The University of the State of New York

284TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, June 17, 1942 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Part II

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

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See instructions for part II on page 1.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Solve the equation $4x^4 + 4x^3 + 13x^2 + 12x + 3 = 0$ [10]

22 Find, correct to the *nearest tenth*, the positive root of $x^3 + x^2 - 5 = 0$ [10]

23 Find the *eighth* term of an arithmetic progression whose first term is 2 and whose second, fourth and eighth terms form a geometric progression. [10]

24 *a* On the same set of axes, plot the graphs of $y^2 = 4(x - 1)$ and $y = x$ [5, 2]

b Prove algebraically that the graph of $y = x$ is tangent to the graph of $y^2 = 4(x - 1)$ at the point (2, 2). [3]

25 *a* Prove: ${}_nC_r = {}_nC_{n-r}$ [6]

b With the aid of *a* find the number of combinations of 100 things taken 98 at a time. [2]

c Write the coefficient of the *98th* term in the expansion $(a + b)^{99}$ [2]

26 *a* Derive the formula $\log_e N = \dfrac{\log_{10} N}{\log_{10} e}$ [6]

b With the aid of *a*, compute, correct to the *nearest tenth*, $\log_e 4$. [$e = 2.718$] [4]

27 A bombing plane with a speed in still air of 200 miles an hour when "loaded" and 220 miles an hour when "unloaded" reaches its objective 420 miles south of its base, "unloads" and returns to its base in 4 hours. Compute the velocity of the wind, which is blowing directly from the north. [10]

*28 Given the function $y = \dfrac{x^3}{3} - x^2 - 3x + 5$

a Find the first derivative of y with respect to x. [2]

b Find the second derivative of y with respect to x. [2]

c Find the coordinates of the maximum point. [1]

d Find the coordinates of the minimum point. [1]

e Find the coordinates of the point of inflection. [2]

f Sketch the curve. [2]

*29 If the graph of $y = m_1 x$ is perpendicular to the graph of $y = m_2 x$, prove that $m_1 = -\dfrac{1}{m_2}$ [10]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Write the *second* term of the expansion $\left(\frac{x}{2} - \frac{2}{x}\right)^4$ 1.....

2 Write in the $a + bi$ form the result of dividing $(10 + 5i)$ by $(2 - i)$. 2.....

3 Do the equations $x + y = 5$ and $2x + 2y = 9$ have any common solutions? [Answer *yes* or *no*.] 3.....

4 If both numerator and denominator of $\frac{h}{h+c}$ are divided by h, the resulting fraction is (*a*) $\frac{1}{1+c}$, (*b*) $\frac{h}{h+c}$ or (*c*) $\frac{1}{c}$. Which is correct, (*a*), (*b*) or (*c*)? 4.....

5 If $f(x) = 3x^{\frac{1}{2}} - 5x^0 + 2x^{-1}$, write the real value of $f(1)$. 5.....

6 Write as an equation the following statement: The number of heat units H required to raise the temperature of a mass of m pounds t degrees varies jointly as m and t. 6.....

7 Given $\log x = a$, $\log y = b$ and $\log z = c$; express $\log \frac{x\sqrt[3]{y}}{z^3}$ in terms of a, b and c. 7.....

8 Given $ar^n = 10$. Express n in terms of $\log a$ and $\log r$. 8.....

9 The second and fifth terms of a geometric series are —2 and 16. Write the *third* term. 9.....

10 A bag contains ten slips of paper numbered from 1 to 10. A slip of paper is drawn and not replaced. Another slip of paper is drawn. What is the probability that the two slips of paper drawn are numbered 1 and 2 respectively? 10.....

11 How many different guards of four soldiers each can be formed from a squad of eight soldiers? 11.....

12 How many different signals consisting of 6 flags can be given, if the flags are arranged vertically and 3 of the flags are white, 2 blue and 1 red? 12.....

13 Write the equation of the straight line parallel to the line represented by $y = 3x + 2$ and passing through the point $(0, 4)$. 13.....

14 In how many points does the graph of $y^2 = x$ intersect the graph of $x^2 = y$? 14.....

[OVER]

15 What must be the positive value of K if, when $x^3 - 4x - 2$ is divided by $x - K$, the remainder is 3? 15.....

16 According to Descartes's rule of signs, must the equation $x^3 - 3x^2 + 5x - 7 = 0$ have three positive roots? [Answer *yes* or *no*.] 16.....

17 If 1 and $1 + i$ are roots of the equation $x^3 + px^2 + qx + r = 0$, in which p, q and r are real, write the value of p. 17.....

18 Is 2 a root of $\sqrt{x + 2} + 4 = x$? [Answer *yes* or *no*.] 18.....

19 If a, b and c are the roots of the equation $x^3 + 6x^2 + 12x + 7 = 0$, write the equation whose roots are $a + 2$, $b + 2$ and $c + 2$. 19.....

20 If a, b and c are the roots of the equation $x^3 - x - 2 = 0$, write the equation whose roots are $2a$, $2b$ and $2c$. 20.....