

The University of the State of New York

288TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, June 16, 1943 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to part II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

Part II

Answer five questions from part II.

21 Find all the roots of the equation $2x^3 - x^2 - 22x - 24 = 0$ [10]

22 Find, correct to the *nearest tenth*, the real root of the equation $5x^3 + 6x^2 + 3x - 83 = 0$ [10]

23 A starts from a certain town and travels at the rate of 5 m.p.h.; $4\frac{1}{2}$ hours later B sets out from the same place to overtake A and travels 3 miles the first hour, $3\frac{1}{2}$ miles the second hour, 4 miles the third hour, and so on. In how many hours will B overtake A? [10]

24 A closed rectangular box whose width, x feet, is one half its length and whose volume is V cubic feet is to be made of two kinds of material. The top and bottom are to cost a cents per square foot and the sides and ends are to cost b cents per square foot. Express in cents the total cost C as a function of a , b , V and x . [10]

25 Solve the equation $13^{2x+2} = 5^x$. Express the root correct to the *nearest tenth*. [10]

26 From three officers and six privates:

a In how many ways can five men be chosen? [2]

b In how many ways can five men be chosen so as to include at least one officer? [5]

c If five men are chosen at random, what is the probability that the group will include at least one officer? [3]

27 Use the same set of axes for both a and b .

a Draw the graph of $y = 2^x$ [5]

b Write the equation $y = \log_2 x$ in exponential form and draw the graph. [5]

*28 Given $f(x) = \frac{x^3}{3} - \frac{3x^2}{2} - 4x + 2$

- a Find the first derivative of $f(x)$. [2]
- b Find the coordinates of the maximum point and of the minimum point of $f(x)$. [4]
- c Find the coordinates of the point of inflection. [2]
- d Sketch the graph of $f(x)$. [2]

*29 Given the equation $x^3 - 1 = 0$

- a Show that either of the two imaginary roots of this equation is equal to the square of the other. [4]
- b Express the three roots of this equation in polar form. [6]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 Find the equation of the line through the point $(-2, 1)$ and parallel to the line whose equation is $2x + 3y = 12$ 1.....
- 2 Find the y intercept of the line whose equation is $0.5x + 3.5y - 1.05 = 0$ 2.....
- 3 If $\log w = a$, $\log z = b$ and $\log y = c$, express $\log \frac{w\sqrt{z}}{y^a}$ in terms of a , b and c . 3.....
- 4 Find, correct to the nearest hundredth, the value of $\sqrt[5]{21.8}$ 4.....
- 5 What is the remainder when $5x^{20} + 9.3$ is divided by $x - 1$? 5.....
- 6 If $x^4 + 2x^3 + kx^2 - 3x + 7$ is divided by $x - 1$, the remainder is 0. What is the value of k ? 6.....
- 7 Express $\frac{2i}{3 + 4i}$ as a fraction with a rational denominator. 7.....
- 8 Write the equation of lowest possible degree with real coefficients which has for two of its roots 3 and $1 - i$. 8.....
- 9 How many imaginary roots has the equation $x^6 + 4x^2 - 10 = 0$? 9.....
- 10 Transform the equation $x^3 - 27x^2 - 36 = 0$ into an equation whose roots are less by 3 than the roots of the given equation. 10.....
- 11 Transform the equation $x^3 - 3x + 3 = 0$ into an equation whose roots are one half the roots of the given equation. 11.....
- 12 Does the equation $2x^3 + 3x^2 - 4x + 1 = 0$ have a rational fractional root? [Answer yes or no.] 12.....
- 13 Find the positive real root of the equation $x^3 + 3x^2 - 4x - 12 = 0$ 13.....
- 14 The roots of the equation $x^3 + px^2 + qx + r = 0$ are 5, 3 and -1 . What is the value of p ? 14.....
- 15 The sum of the first five terms of a geometric progression is 93. If its ratio is 2, what is the first term? 15.....
- 16 Find the fifth term of the expansion $(x - \frac{y}{2})^8$ 16.....
- 17 If ${}_nP_2 = 56$, find n . 17.....
- 18 If the probability of winning a game is $\frac{a}{b}$, what is the probability of losing the game? 18.....
- 19 Given $S = \frac{1}{2}gt^2$ and $V = gt$ where g is a constant. Express S as a function of V . 19.....
- 20 Y varies inversely as the square of x . If $y = 3$ when $x = 5$, find y when $x = 10$ 20.....