The University of the State of New York

312TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, June 20, 1951 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find, to the *nearest tenth*, the real root of the equation $2x^3 - 6x - 7 = 0$. [10]

22 Solve the equation $3x^4 + 4x^3 + 2x^2 - x - 2 = 0$. [10]

23 Using the formula $P_1 = P_2\left(\frac{V_2}{V_1}\right)^{1.41}$ find P_1 to the *nearest tenth* when $P_2 = 18.6$, $V_1 = 37$ and $V_2 = 123$. [10]

24 *a* Using the values $x = 1, 3, 4, 5, 10$, draw the graph of $y = \log_{10} x$ [6]

b On the axes used in answer to *a*, draw the graph of $y = .8$ [2]

c From the graphs made in answer to *a* and *b*, estimate, to the *nearest tenth*, the value of x that satisfies the equation $\log_{10} x = .8$ [2]

25 A man has a 20-gallon container filled with a solution that is 15% alcohol. How many gallons should he drain off and replace by a 40% solution in order to make the 20 gallons a 24% solution? [7, 3]

26 A plane has enough gasoline to fly for t hours. The plane must fly from its base directly into a steady wind whose velocity is v miles per hour and then return over the same course. If the speed of the plane in still air is r miles per hour, express in terms of t, v and r the distance from its base that the plane can fly. [6, 4]

27 *a* State and prove the Remainder Theorem. [6]

b When $x^3 + 2x^2 + mx + n$ is divided by $x - 2$, the remainder is 11; when it is divided by $x + 3$, the remainder is 41. Find m and n. [4]

*28 *a* Which two of the following three numbers have the same modulus?

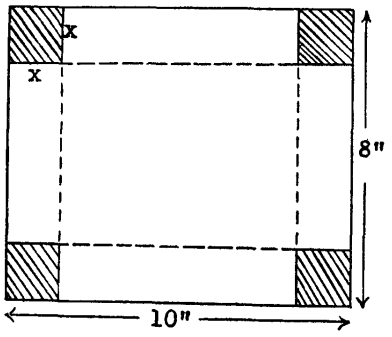
(1) $3 - 4i$ (2) $6 + 7i$ (3) $2 + 9i$ [2]

b Which two of the following three numbers have the same amplitude?

(1) $1 + 2i$ (2) $-1 - 2i$ (3) $2 + 4i$ [3]

c Express $-3i$ in the form $\rho(\cos\theta + i\sin\theta)$. [5]

*29 An open box is made from a rectangular piece of tin by cutting out the equal shaded squares at the four corners and folding along the dotted lines as indicated on the accompanying diagram.



Using the dimensions shown on the diagram,

a express the volume V of the box as a function of x [4]

b find, to the *nearest tenth* of an inch, the value of x for which V is a maximum [6]

* This question is based upon one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil..... Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express $\frac{3}{3-i}$ as an equivalent fraction with a real denominator. 1.....

2 Write an equation of the line passing through the point (5, —2) and parallel to the line $y = 2x - 5$. 2.....

3 Find the coordinates of the point of intersection of the lines $x - 2 = 0$ and $y + 5 = 0$. 3.....

4 When the graphs of $y = x^2 + 1$ and $y = x^2 + x$ are plotted on the same set of axes, how many points do the graphs have in common? 4.....

5 Write in its *simplest form* the third term in the expansion of $(x^2 - x^{-3})^8$ 5.....

6 Is the following statement true or is it false?
If z varies inversely as y and y varies directly as x, then z varies inversely as x. 6.....

7 If $f(x) = 3x^2 + 2\sqrt{x} + 4$, find $f(0)$. 7.....

8 Find the remainder when $x^7 - x$ is divided by $x - 2$. 8.....

9 Solve for x the equation $2^{1-2x} = \frac{1}{4}$ 9.....

10 What is the lowest possible degree of an equation with real coefficients, three of whose roots are 0, 2 and $1 + i$? 10.....

11 Find the positive value of k for which the equation $kx^2 + 9x + 4k = 0$ will have equal roots. 11.....

12 Write an equation whose roots are the roots of $x^3 - 5x^2 + 2x - 1 = 0$ each decreased by 3. 12.....

13 The integer N is a twelve digit number. Find the characteristic of $\log \sqrt[3]{N}$ 13.....

14 Solve for x to the *nearest tenth*: $2^x = 24$ 14.....

[OVER]

15 A boy owns a motion picture projector, 8 different animated cartoon films, 4 different western films and 5 different travel films. How many different groups of films can be selected for programs if each group is to consist of two different cartoons, one western and one travel film? 15.....

16 If the letters a, b, c, d and e are arranged in any order, find the probability that c will be the fifth letter from the left. 16.....

Directions (questions 17–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter a, b or c.

17 Which one of the following equations has exactly one positive root?
$(a)\ x^4 + 2x^3 + x + 1 = 0$ $(b)\ x^4 - x - 1 = 0$
$(c)\ x^4 + 2x^2 - 3x + 1 = 0$ 17.....

18 If the sum of the roots of $x^3 + px^2 + qx + r = 0$ is equal to the product of the roots, then $(a)\ p = q$ $(b)\ p = r$ $(c)\ q = r$ 18.....

19 The value of $\log_b x$ is given by $(a)\ \dfrac{\log_a x}{\log_a b}$ $(b)\ \dfrac{\log_a x}{\log_b a}$ $(c)\ \dfrac{\log_b x}{\log_b a}$ 19.....

20 Which one of the following is a rational, integral function? $(a)\ x^{\frac{2}{3}} - x - 8$
$(b)\ 4x^3 - x\sqrt{3} - 5$ $(c)\ 2x^{-1} + x - 3$ 20.....