The University of the State of New York

309TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, June 21, 1950 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find, to the *nearest tenth*, the real root of the equation $x^3 - 2x^2 + x - 1 = 0$. [10]

22 Solve the equation $x^4 - 5x^3 + 7x^2 + 3x - 10 = 0$. [10]

23 When a load of T tons is put on a cast-iron strut whose length is L feet and whose diameter is D inches, the minimum diameter of the strut necessary to carry the load without crushing is given by the formula $D = \sqrt[3.6]{\frac{T\,L^{1.7}}{50}}$ Find D to the *nearest tenth of an inch* when $T = 6.3$ tons and $L = 20$ feet. [10]

24 *a* Draw the graph of $y = 2^{x+1}$ from $x = -3$ to $x = 2$. [5]

b On the same axes as used in answer to *a*, draw the graph of $y = 4 - x^2$ from $x = -2$ to $x = 2$. [3]

c From the graphs made in answer to *a* and *b*, estimate, to the *nearest tenth*, the roots of the equation $4 - x^2 = 2^{x+1}$. [2]

25 A boat is anchored 3 miles from a straight shore. A camp C is located on the shore 10 miles from A, the point on the shore nearest the boat. A man walks a certain distance from C toward A at 3 miles an hour. He then rows straight to the boat at 4 miles an hour. If the entire trip took him $3\frac{1}{4}$ hours, how many hours did he walk? [6, 4]

26 The circle whose equation is $x^2 + y^2 + ax + by + c = 0$ passes through the points (0, 0), (6, 3) and (3, —6). Find a, b and c. [10]

27 *a* Prove that $\log_b x = \frac{\log_a x}{\log_a b}$ [5]

b If three positive numbers, a, b and c, are in geometric progression, prove that log a, log b and log c are in arithmetic progression. [5]

[OVER]

*28 *a* Find the modulus of $-1 + \frac{4}{3}i$. [2]

b Find, to the *nearest degree,* the amplitude (angle) of $-1 + \frac{4}{3}i$. [3]

c Express $4(\cos 135° + i \sin 135°)$ in the form $a + bi$. [3]

d Write *one* of the imaginary roots of the equation $x^4 - 1 = 0$ in polar form. [2]

*29 Given $f(x) = \frac{x^3}{3} + x^2 - 3x - 5$.

a Find the first derivative of $f(x)$. [2]

b Find the coordinates of the maximum and of the minimum point. [4]

c Find the coordinates of the point of inflection. [2]

d *Sketch* the graph of $y = f(x)$. [2]

* This question is based upon one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil .....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Express $\frac{5+i}{3-2i}$ in the form $a + bi$. 1.....

2 Express the repeating decimal 0.434343 ... as a common fraction. 2.....

3 Using k as the constant of variation, write an equation expressing the relationship: R varies directly as L and inversely as the square of D. 3.....

4 Write in simplest form the *third* term of the expansion $(x^2 - \frac{1}{x})^5$. 4.....

5 Write an equation of the straight line passing through the point $(6, -2)$ and parallel to the line $y = 3x - 5$. 5.....

6 If the graphs of $x^2 + 9y^2 = 25$ and $y = x^2$ are drawn on the same set of axes, how many points do the graphs have in common? 6.....

7 Solve for x: $9^x = \frac{1}{3}$ 7.....

8 If $f(x) = x^{\frac{2}{3}} - 3x^0$, find the value of $f(8)$. 8.....

9 Find the remainder when $x^{25} + 2$ is divided by $x - 1$. 9.....

10 Find the sum of the roots of the equation $5x^4 - 6x^3 - 1 = 0$. 10.....

11 Write an equation, with rational coefficients and of lowest degree possible, two of whose roots are 1 and $3 + \sqrt{2}$. 11.....

12 In which quadrant is the graph of the complex number $-2 + 3i$ located? 12.....

13 Transform the equation $x^3 + 4x^2 - 3x - 1 = 0$ into an equation whose roots are those of the original equation, each *increased* by 1. 13.....

14 Transform the equation $x^3 - 15x^2 + 9x - 108 = 0$ into an equation whose roots are those of the original equation, each *divided* by 3. 14.....

15 Solve for x to the *nearest tenth*: $10^x = 40$ 15.....

16 How many different juries, each of 12 people, can be selected from a panel of 15 people? 16.....

17 How many code words, each of five different letters, can be formed from the letters a, b, c, d and e, if the first and last letters are to be vowels? 17.....

18 If the probability that an event will happen is $\frac{a}{b}$, find the probability that the event will not happen. 18.....

19 Find the abscissa of the point where the graph of $y^2 = 8 - x^3$ crosses the x-axis. 19.....

20 The equation of the axis of symmetry of the parabola $y = x^2 + px + q$ is $x = 3$. Find the value of p. 20.....