ADVANCED ALGEBRA

Wednesday, June 22, 1938 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours. Merely write the answer to each question in the space at the right; no work need be shown.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Group II

Write at top of first page of answer paper to group II (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in advanced algebra.

The minimum time requirement is five recitations a week for half a school year after the completion of intermediate algebra.

The use of the slide rule will be allowed for checking but all computations with tables must be shown on the answer paper.

Fill in the following lines:

Name of school..Name of pupil..

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 If $f(x) = x^2 - 5x + 6$, what is the value of $f(0)$?   1................

2 How many values of x satisfy the equation $x^3 = 1$?   2................

Questions 3–5 refer to the equation $x^5 + 3x^3 + x - 4 = 0$

3 What is the sum of the roots?   3................

4 What is the product of the roots?   4................

5 What is the number of imaginary roots?   5................

6 Write the coefficient of i in the expansion of $(a + bi)^2$ 6................

7 Write the equation of the line passing through the origin and inclined at $45°$ to the x axis.   7................

8 Write the *third* term of the expansion of $(2x - \frac{y}{2})^5$ 8................

9 Given $x = \log_e y$; express y in terms of e and x.   9................

10 What negative real value of x satisfies the equation $x^{-\frac{3}{2}} = \frac{1}{9}$?   10................

11 Transform the equation $x^3 - 6x^2 + 12x + 17 = 0$ into an equation whose roots are less by two than the roots of the given equation.   11................

12 Write the equation whose roots are twice the roots of the equation $8x^3 - 6x + 1 = 0$ 12................

13 How many different groups of 50 people each can be formed from 52 persons?   13................

14 What is the probability that a digit chosen at random from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 will be odd?   14................

15 In how many points does the graph of the equation $4x^2 + 9y^2 = 36$ intersect the graph of the equation $xy = 2$?   15................

16 What is the largest positive number that k must exceed in order that the roots of the equation $x^2 + 2x + k = 0$ shall be imaginary?   16................

17 Write the equation of lowest possible degree with real and rational coefficients which has for two of its roots $\sqrt{2}$ and i.   17................

18 In a geometric progression the first term is 1 and the twentieth term is $(1.05)^{19}$; find the common ratio.   18................

19 Write as an equation the following statement: The time (t) necessary for a train to go a certain distance varies inversely as the rate (r).   19................

20 Does a negative number have a real logarithm? [Answer *yes* or *no*.]   20................

[3]

Group II

Answer five questions from this group. Full credit will not be granted unless all operations (except mental ones) necessary to find results are given; simply indicating the operations is not sufficient. Each answer should be reduced to its simplest form. Purely arithmetical solutions for problems will not be accepted.

21 Find, correct to the *nearest tenth*, the real root of the equation $x^3 - 3x - 3 = 0$ [10]

22 Solve the equation $2x^4 - 3x^3 - 9x^2 + 15x - 5 = 0$ [10]

23 Prove that if $a + bi$ is a root of the equation $x^3 + px + q = 0$, where p and q are real numbers, then $a - bi$ is also a root of the equation. [The statement that this is a special case of a more general theorem will not be accepted as a proof.] [10]

24 Given the geometric progression 3, 6, 12, ... Prove that if k is added to each term of this progression, the resulting terms do not form a geometric progression. [10]

25 The number n of revolutions per minute of a certain type of water turbine is given by the formula $n = \dfrac{400\,h^{1.3}}{61.3\,p^{0.4}}$ where h is the height of the fall in feet and p is the horsepower developed. Compute n to the *nearest integer* when $h = 13$ feet and $p = 86$ horsepower. [10]

26 A number consists of three digits whose sum is 13. The hundreds digit is equal to the tens digit. If the digits are reversed the resulting number is 198 less than the original number. Find the number. [7, 3]

27 a On the same set of axes draw the graphs of the equations
$$y^2 = 4(x - 5) \text{ and } y = 2x + 1 \quad [6, 2]$$
   b From the graphs made in answer to a, determine whether there are any real values of x and y that satisfy the two equations. [2]

*28 The following table shows the frequency distribution of the scores made by a group of students in an intelligence test:

Scores	30–39	40–49	50–59	60–69	70–79	80–89	90–99	100–109	110–119	120–129
Frequency	1	3	4	5	8	12	9	4	2	2

   a Plot the percentile curve. [8]
   b From the curve made in answer to a, determine the median. [2]

*29 The speed V of a point on the rim of a flywheel t seconds after starting is given in feet per second by the formula $V = 18t^2 - t^3$; find the maximum speed. [10]

* This question is based on one of the optional topics in the syllabus.