ADVANCED ALGEBRA

Wednesday, June 23, 1948 — 9.15 a. m. to 12.15 p. m., only

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1. Express $\dfrac{2-3i}{3-2i}$ as a fraction with a real denominator. 1........

2. Write the equation of the straight line parallel to the line $2x - 3y = 4$ and passing through the origin. 2........

3. Given the equation $kx^2 - 4x + k = 0$, find the positive value of k which will make the roots equal. 3........

4. If $f(x) = x^2 - 3x + 2$, find $f(2a)$. 4........

5. Given $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$; solve for R_2. 5........

6. Find the remainder when $x^{75} - 2$ is divided by $x + 1$. 6........

7. Write the equation of lowest degree possible with real coefficients which has for two of its roots 1 and $2 + 5i$. 7........

8. Find the sum of the roots of the equation
$2x^3 - x^2 + 8 = 0$ 8........

9. Transform the equation $2x^3 - x^2 + 8 = 0$ into an equation whose roots are the negatives of the roots of the given equation. 9........

10. The equation $2x^3 - x^2 + 8 = 0$ has one and only one negative root. [Answer *true* or *false*.] 10........

11. Which of the following is a rational integral equation in x? $(a)\, x^2 - \dfrac{1}{x} + 7 = 0$ $(b)\, x^2 - x^{-\frac{1}{2}} + 7 = 0$

$(c)\, x^2 - \frac{1}{2}x - \sqrt{7} = 0$ 11........

12. Solve $3^{x+4} = 9^x$ 12........

13. Find the x-intercept of the graph of $y = \log_{10} x$ 13........

14. If $y = -ax^2 + bx + c$, in which a, b and c are positive integers, then there is always a value of x which will make y negative. [Answer *true* or *false*.] 14........

15. Find the logarithm of $(100)^{1.4}$ 15........

16. Express $.23333\ldots$ as a common fraction. 16........

17. Express in simplest form the *third* term of the expansion of $(2x^3 - \dfrac{5}{3})^6$ 17........

18. If the probability of a man's living for at least 10 years is .8, find the probability of his dying within these 10 years.   18...........

19. If $_nC_2 = 45$, find n.   19...........

20. How many numbers of 3 digits each can be written with the digits 1, 5, 6, 8, 9 if repetition is allowed?   20...........

Part II

Answer five questions from part II.

21. Find to the *nearest tenth* the real root of the equation
$$x^3 + 3x - 20 = 0 \quad [10]$$

22. Solve the equation $2x^4 + 9x^3 + 15x^2 + 14x - 12 = 0$ [10]

23. Given $V = \dfrac{\pi e r^3}{540}$; if $V = 146$ and $e = 173$, find r to the *nearest tenth*. [Use $\pi = 3.14$] [10]

24. Solve graphically the following pair of equations: [Estimate your answer to the *nearest tenth.*] [6, 2, 2]
$$y = 2x$$
$$6x + 7y = 42$$

25. *a.* The amount of electrical current required to melt a fuse wire varies as the three-halves power of the diameter. If the current required to melt a wire of diameter 0.09 in. is 27 amperes, what current will melt a wire of diameter 0.04 in.? [6]

b. Given $G = \dfrac{\pi P H A^4}{10}$; express G in terms of M and A if
$$M = \dfrac{\pi P H A^2}{3} \quad [4]$$

26. The workers in a certain trade have been receiving $77 a week. They are asking for the same weekly wage for 4 fewer hours per week, which will increase their hourly wage by $17\frac{1}{2}$ cents. What is their present hourly wage? [10]

27. Prove that if $\dfrac{p}{q}$, a rational fraction in its lowest terms, is a root of $ax^3 + bx + c = 0$, in which a, b and c are integers, then p is a factor of c. [A statement that this is a special case of the more general theorem will not be accepted.] [10]

*28. *a.* Express in polar form the complex number $2 - 2i$. [3]
b. Express $2(\cos 120° + i \sin 120°)$ in the form $a + bi$. [3]
c. Express in polar form *one* of the imaginary roots of the equation
$$x^4 - 1 = 0 \quad [4]$$

*29. Find the equation of the tangent to the curve $y = x^3 - 2x^2 + 2x - 8$ at the point whose abscissa is 2. [10]

*This question is based on one of the optional topics in the syllabus.