## The University of the State of New York

300TH HIGH SCHOOL EXAMINATION

## ADVANCED ALGEBRA

Wednesday, June 18, 1947 - 9.15 a. m. to 12.15 p. m., only

#### Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied.

(b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is four or five recitations a week for half a school year after the completion of intermediate algebra.

#### Part II

Answer five questions from part II.

- 21 Find, correct to the nearest tenth, the real root of the equation  $x^3 x 3 = 0$  [10]
- 22 Solve the equation  $4x^4 8x^3 5x^2 + 18x 9 = 0$  [10]
- 23 Prove that if  $\frac{p}{q}$ , a rational fraction in its lowest terms, is a root of  $ax^3 + bx + c = 0$ , in which a, b and c are integers, then p is a factor of c. [A statement that this is a special case of the more general theorem will not be accepted.] [10]
  - 24 a Using the values x = .5, 1, 3, 4, 5 and 10, draw the graph of  $y = \log_{10} x$  [5]
    - b On the same set of axes used in answer to a, draw the graph of y = 1.5 .3x [3]
    - c Using the graphs drawn in answer to a and b, find, correct to the nearest tenth, the value of x which satisfies the equation  $\log_{10} x = 1.5 .3x$  [2]
- 25 If  $y = Ae^{kt}$ , find, correct to the nearest hundredth, the value of y if A = .46, e = 2.718. k = 60 and t = .04 [10]
- 26 Telegraph poles are to be placed at equal intervals along a railroad. If there were to be 4 fewer poles per mile, it would be necessary to increase the space between every 2 consecutive poles by 12 feet. Find the number of poles to the mile. [For each mile, include the pole at the beginning but not the one at the end. 1 mile = 5280 ft] [10]
- 27 In a chemical laboratory one carboy contains 12 gallons of acid and 18 gallons of water; another carboy contains 9 gallons of acid and 3 gallons of water. How many gallons must be drawn from each carboy and combined to form a solution that is 7 gallons acid and 7 gallons water? [10]

[1]

OR a Express VI - 6 in polar form. [3]

Express  $4(\cos 270^{\circ} + i \sin 270^{\circ})$  in the form a + bi. Find the modulus of -3 + 4i. [1] [3]

d Find the amplitude (angle) of -3 + 4i. [Give answer correct to the nearest degree.] [3]

\*20 Given  $y = x^3 - 3x^3 - 9x + 2$ 

a Find the coordinates of the maximum point and of the minimum point, [6]

& Find the coordinates of the point of inflection. [2]

c Sketch the curve. [2]

\* This question is based on one of the optional topics in the syllabus.

# ADVANCED ALGEBRA

### Fill in the following lines:

Name of school	
Part I  Answer all questions in this part. Each correct answer will receive 2½ credits. be allowed. Each answer must be reduced to its simplest form.	No partial credit will
1 Express in the form of $a + bi$ the product of $(1 - i\sqrt{3})$ and	
$(2+i\sqrt{3})$	1
2 Expand $(e^x-e^{-x})^2$	2
3 Write in its simplest form the fourth term of the expansion of	3
$(2x + \frac{1}{x})^6$	3
4 Which of the following statements is false?	
$(1)  \sqrt[b]{x^a} = x^{a/b} \qquad (4)  \sqrt[b]{x} \div \sqrt[b]{y} = \sqrt[b]{x}$	
$(2) \sqrt[b]{x^a} = (\sqrt[b]{x})^a $ $(5) \sqrt[b]{x} \cdot \sqrt[b]{y} = \sqrt[b]{xy}$	4
$(3) \sqrt[5]{x} + \sqrt[5]{y} = \sqrt[5]{x+y} $	
5 Write the equation of the straight line passing through the point	5
(2, 3) and parallel to the line $2y = 3x + 4$ 6 The graphs of $y = x^2 - 4x$ and $y = x^2 - 4x - 12$ , plotted on the	
same set of axes, would have the same axis of symmetry. Is this state-	6
ment true or is it false?  7 If the roots of the equation $2x^2 - 3x + 2k = 0$ are equal, what is	
the value of k?	7
[Questions 8–11 refer to the equation $2x^3 - 3x^2 - 4 = 0$ ]	
8 What is the sum of the roots?	8
9 What is the product of the roots?	9
10 How many imaginary roots has the equation?	10
11 Transform the given equation into an equation whose roots are those of the given equation each diminished by 1.	11
12 Find the remainder when $x^8 - x^3 + 2$ is divided by $x + 1$ .	12
13 Find the sum of the infinite series 2, $-\frac{4}{3}$ , $\frac{8}{9}$ ,	13
14 Using $k$ as the constant of variation, write as an equation the following statement: $y$ varies directly as $x$ and inversely as the square of $r$ .	14
15 If $10^x = 25$ , find x correct to the nearest tenth.	15
16 Solve for x the equation $3^{x+1} = 27$	16
17 The graph of $y = x^n + c$ will intersect the x axis if n is a positive odd integer. Is this statement <i>true</i> or is it false?	17
18 How many different numbers of 3 digits each can be written with the	
digits 1, 2, 3, 4, 5 if each number is to begin and end with an even digit and if repetitions are allowed?	18
19 How many different committees of 3 can be chosen from 15 men?	19
20 If an event may happen two ways and fail five ways each countly	
probable, what is the probability that the event will happen?	20