The University of the State of New York

281st High School Examination

ADVANCED ALGEBRA

Wednesday, June 18, 1941 — 9.15 a. m. to 12. 15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Part II

Write at top of first page of answer paper to part II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Write the numerical value of $\frac{(5a)^0}{a^{-\frac{2}{3}}}$ when $a = 8$ 1.....

2 Write $(3 - 2i)^2$ in the form $a + bi$. 2.....

3 Write the *third* term of the expansion $(e^x + e^{-x})^4$ 3.....

4 Insert *three* arithmetic means between -4 and 10. 4.....

5 What must be the slope of the line $y = mx + 9$ if it passes through the point $(2, 3)$? 5.....

6 In how many points does the graph of $y = x^2$ intersect the graph of $y = 1$? 6.....

7 For what value of m is $4x^3 + 2mx^2 + 3x + 1$ exactly divisible by $x - 1$? 7.....

8 Write the equation of the lowest possible degree with real coefficients two of whose roots are 1 and $1 - i$. 8.....

9 Write as an equation the following statement: F varies directly as the product of m and m'. 9.....

Questions 10–13 refer to the equation $f(x) = 6x^3 + px + 35 = 0$ where p is a positive integer.

10 Does $f(x) = 0$ have a positive root? 10.....

11 Is $-\frac{3}{7}$ a possible root of $f(x) = 0$? 11.....

12 For any value of b, could the three roots of $f(x) = 0$ be $\frac{5}{2} + bi$, $\frac{5}{2} - bi$, and $-\frac{7}{3}$? 12.....

13 Does the value of p affect the value of the product of the roots of $f(x) = 0$? 13.....

14 Write the equation whose roots are less by three than the roots of $x^3 - 9x^2 + 30x - 33 = 0$ 14.....

15 Write the equation whose roots are one third the roots of the equation $x^3 + 27x + 81 = 0$ 15.....

16 How many telegraphic characters could be made by using three dots and two dashes in each character? 16.....

17 How many straight lines are determined by six points no three of which lie in the same straight line? 17.....

18 If a letter is taken at random from the word *probability*, what is the probability that the letter will be a b? 18.....

19 Find, correct to the *nearest tenth*, the value of $\sqrt[5]{38.47}$ 19.....

20 Given $f(x) = ax^2 + bx + c$; what must be the value of b if $f(x) = f(-x)$? 20.....

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See instructions for part II on page 1.

Part II

Answer five questions from this part. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Solve the equation $2x^4 + 5x^3 + 3x^2 + x - 2 = 0$ [10]

22 Find, correct to the *nearest tenth*, the real root of the equation $x^3 + 2x - 6 = 0$ [10]

23 Given the set of equations: $x^2 + y^2 = 24$
$x + y = 8$

a Solve for x and y. [8]

b With the aid of the values of x and y obtained in answer to *a*, explain why the set of equations can not be solved graphically. [2]

24 The number of revolutions per minute of a certain water turbine is given by the formula $n = \frac{400\ h^{1.3}}{61.3\ p^{0.4}}$ where n is the number of revolutions per minute, h the height of the fall in feet and p the horsepower developed.

Compute n to the *nearest integer* when $h = 15$ feet and $p = 87$ horsepower. [10]

25 *a* Given $\log_e y - \log_e x = 1$. Express y as a function of x. [6]

b Solve for x correct to the *nearest tenth*: $2^x = 7$ [4]

26 A boy was sent out with just enough money to buy 130 one-cent and two-cent stamps. Inadvertently, when purchasing the stamps, he interchanged the numbers of the two kinds and received 10 cents in change. How many of each kind should he have bought? [10]

27 *a* Show that for the two expressions $a + \sqrt{(a+g)(a-g)}$ and $a - \sqrt{(a+g)(a-g)}$, a is their arithmetic mean and g is their positive geometric mean. [3, 4]

b Show that the difference of the squares of two consecutive integers is always an odd integer. [3]

*28 *a* Obtain the three cube roots of unity by solving the equation $x^3 = 1$ and express these roots in polar form. [6]

b Express $(10, 30°)$ in the form $a + bi$. [4]

*29 The following specifications are given for the construction of a rectangular box: capacity 150 cubic feet; the base a square; the top and front to be made of oak, the remainder of pine. If the price of oak is 20 cents per square foot and that of pine 10 cents per square foot, the cost C in dollars is given by the equation $C = 75x^{-1} + .3x^2$ where x is the edge of the base in feet. What should be the dimensions of the box of minimum cost? [10]

* This question is based on one of the optional topics in the syllabus.