

The University of the State of New York

278TH HIGH SCHOOL EXAMINATION

ADVANCED ALGEBRA

Wednesday, June 19, 1940—9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (a) name of school where you have studied, (b) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

- 1 What is the name of the curve whose equation is $x^2 - y^2 = 25$? 1.....
- 2 Write the equation of the line which passes through the point (3, 0) and has a slope of 2. 2.....
- 3 Solve for x the equation $9^x = 27$ 3.....
- 4 What is the rational root of the equation $x^3 + x^2 + x + 2x + 1 = 0$? 4.....
- 5 How many numbers of three figures each can be formed from the digits 1 to 9 inclusive if no digit is used twice in any number? 5.....
- 6 Given the equation $x^3 + bx^2 + cx + d = 0$ in which the coefficients b , c and d represent real numbers; find the value of d if 1 and $2 + i$ are two roots of the equation. 6.....
- 7 Write the equation whose roots are one third the roots of the equation $x^3 - 18x + 54 = 0$ 7.....
- 8 Write the equation whose roots are less by three than the roots of the equation $x^3 - 9x^2 + 27x - 26 = 0$ 8.....
- 9 Write as an equation the following statement: The pressure (P) of wind on a given sail varies as the square of the wind's velocity (V). 9.....
- Directions (questions 10-14) — Indicate the correct answer to each question by writing *Yes* or *No* on the dotted line at the right.
- 10 Is the quotient of two imaginary numbers always an imaginary number? 10.....
- 11 Is a root of a negative number always an imaginary number? 11.....
- 12 In the function $y = ax^3 + bx^2 + cx + d$, the coefficients a , b , c and d represent real numbers. Does the graph of the function always intersect the x axis? 12.....
- 13 Does the equation $x^6 - 4x^3 - 7 = 0$ have a negative root? 13.....
- 14 Is the graph of $y = 10^x$ symmetric with respect to the y axis? 14.....
- 15 Find, correct to the nearest hundredth, the fifth root of 77.4. 15.....
- 16 Write in simplest form the third term in the expansion of $(1 + \frac{1}{x})^x$. 16.....
- 17 Find the real value of $(x^{\frac{1}{2}} + \frac{x^0}{2})^x$ when $x = 8$ 17.....

18 Given $S = \frac{ab}{a+b}$; express b as a function of S and a .

18.....

19 How many different meals consisting of soup, meat, salad, dessert and a drink could a person choose from a menu offering two soups, five meat courses, two salads, four desserts and three drinks?

19.....

20 One letter is to be taken at random from each of the words *factor* and *father*. What is the probability that the same letter will be taken from each?

20.....

See instructions for group II on page 1.

Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Solve the equation $2x^4 - x^3 - 14x^2 - 5x + 6 = 0$ [10]

22 Find, correct to the nearest tenth, the positive root of the equation $x^3 + x^2 - 4x - 3 = 0$ [10]

23 In how many years will a sum of money double itself if the interest rate is 5%, compounded semiannually? [Use $A = P(1 + \frac{r}{2})^{2n}$] [10]

24 a On the same set of axes, plot the graphs of $(x-2)^2 + y^2 = 4$ and $y = \frac{(x-2)^2}{2}$ [4, 4]

b From the graphs made in answer to a, estimate, correct to the nearest tenth, two values of x and of y , common to both equations. [2]

25 A cyclist and an autoist start at the same time from A for B , a town 60 miles away. They travel over the same route at 8 and 36 miles an hour respectively. When the autoist reaches B , he stops 20 minutes for lunch and then starts back. How many hours has the cyclist traveled when he meets the autoist on the return trip? [10]

26 a State and prove the Remainder Theorem. [1, 4]

b State and prove the Factor Theorem. [1, 2]

c Show in two different ways that when $x^3 - 3x^2 + 5x - 3$ is divided by $x - 2$ the remainder is 3. [2]

27 If x , y and z are in geometric progression, prove that $\frac{1}{y-x}$, $\frac{1}{2y}$ and $\frac{1}{y-z}$ are in arithmetic progression. [10]

*28 Given: $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x + 5$

a Find the coordinates of the maximum and the minimum points. [6]

b Find the coordinates of the point of inflection. [2]

c Sketch the curve. [2]

*29 a Express $4\sqrt{2} - 4i\sqrt{2}$ in polar form. [4]

b Using the relation $[\rho(\cos \theta + i \sin \theta)]^2 = \rho^2(\cos 2\theta + i \sin 2\theta)$, show that

(1) $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

(2) $\sin 2\theta = 2 \sin \theta \cos \theta$ [6]

* This question is based on one of the optional topics in the syllabus.