The University of the State of New York

297th High School Examination

ADVANCED ALGEBRA

Wednesday, June 19, 1946 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Solve completely: $3x^4 + 8x^3 + 6x^2 + 3x - 2 = 0$ [10]

22 Find, correct to the *nearest tenth*, the root of $x^3 - 9x^2 + 23x - 16 = 0$ which lies between 1 and 2. [10]

23 *a* Find, correct to the *nearest thousandth*, the value of $\sqrt[5]{\dfrac{(.032)^3 \times 823.8}{3680}}$ [6]

b Solve for x correct to the *nearest tenth*: $2.3^x = 43$ [4]

24 By lowering the price of oranges 5 cents per dozen, a merchant finds he can sell 24 more oranges for $3 than he used to sell for that amount. At what price per dozen did he sell them at first? [10]

25 Given a geometric progression in which the first term is *a* and the common ratio is *r*; prove that the logarithms of the terms of this progression, taken in order, form an arithmetic progression. [10]

26 *a* Using the same set of axes, draw the graphs of

$x^2 + (y - 4)^2 = 16$ [5]

$y = x^2 + 2$ [3]

b From the graphs made in answer to *a*, read to the *nearest tenth* the coordinates of the points of intersection. [2]

27 Given $x^2 - mx + m + 3 = 0$

a Express the discriminant D as a function of m. [3]

b Using the vertical axis to represent D and the horizontal axis to represent m, sketch the graph of the function found in answer to *a*. [2]

c Using the graph made in answer to *b*, determine the values of m for which the roots of $x^2 - mx + m + 3 = 0$ are

(1) real and equal [1]

(2) real and unequal [2]

(3) imaginary [2]

*28 Find the equation of the straight line tangent to the parabola $y = -x^2 + 3$ at the point whose abscissa is -1. [10]

*29 *a* Express $2i$ in polar form. [2]

b Express $\sqrt{2}(\cos 315° + i \sin 315°)$ in the form $a + bi$. [2]

c Express the product of $2(\cos 60° + i \sin 60°)$ and $(\cos 90° + i \sin 90°)$ in the form $a + bi$. [6]

*This question is based on one of the optional topics in the syllabus.

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Fill in the following lines:

Name of school...Name of pupil...

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 What is the x intercept of the graph of $y = 2x + 4$? 1...............

2 Write the equation of the line parallel to $3x + 2y = 1$ and passing through the point $(4, -1)$. 2...............

3 What is the sum of the roots of the following equation? $5x^3 - 10x^2 + 8 = 0$ 3...............

4 Transform the equation $2x^3 - 6x^2 - 3x + 8 = 0$ into an equation whose roots are less by 1 than those of the given equation. 4...............

5 Transform the equation $x^3 + 2x - 6 = 0$ into an equation whose roots are the roots of the given equation each multiplied by 2. 5...............

6 How many imaginary roots has the equation $x^5 + 2x^2 + 1 = 0$? 6...............

7 Express $\dfrac{2 + 3i}{2 - 3i}$ as a fraction with a real denominator. 7...............

8 The electrical resistance of a wire varies inversely as the square of its diameter. Find the constant of variation if the resistance is 24 when the diameter is 2. 8...............

9 Write the *seventh* term of $(x^2 + \dfrac{1}{x})^9$ 9...............

10 Find the value of $4x^0 + (\sqrt[3]{x})^2 - (\frac{1}{4})^{-\frac{1}{2}}$ when $x = 8$. 10...............

11 Find the remainder when $x^{36} - 2x^{18} + 5$ is divided by $x - 1$. 11...............

12 Write $\log_b N = a$ in exponential form. 12...............

13 Find the value of x which satisfies the equation $4^{x+2} = 8^x$ 13...............

14 Find the value of $\sqrt[3]{25.1}$ correct to the *nearest hundredth*. 14...............

15 If x, y and z are three consecutive terms of a geometric progression, write x as a function of y and z. 15...............

16 How many whole numbers greater than 300 and less than 1000 can be made with the digits 1, 2, 3, 4 and 5 if no digit is repeated in any number? 16...............

17 How many different committees of 3 each can be chosen from a class of 11 boys if a certain boy must be included in each committee? 17...............

18 What is the probability of getting one head and one tail when two coins are tossed? 18...............

19 In how many points do the graphs of $x^2 - y^2 = 16$ and $x^2 + y^2 = 25$ intersect? 19...............

20 Solve for x the equation $x + \sqrt{x^2 - 3} = 2$ 20...............