10]

22 Solve the equation $3x^4 + 8x^3 + 9x^2 + 2x - 2 = 0$ [10]

23 a Find to the *nearest tenth* the value of $\dfrac{3s^{1.5}}{\sqrt{x}}$ when $s = 8.43$, $x = 3.95$ and $r = 2.4$ [7]

   b Solve for x to the *nearest tenth*: $2^x = 90$ [3]

24 a Draw the graph of $(x - 3)^2 + y^2 = 16$.   [4]

   b On the same set of axes as used in a draw the graph of $x^2 - y^2 = 9$.   [4]

   c From the graphs made in answer to a and b, estimate to the *nearest tenth* the real values of x and y that satisfy both equations.   [2]

25 a Prove that $_nC_r = {}_nC_{n-r}$ [7]

   b Find the value of $_{45}C_{42}$ [3]

26 Two stations, A and B, are 300 miles apart. At a certain time a passenger train leaves A for B, and at the same time a freight train leaves B for A. The two trains meet at a point 100 miles from B. Had the speed of the passenger train been 10 miles an hour greater, it would have reached B 9 hours before the freight train reached A. Find the speed of each train.   [10]

27 Sand and gravel have been mixed in two separate piles. In the first pile the ratio of sand to gravel is 1:1, and in the second pile the ratio of sand to gravel is 1:4. A third pile, in which the ratio of sand to gravel is 1:3, is to be formed from the first two piles. If the third pile is to contain 15 cubic yards, how many cubic yards must be taken from *each* of the first two piles?   [10]

[1]   [OVER]

*28 a Find the modulus of $8 + 15\ i$.   [1]

$\quad b$ Find to the *nearest degree* the amplitude (angle) of $-1 + 3i$.   [3]

$\quad c$ Express $6(\cos 240° + i \sin 240°)$ in $a + bi$ form.   [3]

$\quad d$ Write in polar form the root of $x^5 - 32 = 0$ that when represented graphically lies in the fourth quadrant.   [3]

*29 For *each* of the following find the derivative of y with respect to x:

$\quad$ (a) $y = x^3 - 2x^2 + 4x - 5$ [2]

$\quad$ (b) $y = (x^2 - 5)^6$ [2]

$\quad$ (c) $y = \sqrt{3x^2 - 1}$ [3]

$\quad$ (d) $y = \dfrac{x^2}{3x + 1}$ [3]

* This question is based upon one of the optional topics in the syllabus.

To High School Principals and Mathematics Teachers:

IMPORTANT NOTICE

ADVANCED ALGEBRA REGENTS EXAMINATION

Wednesday, June 17, 1953 — 9.15 a. m. to 12.15 p. m.

At the beginning of this examination, please announce to the candidates that the fraction in question 23 *a* (Part II) should appear as follows:

$$\frac{3s^{1.5}}{\sqrt[r]{x}}$$

When the examination papers are opened on Wednesday, June 17th at 9.15 a. m., you will observe that only a vestige of the radical index "*r*" appears on the printed copy.

Peter P. Muirhead

Peter P. Muirhead
Chief

Fill in the following lines:

Name of pupil...Name of school...

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express $\dfrac{1+\sqrt{-2}}{2+\sqrt{-2}}$ as a fraction with a real denominator.   1................

2 Write the equation of the straight line parallel to the line $3x + 2y = 7$ and passing through the point $(4, -1)$.   2................

3 Find the value of k if $x^3 - 3x^2 + kx - 9$ is exactly divisible by $x - 3$.   3................

4 Express $.4343\ldots$ as a common fraction.   4................

5 Transform the equation $2x^3 - 3x^2 + 1 = 0$ into an equation whose roots are those of the original equation each multiplied by 2.   5................

6 Transform the equation $x^3 + x^2 - 5x + 2 = 0$ into an equation whose roots are those of the original equation each decreased by 2.   6................

7 How many imaginary roots has the equation $x^5 + x^3 + 1 = 0$?   7................

8 Find the value of $x^{\frac{3}{2}} - 2x^0 + x^{-1}$ if $x = 4$.   8................

9 Write an equation with real coefficients and of lowest possible degree, two of whose roots are 2 and $1 - i$.   9................

10 Find the positive value of k that will make the roots of $x^2 - 2kx + k + 2 = 0$ equal.   10................

11 Using k as the constant of variation, write as an equation: a varies directly as b and inversely as the square of c.   11................

12 Two of the roots of the equation $x^3 + px + q = 0$ are -4 and 1. Find the third root.   12................

13 The coefficient of the third term in the expansion of $(x + y)^n$, where n is positive, is 6. Find the value of n.   13................

[3]   [OVER]

14 If $y^2 - xy + 5 = 0$, express y as a function of x. 14................

15 If $\log_a \sqrt[3]{7} = 0.9358$, find the $\log_a 7$. 15................

16 A fisherman has 10 different trout flies. How many selections of two flies each can he make? 16................

17 How many different integers between 300 and 600 can be made with the digits 2, 3, 4, 5 and 6 if no digits are repeated? 17................

18 If the letters of the word *pencil* are arranged at random, what is the probability that the first and last letters will be vowels? 18................

Directions (19, 20): Indicate the correct answer for *each* of the following questions by writing on the line at the right the letter *a, b* or *c*.

19 If the graphs of $4x^2 + y^2 = 16$ and $2x^2 - 2 = y$ are drawn on the same set of axes, how many points will they have in common? (*a*) 0 (*b*) 2 (*c*) 4 19......

20 Which one of the following is a rational integral function in x?

(*a*) $x^2 - \dfrac{2}{x} - 7$ (*b*) $3x^2 - \sqrt{x} - 5$ (*c*) $\frac{1}{2}x^2 - x\sqrt{3} - 1$ 20......